

CHAPTER 4

PARAMETRIC ANALYSIS OF SHEAR LAG IN BOX BEAMS

4.1 INTRODUCTION

The bending stress distribution across wide flanges of a box beam under symmetrical flexure is not uniform. This phenomenon has long been recognized as shear lag. The stress at the junctions of the web and flange is much higher than that at the center of the flange. This phenomenon is renowned as positive shear lag and opposite to this is negative shear lag.

Various analysis of shear lag phenomenon is carried out by the variational approach. The potential energy of a box beam consists of (i) potential energy of the load system (ii) strain energy of side webs and flanges and (iii) strain energy of the two cover sheets. The longitudinal displacement (normal stress distribution) of the flange usually assumed as parabolic variation, cubic parabolic variation, quartic parabolic variation and pentic parabolic variation as short out in literature survey. The two Reissner parameters n and k have different values according to the assumed longitudinal displacement of the flange. The variational principle was applied first time by Reissner (1945) in analyzing the box beam (Fig. 4.1).

In the present chapter, more detailed investigation of various parameters is carried out. The longitudinal displacement of the flange is assumed a polynomial of order 'a' (where 'a' is an integer ≥ 2) and the potential energy of box beam (Fig. 4.1) is evaluated.

The differential equation is solved by using the principle of minimum potential energy in a way as originally proposed by Reissner (1945).

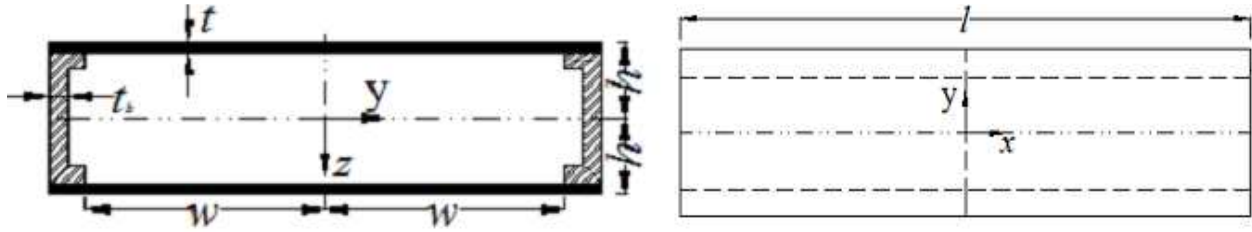


Fig. 4.1. Rectangular box beam with doubly symmetric cross section
(After Reissner 1945)

4.2 FORMULATION AND SOLUTIONS

In a box beam, shown in Fig. 4.1, the top and bottom slab has a width of $2w$ and a uniform thickness t . The web has thickness t_b and depth $2h$. A given distribution of load is applied normal to the plane of the top cover sheet along the span length l . A distribution of bending moments $M(x)$ corresponds to the load distribution. The span-wise coordinate is x , the coordinate in the plane of the cover sheets perpendicular to the x direction be y and $z(x)$ the deflection of the neutral axis of the beam.

The elastic potential energy inducing in the structure by the load system is

$$\Pi_l = \int M(x) \frac{d^2 z}{dx^2} dx \quad (1)$$

The second part is the strain energy of side webs,

$$\Pi_w = \frac{1}{2} \int EI_w \left(\frac{d^2 z}{dx^2} \right)^2 dx \quad (2)$$

The quantity I_w denotes the principal moment of inertia of the two side web

The third part is the strain energy of the two cover sheets. With the assumption that the normal strain in the chord wise direction in the sheet is negligibly small (Reissner 1945), the strain energy of the two sheets is given by the integral

$$\Pi_s = \frac{1}{2} \iint 2t[E\varepsilon_x^2 dx + G\gamma^2] dx dy \quad (3)$$

where E and G denote effective modulus of elasticity and rigidity respectively. Span-wise normal strain ε_{xx} shear strain λ_{yz} (neglecting the other term, i.e., $\partial v/\partial z$ (Reissner 1945)) are then expressed as $\partial v/\partial z$

$$\varepsilon_{xx} = \frac{\partial u}{\partial x}, \quad \lambda_{yz} = \frac{\partial u}{\partial y} \quad (4)$$

The total potential energy can be represented as

$$\Pi = \Pi_l + \Pi_w + \Pi_s \quad (5)$$

The theorem of minimum potential energy states that the total potential energy becomes minimum for the correct displacement functions u and z , if and only if, such displacement function are compared which satisfy all conditions of support and continuity imposed on the displacements [Reissner 1945].

The assumptions for the span-wise sheet displacement is

$$u(x, y) = \pm h \left[\frac{dz}{dx} + \left(1 - \frac{y^a}{w^a}\right) U(x) \right] \quad (6)$$

where $U(x)$ represents the correction due to shear lag. Instead of the vanishing chord-wise variation of the sheet displacements of elementary beam theory, the relative magnitude of the function $U(x)$ is a measure for the magnitude of the shear lag effect. The correction is such that the continuity of the displacements at the junction of web and flange along the flanges, that is along $y = \pm w$ is preserved [Reissner 1945].

From Eqs. (6) and (4) we obtain the following expressions for the strains in the sheets,

$$\varepsilon_x = \mp h \left[z'' + \left(1 - \frac{y^a}{w^a}\right) U' \right] \quad (7)$$

$$\gamma = \pm h \frac{a}{w} \frac{y^{a-1}}{w^{a-1}} U \quad (8)$$

The expression for the strain energy of the cover sheets is obtained as

$$\Pi_s = \iint th^2 \left\{ E \left[z'' + \left(1 - \frac{y^a}{w^a} \right) U \right]^2 + G \left[\frac{a}{w} \frac{y^{a-1}}{w^{a-1}} U \right]^2 \right\} dx dy \quad (9)$$

In Eq. (9) integrating with respect to y and substituting

$$I_s = 4wth^2 \quad I = I_s + I_w \quad (10)$$

we have

$$\Pi_s = \frac{1}{2} \int EI_s \left\{ z'' + \left(1 + \frac{1}{2a+1} - \frac{2}{a+1} \right) U \right\}^2 + \left(\frac{2a}{a+1} \right) z U' + \left(\frac{a^2}{2a-1} \right) \frac{E}{G} \frac{1}{w^2} U^2 \right\} dx \quad (11a)$$

