CHAPTER 2

LITERATURE REVIEW

2.1 INTRODUCTION

The classical theory i.e. simple bending theory (SBT) assumes that a plane section remains plane after bending. This assumption results in a linear distribution of bending stress in the cross section of the beam. The assumption can only be true in a box section if the shear stiffness of the cross section is infinite. Due to the shear flow between the flange and the web of the box, the panels displace longitudinally in the way that the middle portion of the flange and web lag behind that of the portion closer to the corner of the box section. This longitudinal displacement of the flange and the web results in the nonuniform bending stress distribution. The unequal distribution of bending stress addressed to shear lag phenomenon. Shear lag phenomenon does not occur if there is no gradient in shear flow.

The shear lag effect reduces the effectiveness of the box structure by increasing/decreasing the stress concentration at the web-flange junctions, reducing/increasing the axial stresses at the middle of the frame panels, which accumulates to increased lateral deflections of the structure. The cantilever box girder bridge and tubular buildings have closely resembled the cantilever box beams having idealized box section (Fig. A.1) hence the distribution of bending stresses greatly affected by shear lag in the flange. In skyscrapers, certain structural form especially tube frame structures are widely used as an economical system (Fig. A.3). The system consists of closely spaced exterior columns along the periphery interconnected by deep spandrel beams of each floor level in its basic form. This produces a system of rigidly jointed orthogonal frame panels forming a rectangular tube which acts as a cantilever hollow box. Framed tube acts like a hollow boxed beam under lateral loads such as wind and earthquake. Thus, the tubular buildings are also under in influence of shear lag. In tubular buildings, the flexibility of spandrel beams produces shear lag phenomenon with the effect of increasing the axial stresses in the corner columns and of reducing axial stress in the columns toward the center of flange panel of the orthogonal frame in the bottom of the structure.

The bending stress distribution across wide flanges of a box beam under symmetrical flexure is not uniform. This phenomenon has long been recognized as shear lag. The stress at the junctions of the web and flange is much higher than that at the center of the flange. This phenomenon is usually notified as positive shear lag [Coull and Bose 1975; Coull and Ahmed 1978; Foutch and Chang 1982; Ha et al. 1978; Foutch and Chang 1982; Kwan 1994, 1996; Reissner 1945]. Reversal of this phenomenon is known as negative shear lag [Chang and Zheng 1987; Mahjoub et al. 2011; Shushkewich, 1991; Singh and Nagpal 1994]. This effect of positive shear lag was originally reported by Reissner (1945) while analyzing a box beam.

The analysis of shear lag phenomenon in box beam was based on the principle of minimum potential energy. Analysis of shear lag phenomenon has also been carried out by using energy methods. The stress compatibly equilibrium equations considered in the least work solution method [Coull and Bose 1975, 1976; Lin and Zhao 2011], primarily introduced by Reissner (1941). The governing equations in variational principle i.e., principle of minimum potential energy, are committed to the strain compatibility [Chang 1984; Chang and Zheng 1987; Foutch and Chang 1982; Kuzmanovic and Graham 1981;

Kwan 1994; Lee et al.2000, 2001; Lin and Zhao 2011; Mahjoub et al. 2011; Reissner 1945]. In the continuity of literatures published after Reissner (1945), many other authors used principle of minimum potential energy to analyze the shear lag phenomenon in tubular structures [Chang 1984; Chang and Zheng 1987; Foutch and Chang 1982; Kuzmanovic and Graham 1981; Kwan 1994; Lee et al.2000, 2001; Lin and Zhao 2011; Mahjoub et al. 2011]. Various methods were dealt to analyze the shear lag in box beams, box girder bridges (linear and curve); cable-stayed bridges; prestressed concrete bridges; composite box girder bridges; thin-walled structures; framed tube structures (RCC and steel structures); I and T-section beams; pipe connections; axial member connections and laminated plate structures.

An orthotropic membrane analogy of transforming the framework panels into equivalent orthotropic membranes, each with elastic properties so chosen, to represent the axial and shear behavior of the actual frame work, has been developed [Coull and Bose 1975, 1976; Coull and Ahmed 1978). Before this philosophy, Chan et al. (1974) proposed to evaluate the shear lag effects in the cantilever box structure with the solid shear wall as web panels and rigidly jointed beam-column frames as flange panels. This methodology may also be applied in frame tube structures by including the shear deformation of the frame members and the deformation of the beam-column joints in the derivation of the equivalent elastic properties [Khan and Stafford 1976; Ha et al. 1978]. In the series of development, Grillage analogy method [Hambly and Pennells 1975; Kissane and Beal 1975; Evans and Shanmugam 1984; Sennah and Kennedy 2002]; Folded plate method [Meyer and Scordelis 1971; Marsh and Tayler 1990; Sennah and Kennedy 2002] and Finite strip method [Cheung and Cheung 1971; Cusens and Loo 1974; Cheung and Chan 1978; Sennah and Kennedy 2002] were used for analysis of shear lag in tubular structures. The finite element method was firstly introduced by Moffat and Dowling (1975) to analyze the shear lag in steel box girders. Another researcher also dealt with this method for analysis of shear lag phenomenon in the tubular structures [Chang and Zheng 1987; Chang and Gang 1990; Cook 1991; Evans and Al-Rifaie 1975; Li et al. 1988; Shang-min and Shui 2014; Sennah and Kennedy 2002; Shushkewich 1988, 1991].

The different approaches of analyzing the shear lag phenomenon in the structures and their limitations along with applications have been reviewed in the present chapter.

2.2 SHEAR LAG IN BOX BEAMS

2.2.1 Thin Walled Box Beams

A thin walled box beam was analyzed by Reissner (1941, 1945) applying analytical method. Initially, the stress variations in the box beam were dealt with least work solution method after that a variational principle adopted considering the longitudinal displacement of the cover sheet of the box beam. The corresponding unequal stress distributions were termed as shear lag in box beam. The least work method governs by stress compatibility equations whereas the principle of minimum potential energy method, govern by strain compatibility equilibrium equations. Various authors analysed box beam by variational approach [Chen 2014; Dezi and Mentrasti 1985; Koo and Cheung 1989; Mentrasti 1991]. Non uniform normal longitudinal stress distribution in a trapezoidal box beam with lateral cantilever, have been discussed. The state of stress in each structural element was analyzed considering warping of the horizontal flanges [Dezi and Mentrasti 1985].

Koo and Cheung (1989) formulated a mixed variational principle for thin-walled prismatic structures and a solution method is proposed. In this principle, the cross section of the thin-walled prismatic structure is assumed to be rigid in its own plane, but flexible out of plane, and both the stresses and displacements are considered to be unknown variables in this function. Mentrasti (1991) studied the thin-walled trapezoidal cross-section beams with lateral cantilevers subjected to a torsional load, considering the distortion of the cross section. Transverse and longitudinal displacements of the walls are governed by six shape functions, which describe bending in the horizontal plane, as well as torsion and distortion of the cross section; the shearing strain in the middle plane of the walls is considered; the shear center is not used. A wide set of constraints is studied (rigid or flexible transverse diaphragms, mixed kinematic and static boundary conditions) reviewing the inversion of the curvature of the distribution of longitudinal stress (shear lag anomaly). A harmonic analysis was performed using the variable separation method for a simply supported and cantilever beam under a given load [Chen 2014] and solutions obtained using superposition.