Denoting the coefficients for the assumed polynomial of the sheet displacement as

$$A = \left(1 + \frac{1}{2a+1} - \frac{2}{a+1} \right), B = \left(\frac{2a}{a+1} \right), C = \left(\frac{a^2}{2a-1} \right) \quad (11b)$$

the Eq. (11a) can be written

$$\Pi_s = \frac{1}{2} \int EI_s \left\{ z'' + AU^2 + BU' + C \frac{E}{G} \frac{1}{w^2} U^2 \right\} dx \quad (12)$$

substituting Eqs. (12), (2), and (1) into Eq. (5), the expression for the potential energy of the system becomes

$$\Pi = \int \left\{ \frac{1}{2} EI (z')^2 + Mz \right\} dx + \int \frac{1}{2} EI_s \left\{ AU^2 + BU' + C \frac{E}{G} \frac{1}{w^2} U^2 \right\} dx \quad (13)$$

Differential equation and boundary conditions for z and U are obtained by making

$$\partial \Pi = 0 \quad (14)$$

Thus, with x_1 and x_2 denoting the interval of integration,

$$\partial \Pi = \int \left\{ [EI_s z'' + M + \frac{B}{2} EI_s U] \delta z + EI_s [-AU - \frac{B}{2} z + C \frac{E}{G} \frac{1}{w^2} U] \delta U \right\} dx + \left\{ EI_s [AU + \frac{B}{2} z'] \delta U \right\}_{x_1}^{x_2} = 0 \quad (15)$$

As $\delta z''$ and δU are arbitrary in the interval $[x_1, x_2]$ the terms multiplying them must vanish.

The following two differential equations

$$z'' + \frac{B}{2} \frac{I_s}{I} U + \frac{M}{EI} = 0 \quad (16)$$

$$EI_s [U'' - \frac{C}{A} \frac{G}{E} \frac{1}{w^2} U + \frac{B}{2A} z'] = 0 \quad (17)$$

The integrated portion of Eq. (15) defines the boundary and transition conditions for the function U . At a section where the sheet is fixed,

$$\delta U = 0 \text{ and } U = 0 \quad (18)$$

At a section, where the sheet is not fixed and consequently δU is arbitrary,

$$EI_s [U' + \frac{B}{2A} z'] = 0 \quad (19)$$

Transition conditions for adjacent bays with different stiffness are

$$U \text{ and } EI_s [U' + \frac{B}{2A} z'] \text{ continuous [Reissner 1945]} \quad (20)$$

The above boundary and transition conditions are in addition to those imposed on z and M in elementary beam theory.

The quantity U is eliminated from Eqs. (16) to (20), and a system of relations containing the beam deflection, z only, are obtained. The differential equation for z is derived by differentiating Eq. (19) and substituting U' from Eq. (18) as

$$z'' + \frac{M}{EI} - w^2 \frac{E}{G} \frac{A}{C} \left(z' + \frac{M}{EI} \right) - \frac{B^2}{4C} \frac{I_s}{I} z^{IV} = 0 \quad (21)$$

Equation (21) is written as

$$z'' - \frac{E}{G} w^2 \left(\frac{A}{C}\right) \left(1 - \frac{B^2 I_s}{4A I}\right) z^{IV} = -\frac{M}{EI} + w^2 \frac{E}{G} \left(\frac{A}{C}\right) \frac{M''}{EI} \quad (22)$$

With the help of Eqs. (16) and (17), the boundary condition (18), which helps when the sheet is attached to the support, is transformed into

$$\left(1 - \frac{B^2 I_s}{4A I}\right) z'' + \frac{M'}{EI} = 0 \quad (\text{Reissner 1945}) \quad (23)$$

Similarly, the boundary condition (19), which holds when the sheet is not attached to the support, becomes

$$\left(1 - \frac{B^2 I_s}{4A I}\right) z'' + \frac{M}{EI} = 0 \quad (24)$$

The continuity condition (20) may be transformed in an analogous manner. The values of the flange stress are given by

$$\sigma_f = \pm E h z'' \quad (25)$$

For the application of the results, it may be noted that the differential equation (21) can be solved for the value of z'' which, according to (25), gives directly the appropriate value of the flange stress σ_f . The magnitude of the deflection z can then be found from the value of z'' as in elementary beam theory.

For the evaluation of the solution the two Reissner's parameters are

$$n = \frac{1}{1 - \frac{B^2 I_s}{4A I}}, \quad k = \frac{1}{w} \sqrt{n \frac{C G}{A E}} \quad (26, 27)$$

Substituting the values of Reissner's parameters, the differential equation (22) transform as

$$z'' - \frac{1}{k^2} z^{IV} = -\frac{M}{EI} + \frac{n}{k^2} \frac{M''}{EI} \quad (28)$$

The boundary condition at an end section where the sheet is attached to the support becomes

$$z'' = -n \frac{M'}{EI} \quad (29)$$

and the boundary condition at an end section where the sheet is not attached to the support becomes

$$z'' = -n \frac{M}{EI} \quad (30)$$

The value of the parameter n and k have been found same as originally by E. Reissner (1945) for the degree of polynomial two. The values of corresponding polynomial coefficient A , B and C in the present study are $8/15$, $4/3$ and $4/3$ respectively. The values of two Reissner's parameter are tabulated (Table 4.1.) for assumed degree polynomial 2 and 4. The two Reissner's parameters are well agreed with the Chang and Zheng (1987), in which the polynomials of degree two, three and four are assumed for the slab displacement of the box girder bridge.

Table 4.1. Values of Reissner's parameters for degree of polynomial 2 and 3

| Degree of polynomial 'a' | Polynomial coefficients | | | Reissner's parameters | |
|--------------------------|-------------------------|-----|------|-----------------------|------------------------|
| | A | B | C | n | K |
| 3 | 9/14 | 3/2 | 9/5 | $1/(1-7I_s/8I)$ | $1/w\sqrt{(14nG/5E)}$ |
| 4 | 32/45 | 8/5 | 16/7 | $1/(1-9I_s/10I)$ | $1/w\sqrt{(45nG/14E)}$ |

4.3 APPLICATION EXAMPLES

A cantilever beam with uniform load distribution and cover sheet fixed at support, a beam with simply supported and loaded according to cosine law and a built-up beam with uniform loads distribution is analysed as originally by Reissner (1945).