A stress resultant theory was formulated for elastic and elastoplastic warping [Hjelmstad 1987]. The constitutive relationships retain the interesting feature of coupling the shear strains (shear and bi-shear) with the warping intensity. This coupling gives rise to the so-called shear lag anomaly. A thin-walled beam (open or closed cross sections) analysed, undergoing elastoplastic deformations [Prokic 1994]**.** A numerical method based on the finite-element method and incremental solution techniques was used. A model of describing the warping and shear deformation was applied. Von Mises yield condition with isotropic hardening and shear lag phenomenon, as a special case of general analysis, was considered as well. In addition, it is adopted a general approach to the solution of the problem based on the finite element method. The principle of virtual displacements has been used to give a new linear stiffness matrix. By applying warping functions and general approach to the solution as stated above, shear lag phenomenon (positive or negative) was described in wide flanges of thin-walled box girders [Prokic 2002]**.** Tenchev (1996) performed an FEM parametric study of the stress distribution in orthotropic beam flanges. In the FE analyses, a two-dimensional plane stress model was used. An empirical formula was established for the shear lag coefficient which is used for computing the effective flange width for stress calculations. Longitudinal flange stiffeners can be accounted for by modifying the modular ratio. General displacement field of thin-walled section beams considering the uniform and non-uniform shear warping deformation by using step-by-step application of St. Venant's semi-inverse method was proposed [Park et al. 1997]. A beam model was developed. For the analysis of the general flexural-torsional response on the unsymmetrical section beams, a continuous finite element using boundary element contraction technique was formulated and applied. The ultimate load of thin-walled box beam undergoing limited plastic strain with consideration of the shear lag effect on the basis of the stress–strain relationship of elastic, linearly hardening materials was investigated [Wu et al. 2004]. In the procedure, calculation formulae for strength increase coefficient, flange effective width ratio, critical values of plastic strain and shear lag coefficient are obtained for thin-walled box beams with elastic, linearly hardening materials.

Lin and Zhao (2012) described inelastic shear lag behavior in steel box beams. Shear lag effects in flanged flexural members are usually recognized as the uneven longitudinal deformation and normal stresses along the flanges. Elastic shear lag behavior has been extensively studied and considered in structural design while the study of inelastic shear lag is limited. The analytical method was verified using laboratory tests of two steel box beams. A least-work based method was developed for modeling inelastic shear lag behavior. An effective modulus was formulated and the Poisson's ratio following the theory of plasticity was used in the inelastic shear lag model.

2.2.2 Composite Box Beams

Gjelsvik (1991) analysed wide flange composite beam with shear lag in which the real beam is replaced by equivalent beam where the shear deformations are concentrated in the thin layer. The stress distributions were found by simple formulas. The warping solutions for thin walled prismatic orthotropic composite beam were generated [Anido and Ganga Rao 1996; Dikaros and Sapountzakis 2014 I, II; Sun and Bursi 2005]. The elastic coefficient that account for warping effect, are introduced and solution correlated with an elastic solution by Lekhnitskii (anti-symmetric shear lag), and for a box beam with an analytical solution of Reissner (symmetric shear lag). Sun and Bursi (2005) proposed displacement-based and two-field mixed beam elements for the linear analysis of steel–concrete composite beams with shear lag and deformable shear connection and the results were compared with ABAQUS code.

Dikaros and Sapountzakis (2014 I, II) presented theoretical development in part I and numerical implementation of the solution method, whereas in part II discussed the examined numerical applications illustrating the efficiency, accuracy, and range of applications of the proposed method as well as the effects arising because of non-uniform warping. Nagaraj and Gangarao (1997) presented experimental and theoretical characterizations of mechanical properties of glass fiber reinforced plastic (GFRP) wide flange and box beams. Experimental results revealed that fiber asymmetry and interfacial slip between fabric layers affect the beam stiffness. Shear deflections were found to be significant in relation to bending deflections. Dezi et al. (2001) proposed a model for analyzing the shear-lag effect in composite beams with flexible shear connection taking into account the long-term behavior of the concrete. A variational balance condition is imposed by the virtual work theorem for three-dimensional bodies. Lenwari (2005) also used virtual work theorem to analyzed the flexural behavior of rolled steel beams that were strengthened with partial-length, adhesive-bonded carbon fiber-reinforced polymer (CFRP) plates. The analytical methods used include shear lag analysis, section analysis and discussed the agreement between the experimental results and the analytical predictions.

The symmetrically laminated thin-walled composite box beams under bending load were analyzed by the principle of minimum potential energy [Yaping et al. 2002] to account for shear lag and shear deformation effects. A comprehensive analysis on the effects of shear lag and shear deformation were presented for a simply supported thin-walled composite box beam subjected to a centralized load at mid-span. The initial value solutions of static equilibrium differential equations of single-cell thin-walled composite-laminated box beams under bending loads with consideration of both shear lag and shear deformation were established. The results were verified by numerical methods using finite beam element [Wu et al. 2004b]**.** The procedure presented can be used to analyze effectively the mechanical behaviors of some complex composite structures (such as continuous laminated box girders, etc.).

Chen et al. (2013) put forward a shear lag model for fiber-reinforced plastics (FRP) composite simply supported II-beams under bending loads using the energy method and results were validate with finite element analysis. For the nonlinear behavior of wide flange steel and steel-concrete composite beam up to collapse, Henriques et al. (2015) introduced an accurate and computationally efficient GBT-based finite element. The element incorporates reinforced concrete cracking/crushing, shear lag effects and steel beam plasticity (including shear deformation of the steel web). In addition, analytical solutions for elastic shear lag are derived and the GBT modal decomposition features are employed to extract valuable information concerning the effect of shear lag phenomena up to the collapse.

The metallic ship structures, for the special case of composite hull Ghelardi et al. (2015) study the concept of shear lag effective breadth of plating, widely applied in the scantling assessments of metallic ship structures. An empirical formulation was proposed for effective breadth evaluations. Thus, suitable finite element models were created and validated to investigate the behavior of the effective breadth of stiffened laminates when varying geometrical and other typical parameters of composite made ship structures. Milner and Peczkis (2007) reported the difficulties of modeling the vertical flexure of mechanically fastened wooden ship hulls as that of box beams, a mainstay of naval architecture when applied to iron and steel ships, can largely be overcome by factoring the incomplete composite action of timber components in terms of a reduced shear modulus, an increased shear lag, and a reduced sectional area in tension (owing to butt joints).

2.2.3 Multi-Cellular Box Beams

A constant-amplitude cyclic loading test conducted in complex welded box beams made of high-strength low-alloy steel [Nussbaumer et al. 1999]. A box beam was designed to simulate the cellular structure of a double-hull ship, but the results of the experiments are equally applicable to other box systems such as bridges. The experiment results demonstrated the good crack tolerance of cellular structures and studied the propagation of long cracks. The stresses varied across the flange due to shear lag, but this effect was small. A finite curved beam element considering both shear lag and warping torsion effects, especially for the dynamical analysis of curved thin-walled multi cell box beams [Yaping 2002].The contribution of rotational inertia to mass matrix used to analyze accurately the static and dynamic behaviors of curved thin-walled box bridge with a single cell or multi-cell.

2.3 SHEAR LAG IN BRIDGES

The shear lag was analyzed in bridges particularly in box bridges having different shape i.e. linear and curve; with a different cross section i.e. single cell or multicellular cross sections. The different types of box bridges were considered for analysis which was box girder bridges; cablestayed bridges and prestressed concrete bridges. The shear lag was also considered in girders. The various investigation of shear lag in box bridges and girders are presented below systematically.

2.3.1 Box Girder Bridges

The shear lag was analyzed in box girder bridges by the various analytical methods, numerical method and performed the test to verify the result obtained. Shushkewich (1986) utilized the strength of material relationship for membrane forces in symmetrically loaded box girder bridges. The shear lag was ignored for the calculation of compressive stress in the top slab. But further investigations were made considering shear lag effect. Primarily the shear lag was analyzed through variational approach [Chang and Zheng 1987; Luo et al. 2002a, 2003; Wang et al. 2005]. The span –wise displacement was assumed parabola, cubic parabola, quartic parabola etc. The regions of negative shear lag effect with the interrelation of span/width parameters are determined [Chang and Zheng 1987]. The theoretical results thus obtained are compared with a plexiglass model test and are in close agreement with the test results.