4.3.1 Expression for SLF for Cantilever Beam Subjected to Uniform Loading

Assuming that the free end of the beam has co-ordinate $x = 0$ and the fixed end of the beam the co-ordinate $x = l$. The distribution of bending moment may write in the form

$$M = M_0 \left(\frac{x}{l}\right)^2 \quad (31)$$

The differential equation (28) then becomes

$$z'' - \frac{1}{k^2} z^{IV} = -\frac{M_0}{EI} \left[\left(\frac{x}{l}\right)^2 - \frac{2n}{(kl)^2} \right] \quad (32)$$

Solving for z'' we find

$$z'' = \frac{M_0}{EI} \left\{ D_1 \sinh kx + D_2 \cosh kx - \left(\frac{x}{l}\right)^2 + \frac{2(n-1)}{(kl)^2} \right\} \quad (33)$$

Satisfying the boundary condition (30) when $x = 0$ and (29) when $x = l$, we obtain

$$z'' = -\frac{M_0}{EI} \left\{ \left(\frac{x}{l}\right)^2 - \frac{2(n-1)}{(kl)^2} \left[(\cosh kx - 1) - \frac{\sinh kl - kl}{\cosh kl} \sinh kx \right] \right\} \quad (34)$$

According to Eq. (25), the flange stress at the fixed end of the beam becomes

$$\sigma_f(l) = \pm \frac{M_0 h}{I} \left\{ 1 + \frac{2(n-1)}{kl} \left[\tanh kl - \frac{1}{kl} + \frac{1}{kl \cosh kl} \right] \right\} \quad (35)$$

Setting the equation (35) in the dimensionless form, the shear lag factor may be finally expressed as

$$F(s) = \frac{\sigma_f(l)}{\pm \frac{M_0 h}{I}} - 1 = \left\{ \frac{2(n-1)}{kl} \left[\tanh kl - \frac{1}{kl} + \frac{1}{kl \cosh kl} \right] \right\} \quad (36)$$

4.3.2 Expression for SLF for Simply Supported Beam Subjected to Load Variation as Cosine Law

In simply supported beam of span length l , assuming the origin of the coordinate system at the center of the beam. The moment distribution may be written

$$M = M_0 \cos \pi \frac{x}{l} \quad (37)$$

A particular solution of Eq. (28) is

$$z = \left(\frac{l}{\pi}\right)^2 \frac{M_0}{EI} \frac{1 + n\left(\frac{\pi}{kl}\right)^2}{1 + \left(\frac{\pi}{kl}\right)^2} \cos \pi \frac{x}{l} \quad (38)$$

As Eq. (38) satisfies the boundary condition (30) and the condition of vanishing deflection at the ends of the beam, it is the complete expression for the deflection function. When $l/k = 0$, Eq. (38) reduces to the expression for z in the case where shear lag is not taken into account. The factor may be derived on the basis of above conditions as

$$\frac{1 + n\left(\frac{\pi}{kl}\right)^2}{1 + \left(\frac{\pi}{kl}\right)^2} = \frac{1 + \pi^2 \frac{E}{G} \frac{A}{C} \frac{w^2}{l^2}}{1 + \pi^2 \frac{E}{G} \frac{A}{C} \frac{w^2}{l^2} \left(1 - \frac{B^2}{4A} \frac{I_s}{I}\right)} \quad (39)$$

The shear lag factor can be estimated as

$$F(s) = \frac{1 + n\left(\frac{\pi}{kl}\right)^2}{1 + \left(\frac{\pi}{kl}\right)^2} - 1 = \frac{1 + \pi^2 \frac{E}{G} \frac{A}{C} \frac{w^2}{l^2}}{1 + \pi^2 \frac{E}{G} \frac{A}{C} \frac{w^2}{l^2} \left(1 - \frac{B^2}{4A} \frac{I_s}{I}\right)} - 1 \quad (40)$$

4.3.3 Expression for SLF for Built-up Beam Subjected to Uniform Loading

For a beam with both ends built-in subjected to uniformly distributed load, assuming origin at the center of the span, the distribution of bending moments may be written as

$$M = M_0 \left[\left(\frac{x}{l} \right)^2 - \frac{1}{12} \right] \quad (41)$$

With this value of M , the Eq. (28) is solved in the form

$$z'' = -\frac{M_0}{EI} \left\{ \left(\frac{x}{l} \right)^2 - \frac{1}{12} - \frac{2(n-1)}{(kl)^2} + D_2 \cosh kx \right\} \quad (42)$$

Determining the constant D_2 (Reissner 1945) the Eq. (42) result in

$$z'' = -\frac{M_0}{EI} \left\{ \left(\frac{x}{l} \right)^2 - \frac{1}{12} - \frac{(n-1)}{(kl)^2} + \left[\frac{\cosh kx}{\sinh \frac{kl}{2}} - \frac{1}{\frac{kl}{2}} \right] \right\} \quad (43)$$

The stresses in the flange is written as

$$\sigma_f = \pm \frac{M_0}{I} h \left\{ \left(\frac{x}{l} \right)^2 - \frac{1}{12} + \frac{(n-1)}{kl} \left[\frac{\cosh kx}{\sinh \frac{kl}{2}} - \frac{1}{\frac{kl}{2}} \right] \right\}$$

(44)

The shear lag factor at the end of the span is

$$F(s) = \frac{6(n-1)}{(kl)^2} \left[\tanh \frac{kl}{2} - \frac{2}{kl} \right] \quad (45)$$

And at the centre of the beam is

$$F(s) = -\frac{12(n-1)}{(kl)^2} \left[\frac{1}{\sinh \frac{kl}{2}} - \frac{1}{\frac{kl}{2}} \right] \quad (46)$$

4.4 PARAMETRIC ANALYSIS OF SHEAR LAG FACTOR (SLF)

Shear lag factor (SLF) can be expressed and evaluated in several ways as

(i) Lee et al. (2002) defined SLF as $\sigma_{\max}/\sigma_{\min}$, where, σ_{\max} is the maximum stress at the center of the flange, and σ_{\min} is the minimum stress at web- flange junction to define degree of negative shear lag.

(ii) Ratio of axial stress at web-flange junction with shear lag to axial stress at web flange junction without shear lag, denoted as λ_s by Kwan (1996) and Song et al. (1990).

In the present study, the shear lag factor is defined as $F(s) = \sigma_f / (\pm M_0 h / I) - 1$, which could be understood physically as the ratio of maximum flange stress to its average value minus one at any section, located at any distance from the support. The support condition adopted are as (i) simply supported beam, (ii) cantilever beam (iii) built-up beam (at the fixed end) and (iv) built-up beam (at midspan). For all these cases, the load applied are as shown in Fig. 4.2. For all such different loading and support conditions, the expressions for SLF have been derived (Eqs. 36, 40, 45 and 46). The variation of two Reissner's parameters n and k which varies along with the variation in the degree of polynomial 'a' is study.