Khristek and Bazant (1987) analyzed the effect of shear lag on creep deformations and stress redistributions in concrete box girder bridges, both deterministically and stochastically. It is found that the shear lag is as important for creep as it is for elastic deformations. The shear lag causes a significant increase in the maximum longitudinal normal stress, alters the stress redistributions due to a change in the structural system during the construction stage, and substantially increases the deflections. A finite difference solution for shear lag effect on box girder bridges was presented by Chang and Yun (1988). The span wise displacement is assumed to be a quartic parabolic curve on the cross section instead of a quadratic. Shear lag effects are computed on a cantilever box girder with linearly varying depth. The results are compared with model tests. Shushkewich (1988) perform plane frame analysis to demonstrate actual three dimensional behaviors by using membrane equations. The method also allows the reinforcing and pre-stressing. Kristek and Studnicka (1991) have described the main features of the negative shear-lag phenomenon. It was concluded that negative shear lag was not a particularly exceptional phenomenon but an effect that can occur at a wide variety of arrangements of plated structures with a considerable degree of significance. The negative shear lag occurs along a considerable portion of the length of such structures. It is confirmed that the shear lag depends not only on the shear flow magnitude itself (or on the magnitude of shear force) but especially on the shear-flow gradient as well, which is influenced primarily by the type of loading.

Shushkewich (1991) has been explained the problem of negative shear lag in a cantilever beam subjected to a uniform load. In essence, the problem was one of two components, each of which was subjected to positive shear lag being combined such that negative shear lag is produced. The key ingredient for negative shear lag was that the predominant component dampened faster than the less predominant component. In the generalized way negative shear lag can occur whenever two or more components have the shear lag that dampens at varying rates. The negative shear lag in box girder with varying depth was analyzed [Luo et al. 2001] using modified finite segment method. With various boundary conditions and support conditions in the box girder, it was concluded that the negative shear lag effect not only exists in cantilever and continuous box girder but also occurs in simply supported box girders. Stochastic variational principle (SVP) and stochastic finite segment method (SFSM) was developed for analysis of the shear lag effect on a box-girder by Yang et al. (2001). Then the second order perturbation techniques are employed to develop a set of expanded deterministic expressions of the functional. Then the recursive stiffness equations of the SFSM are obtained by the stationary conditions of the second order energy functional. The origin of the negative shear lag phenomenon was explained according to the physical point of view by Lee et al. 2002. It is shown that, at any given location, negative shear lag can take place whenever the portion of the shear flow acting along the flange edges, which produces the shear lag-aftereffect, is larger than the remaining portion of shear flow caused by positive shear lag. In box girders with wide flanges, an uneven flange stress distribution, the flexural normal stress is greater at the edges than that at the center of the flange, so-called shear lag (or positive shear lag), may occur following the characteristics of stress concentration problems.

Luo et al. (2002a, c) and Luo et al. (2004a) presented a finite-segment method for analyzing shear-lag effects in box girders, with an assumption that the spanwise displacements of the flange plates were described by a third-power parabolic function. The experimental studies were conducted on shear lag effect for box girders with constant and varying depth in cross section. The experiments intend to address two issues, the beam–column action and the effect of varying depth upon the shear lag of box girders. Finite element analysis is also conducted to check the accuracy of the numerical methods in predicting the shear lag effect. In order to obtain the longitudinal stresses under the shear-lag effect, the element stiffness equations were developed based on the variational principle by taking the homogeneous solutions of the differential equations as the displacement functions of the finite segment. Luo et al (2003) presented a new systematic approach to the shear lag analysis of structures that are subjected to simultaneous bending and axial forces. The shear lag effects in beam action and column action are considered separately in analogy to the stress calculation of beam columns, using box girder as an illustrating example. The analytical methods i.e. principle of minimum potential energy and a numerical technique i.e. finite strip method were adopted in the analysis. A Perspex glass model of a three-span continuous box girder with varying depth was tested to provide experimental results for verifying the accuracy of the proposed method [Luo et al. 2004a]. Luo et al. 2004b proposed a new method for the determination of membrane forces acting on box girder bridges considering shear lag effect. The box girder was divided into the plates and the equilibrium condition is applied. The analytical formulas for calculating the membrane normal, transverse and shear forces of each plate element considering shear lag effect are derived.

The finite element method was introduced for the analysis of steel box beam by Moffat and Doweling (1975). The effects of longitudinal and transverse stiffeners on shear lag phenomenon were concluded. Lertsima et al. (2004) studied the effect of finite element mesh on the shear lag phenomenon in box girder bridges which may be a source for the discrepancy of the stress concentration factors. The structures were discretized by the three-dimensional finite element method using shell elements. To characterize the geometry of box girder bridges, the multi-mesh extrapolation was employed. The accuracy ensured to investigate the deflection at the mid-span for the effect of the shear lag [Lertsima et al. 2005] by using three-dimensional finite element analyzes for various box girders.

Wang et al. (2005) presented an efficient finite segment method for the analysis of curved box girders with corner stiffeners for analysing the shear lag effect. The deck was modelled with modified curved beam element; bottom plate using normal curved beam element; the webs and the corner stiffeners using a spatial displacement field. Sa-nguanmanasaka (2007) analysed shear lag effect in box girder bridges by carefully treating influence of finite element meshes. The effect of shear lag was investigated in a continuous box girder by using the three-dimensional finite element method using shell elements. Based on the numerical results, empirical formulas were proposed to compute stress concentration factors that include the shear lag effect. The various box girder cross-sections i.e. rectangular, trapezoidal and circular were studied in detail [Gupta 2010] using three-dimensional 4-noded shell elements. The linear analysis has been carried out for the Dead Load (Self Weight) and Live Load of Indian Road Congress Class 70R loading. The paper presents a parametric study for deflections, longitudinal and transverse bending stresses and shear lag for these cross-sections. Qin and Liu (2010) proposed a new method based on the symplectic elasticity method for the determination of effective flange in box girder bridges considering shear lag effect. The flange slab of the box girder was simplified into a plane stress plate. Using equilibrium conditions of the plates, the Hamilton dual equations for top plate element is established. Through examples using the finite element method, the results obtained by the proposed method are examined.

Zhou (2010) established a finite-element method considering the interaction of the bending and shear-lag deformation of a box girder. A stiffness matrix induced by shear-lag-was defined and deduced. At each node of the beam element, two shear-lag degrees of freedom were used as boundary conditions for the box girders. The effective flange width provisions in the current AASHTO specifications evaluated [Lin and Zhao 2011] and the shear-lag effects in box girders were studied systematically using a variation analysis method. Al-Sherrawi and Fadhil (2012) studied the effects of stiffeners on shear lag in steel box girders with stiffened flanges using three-dimensional linear finite element methods. The actual top flange stress distribution and effective width in steel box girders were determined. The steel plates of the flanges and webs have been modeled by four-node iso-parametric shell elements while the stiffeners have been modeled as beam elements. The top flange longitudinal stresses were reduced using stiffeners but didn't affect the shear lag. Bažant et al. (2012 II) presented numerical procedure and reviews on box girder of the thick shell that discretized by eight-node, three-dimensional (3D) finite elements. The environmental effect on shrinkage strain except concrete creep assumed to follow linear Viscoelasticity and modeled by a rate-type law based on the Kelvin chain, the properties of which are adjusted for humidity conditions and temperature.

Zhang (2012) also established an improved finite-segment method for shear lag warping in a box girder with cantilever slabs. The homogeneous solution of the governing differential equation for shear lag was adopted as the element displacement function. The formulas of the element stiffness matrix and the equivalent nodal force vector were derived. Zhou (2014) deduced the theoretical solution method for the shear lag effect of cantilever box girder through exerting the principle of minimum potential energy. The expressions of a longitudinal displacement function, maximum angle displacement difference function and computational formulas of the shear lag coefficient, additional bending moment and deflection were deduced for concentrated and uniformly distributed load. The model was analysed by finite element method and the analysis indicates that if the external load produces a constant shear flow within the section of the cantilever beam, the positive shear lag effect will be produced only; if such load produces a varying or reverse shear flow within the section of the cantilever beam, the negative shear lag effect will be produced; the magnitude of the shear lag coefficient is proportional to the wide-span ratio; either the positive or negative shear lag effect will increase the structural deflection.