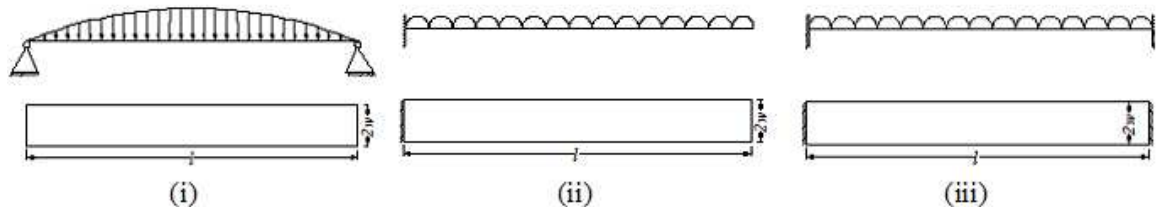


Fig. 4.2. Different loading and support condition for box beam: (i) simply supported beam (cosine law); (ii) cantilever beam (udl); (iii) built-up beam (udl)

The variation of $F(s)$ have been investigated with respect to three parameters I_s/I , l/w and G/E . The parameter I_s/I defined as relative stiffness (r_s) of the cover sheet (flange) over entire cross section. The value of these parameters originally adopted by Reissner is $r_s = 0.5$, $l/w = 5$ and $G/E = 3/8$. The parameter r_s have been varied from 0.5 to 0.92 based on some existing box girder bridges [Menn 1990]. The corresponding results are plotted in Fig. 4.4, adopting the other two parameters as $l/w = 5$ and $G/E = 3/8$. Similarly, the variation of $F(s)$ with respect to l/w has been carried out for the values of parameters $r_s = 0.5$, $G/E = 3/8$. Also, to study the effect of material properties on $F(s)$, the ratio of shear

modulus and Young's modulus is varied from 0.33 to 0.5 based on the Poisson's ratio μ varying from 0 to 0.5.

The objective of this parametric study is to develop a set of monographs which can be readily used for practical design purposes of such box sections used either as building or bridge.

4.5 RESULTS AND DISCUSSION

The degree of polynomial 'a' significantly affects the variation of Reissner's parameters n and k (Fig. 4.3 (i) and 4.3 (ii)). Here, the coefficient (kl) is expressed as the ratio of span over width of the box beam [Chang and Zheng 1987].

It can be seen from Fig. 4.3, with increasing value of the degree of the polynomial, the value of n converges at a value of 2 (approximately), whereas, the value of kl keeps on increasing almost linearly with 'a'. So, the above graphs can be used to find out the Reissner's parameters for the corresponding degree of the polynomial.

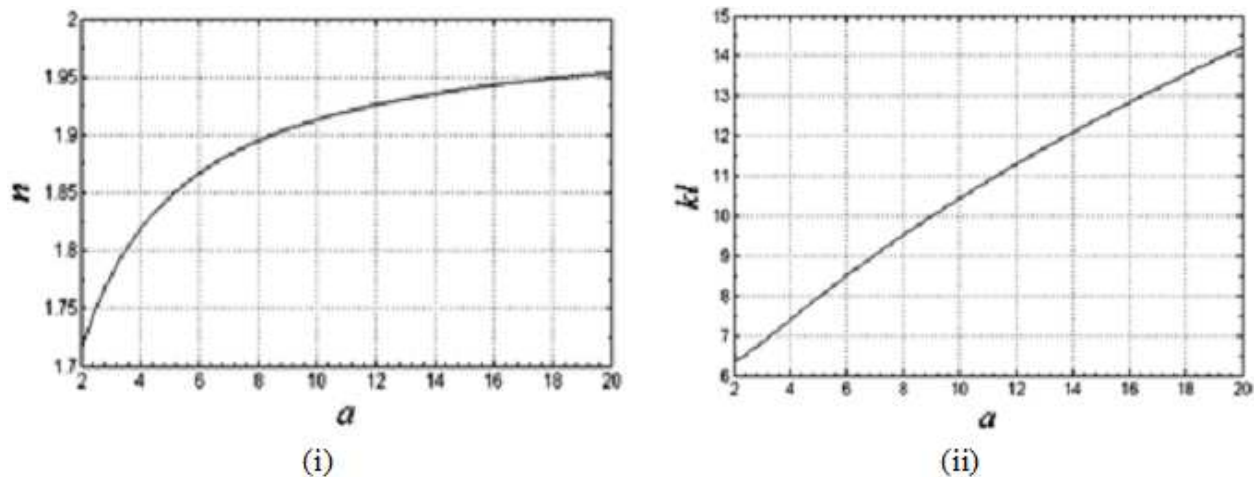


Fig. 4.3. Variations of Reissner's parameters with degree of polynomial 'a':
(i) variation of 'n'; (ii) variation of 'kl'

4.5.1 Variation of SLF with Varying r_s and Degree of Polynomial 'a'

The relative stiffness r_s , influence significantly the variation of SLF. In Fig. 4.4, such variation of shear lag factor has been depicted for different cases. In a simply supported

beam subjected to cosine loading, the shear lag factor $F(s)$ increases monotonically with increasing r_s and converges rapidly for the lower value of stiffness ratio. Also, it can be noted that the value of $F(s)$ is higher for increasing degree of polynomial. For a cantilever beam subjected to uniform loading, shear lag factor $F(s)$ remains less than 1, Fig. 4.4 (ii). The value is comparatively much lower than that of simply supported case.

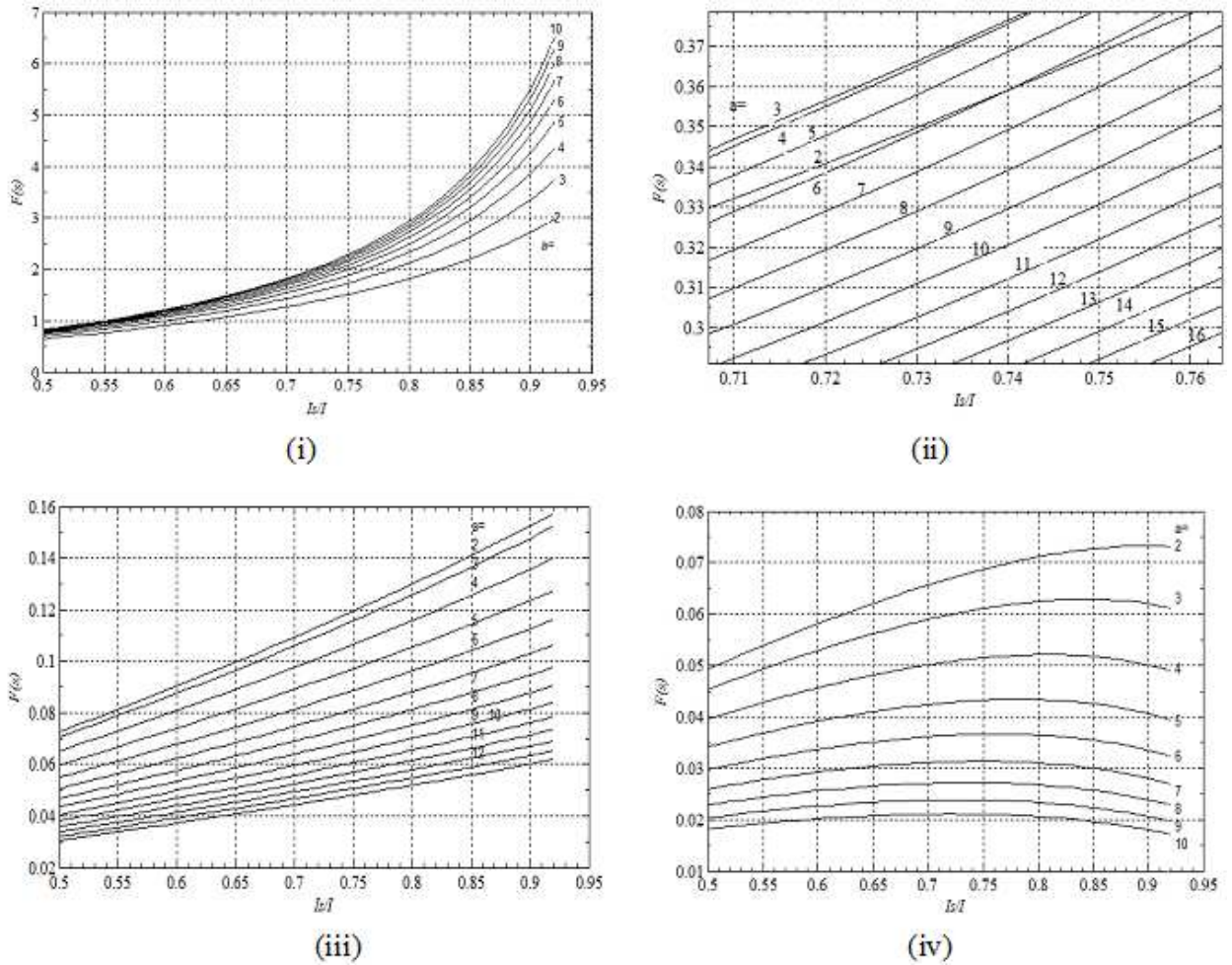


Fig. 4.4. Monograph for shear lag factor with varying I_s/I along with the degree of polynomial: (i) simply supported beam; (ii) cantilever beam; (iii) built-up beam (at fixed end); (iv) built-up beam (at mid span)

Fig. 4.4 (iii) and (iv), exhibit the case of a fixed beam subjected to uniform loading. At the fixed end, the value of SLF is 16% corresponding to the 2 degree polynomial and 8% at mid-span. The shear lag factor is negligibly small in the case of fixed end beams at the support and mid span, Fig. 4.4. The results are justified with Moffat and Dowling (1975). In a box girder bridge shear lag is more pronounced by addition of stiffener in the flange panel as relative stiffness of the flange increases.

4.5.2 Variation of SLF with Varying l/w and Degree of Polynomial ‘ a ’

As ‘ l/w ’ increases, the effect of positive and negative shear lags decreases [Chang and Zheng 1987]. The study emphasized that the parameter significantly affects the negative shear lag, and more detailed parametric study should be undertaken. Therefore, the present study reinvestigates the variation of the shear lag factor with respect to ‘ l/w ’. The same trend for cantilever beam could be noted as presented in Fig. 4.5 (ii). The present study provides a data set which is more versatile in terms of the following: (i) the degree of polynomial considered by other authors are only between 2 to 5, whereas, the present study ‘ a ’ varies from 2 to 10 or 20. (ii) For any value of ‘ a ’, $F(s)$ can be found. Therefore, this monograph will be useful significantly for designers. It can be noted from Fig. 4.5, that $F(s)$ is independent of l/w for simply supported beams with cosine loading. It increases with the degree of polynomial. For the case of a cantilever beam, initially the value of $F(s)$ increases for value ‘ a ’ varying from 2 to 4. Thereafter, the $F(s)$ decreases for higher values of ‘ a ’. In Fig. 4.5 (iii), it would be interesting to see detailed variation.

Some typical plots showing the variation of $F(s)$ for certain practical values of r_s and l/w with the degree of polynomial (a), are given in Figs. 4.6 and 4.7. It can be seen that, in simply supported box beam $F(s)$ increases monotonically for a fixed value of r_s and converses for the higher value of ‘ a ’. In the cantilever box beam, the shear lag factor firstly

increases along with 'a' up to four and afterward it decreases linearly for the higher value of 'a'. However, in the built-up beam, this variation exhibit decreasing trend.

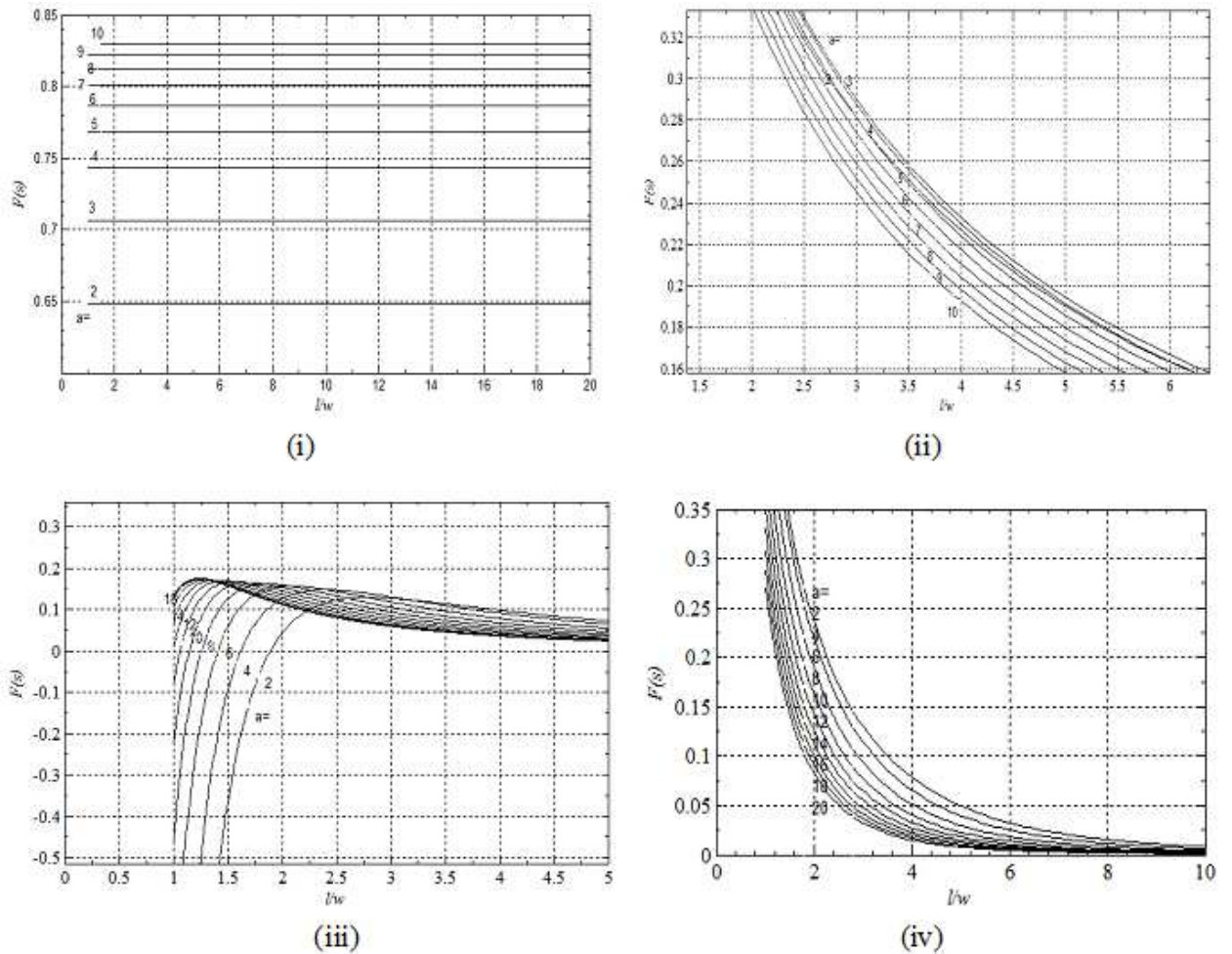


Fig. 4.5. Monograph for shear lag factor with varying l/w along with the degree of polynomial: (i) simply supported beam; (ii) cantilever beam; (iii) built-up beam (at fixed end); (iv) built-up beam (at mid span)

Another variation of the shear lag factor for a particular value of l/w represents the same trend for simply supported box beam. Whereas, in cantilever box beam value of $F(s)$ increases for the value of 'a' from 2 to 4. This variation is not significant and after that shear lag factor falls linearly for the higher value of 'a' but at the slower rate. In the built-up beam, the variation in the shear lag factor is similar as in the case of a fixed value of r_s .

At the fixed support, for the value of $l/w = 2.5$ and lower, shear lag factor has increased linearly for the value of 'a' from 2 to 4 and then reduces accordingly.

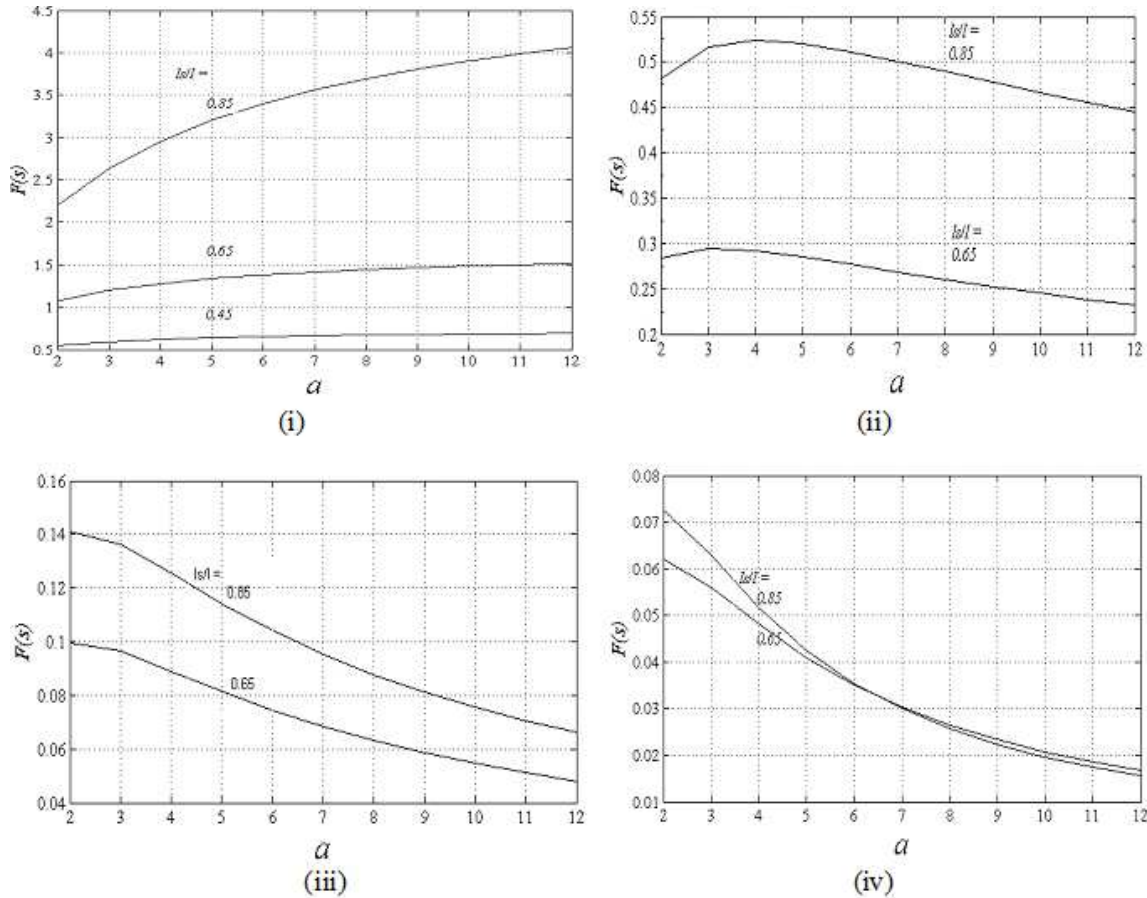


Fig. 4.6. Variation of shear lag factor with degree of polynomial for $I_s/I = 0.65, 0.85$: (i) simply supported beam; (ii) cantilever beam; (iii) built-up beam (at fixed end); (iv) built-up beam (at mid span)

4.5.3 Variation of SLF with Varying G/E and Degree of Polynomial 'a'

The effect of material property has also been reinvestigated (Figs. 4.8, 4.9). In the cantilever box beam, it is clear from the figure that, for a fixed value of G/E , the shear lag factor increases sharply for the degree of polynomial (a) 2 to 3 and decreases almost linearly for the higher value of 'a'. The variation for the other type of beams is presented in

the figure clearly. Therefore, these monographs can also be used to interpret the effect of material properties in terms of varying poisson's ratio.

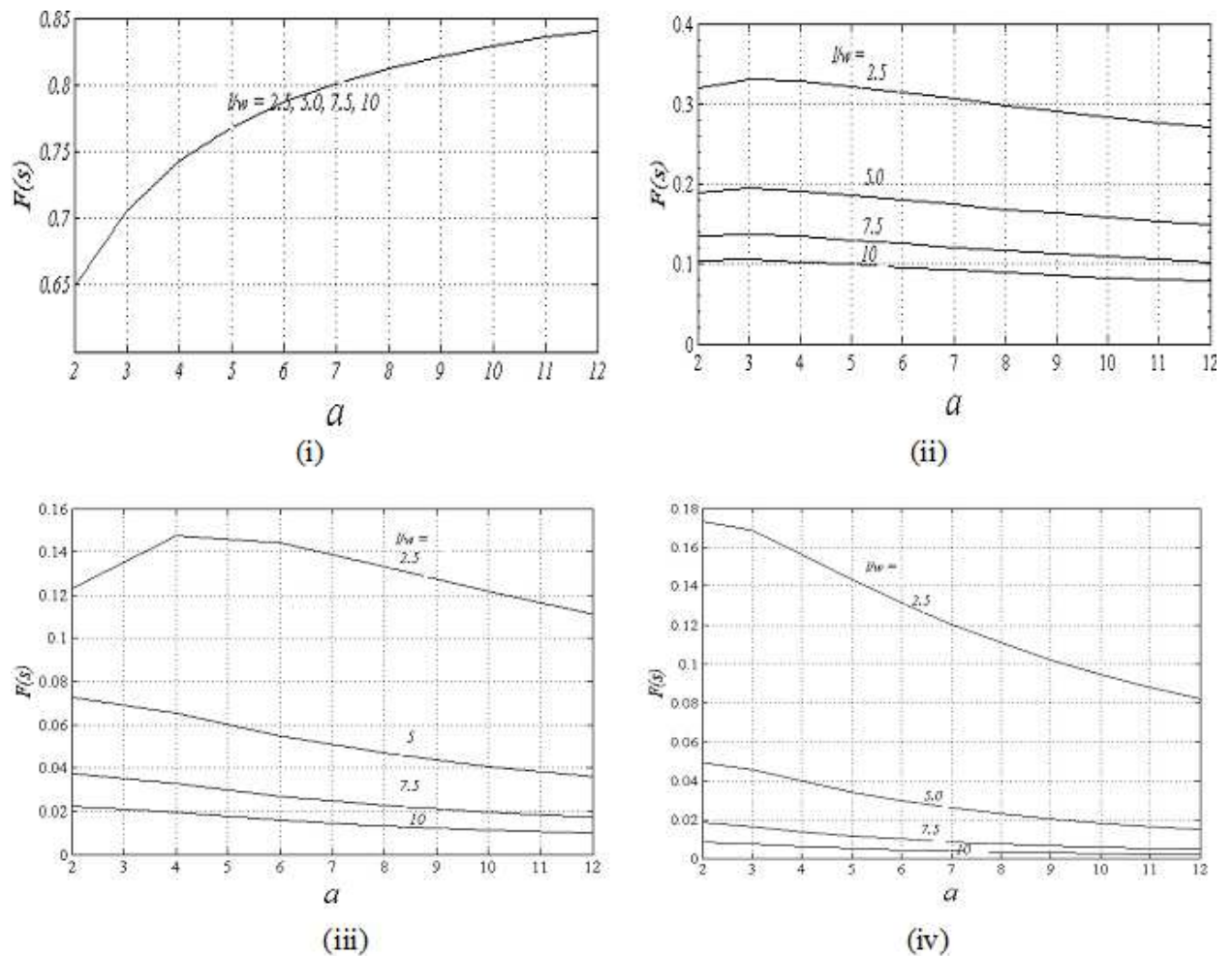
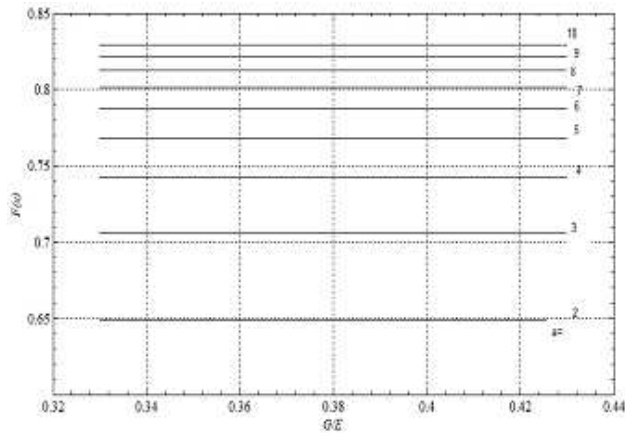
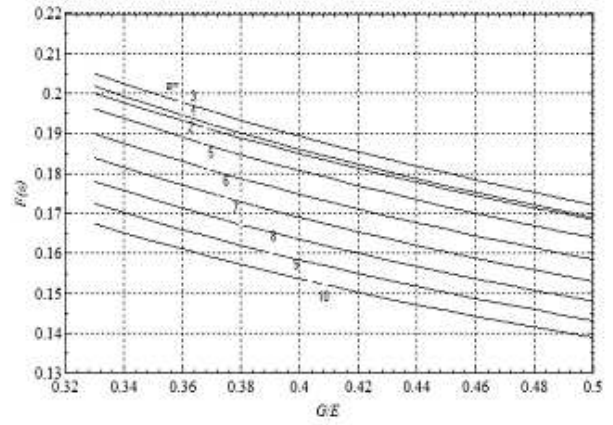


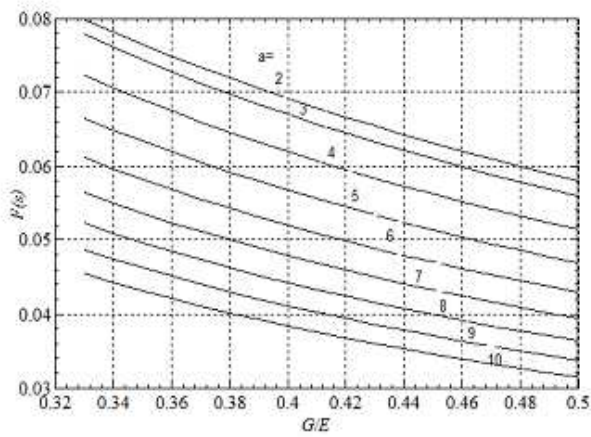
Fig. 4.7. Variation of shear lag factor with degree of polynomial for $l/w = 2.5, 5, 7.5$ and 10 : (i) simply supported beam; (ii) cantilever beam; (iii) built-up beam (at fixed end); (iv) built-up beam (at mid span)



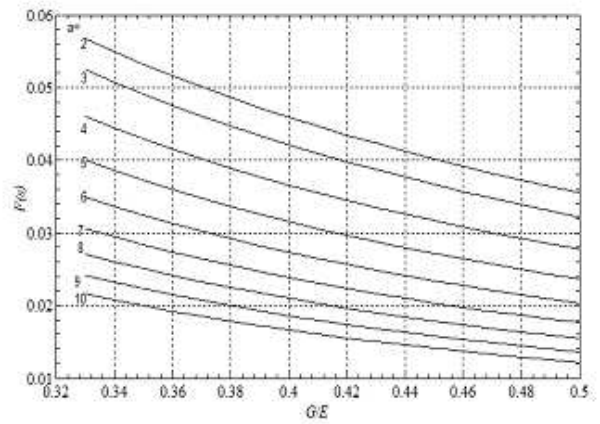
(i)



(ii)

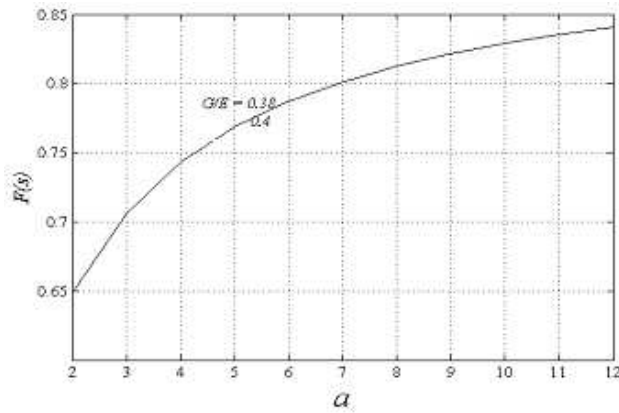


(iii)

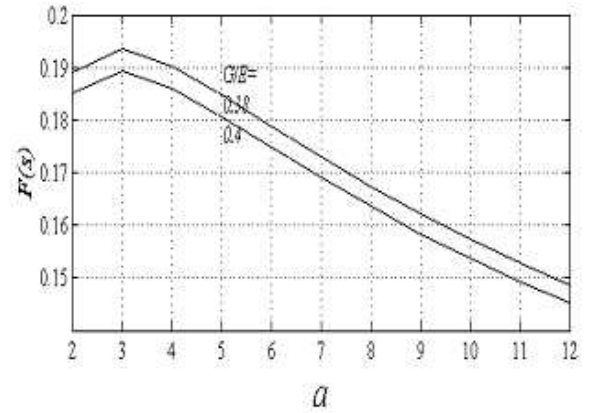


(iv)

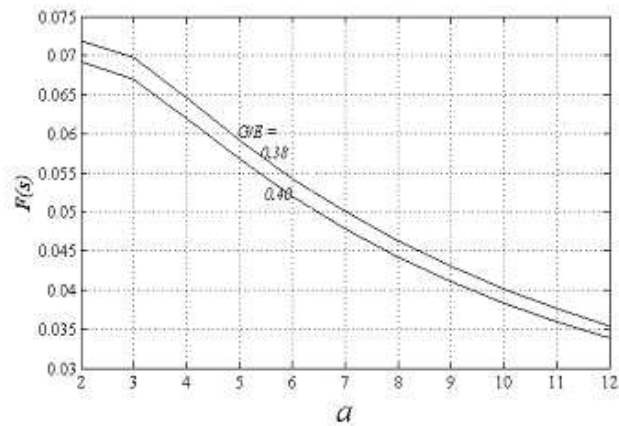
Fig. 4.8. Monograph for shear lag factor with varying G/E along with the degree of polynomial: (i) simply supported beam; (ii) cantilever beam; (iii) built-up beam (at fixed end); (iv) built-up beam (at mid span)



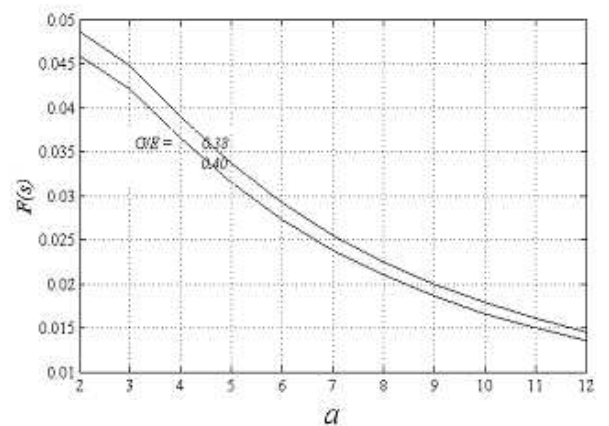
(i)



(ii)



(iii)



(iv)

Fig. 4.9. Variation of shear lag factor with degree of polynomial for $G/E = 0.38, 0.40$: (i) simply supported beam; (ii) cantilever beam; (iii) built-up beam (at fixed end); (iv) built-up beam (at mid span)

4.6 SUMMARY AND CONCLUDING REMARKS

A clear understanding of the effects of variation of parameters is essential to effectively control the shear lag phenomenon. However, the following conclusions are drawn:

The present study involves extensive parametric study over a wide range of variation of assumed cover sheet displacement. It elaborates the different type of loading and support conditions. Shear lag is more pronounced as the relative stiffness of cover sheet increases. In the simply supported case, the r_s and the parameter l/w in the case of built-up beam, which have a great impact on shear lag. The parameters r_s , l/w and G/E are the main features in the design of

tubular structures. The effect of variation of all these parameters in the design can be estimated precisely and accurately. The monographs presented herein are applicable to the both tubular framed buildings and box girder bridges as well.

Ductility of a composite tubular structure is difficult to assess, however, the effect of material properties on shear lag phenomenon can be assessed. Therefore, the present study is useful towards solving the assumption of polynomial of the flange panels.