2.3.2 Thin Walled Box Girder Bridges

Razaqpur and Li (1991) developed a thin-walled-box-girder finite element that can model extension, flexure, torsion, torsional warping, distortion, distortional warping, and shear lag effects, using an extended version of Vlasov's thin-walled beam theory. The torsion-bending behavior of thin walled box beams under consideration of shear lag components was also analysed by Tesar (1996). Besides the six nodal degrees of freedom of a conventional beam element (two end nodes) it has additional degrees of freedom to account for torsional warping, distortion, distortional warping, and shear lag [Razaqpur and Li 1991]. The governing differential equation pertaining to each action was used to derive the exact shape functions and the stiffness matrix and nodal load vector of the element. An orthogonalization procedure was employed to uncouple the various distortional and shear lag modes. Tesar (1996) used the FETM-method as problem-oriented combination of transfer matrix and finite element techniques. An adaptive finite element analysis procedure is used to deal with the problem of shear lag effects of plated structures and thin-walled structures [Lee and Wu 2000a]. This study consists of shear lag effects in simple plated structures, such as straight rectangular, single-cell box girders.

Luo and Li (2000) investigated the influence of shear lag for thin-walled curved box girders, including longitudinal warping of the flange in addition with displacement functions of the flange slabs are approximated by a cubic parabolic curve instead of a quadratic curve. On the basis of the thin-walled curved bar theory and the potential variational principle, the equations of equilibrium considering the shear lag, bending, and torsion (St.Venant and warping) for a thin-walled curved box girder are established. The closed-form solutions of the equations are derived, and Vlasov's equation is further developed. 3-bar simulation-transfer matrix method for shear lag analysis, in which the influence of section size on internal forces and stress were taken into consideration, was developed [Li et al 2011]. The inner force, stress of thin-walled box girder considering the shear lag effects are recursively calculated and verified with those from the ANSYS program. Lin and Zhao (2011) developed a method based on energy for quantifying elastic shear lag effects in thin-walled flexural members such as box girders. The method uses the summation of infinite number of binomials with unknown parameters to approach the longitudinal displacement in the flange of the box girders. The experimental tests were conducted of three rectangular box girders for critically determining the shear lag design parameters.

A new method for analyzing shear lag effect in thin- walled box girders was proposed in which additional deflection induced by shear lag effect is adopted as the generalized displacement [Zhang and Lin 2014a, b]. Based on the generalized moment the shear lag deformation state is separated from the flexural deformation state of the corresponding elementary beam and analyzed as a fundamental deformation state. The quadratic parabola is demonstrated to be the reasonable curve of the warping displacement function in the shear lag effect analysis of a box girder and the accuracy of the degrees of the warping functions is evaluated. The negative shear lag was illustrated through the generalized moment. The principle of minimum potential energy as an analytical method and finite beam segment element with 8 degrees of freedom as a numerical tools were applied to analyzed the structure and to verified the result respectably. A test was also conducted for two plexi-glass models of continuous box girders [Zhang and Lin 2014b].

2.3.3 Composite Box Girder Bridges

The shear lag phenomenon was discretized by various authors in composite box girder bridges [Branco and Green 1985; Lee and Wu 2000b; Tahan et al. 1997; Gara et al 2011; Abbu et al. 2013]. Various numerical tools, using finite strip analysis [Branco and Green 1985]; adaptive finite element [Lee and Wu 2000b]; finite element formulation and technique [Gara et al 2011; Abbu et al. 2013] were put forward systematically. Tahan et al. (1997) tackled the problem of shear lag by means of a classical technique and some of the salient features are highlighted, especially as an aid to engineers at the preliminary design stage of plated structures. The influence of bracing systems the on open box behavior using finite strip analysis results have and a one-quarter scale girder model test data been discussed [Branco and Green 1985]. Tie and distortional bracing were found effective in preventing distortion, with web stiffening found to be more effective than interior cross bracing. Horizontal bracing at the flange level was considered to reduce twisting of the section. This bracing can take the form of either torsion boxes or top chord bracing, both being effective. A torsion-bending analysis based on a rigid section behavior is reviewed. Lee and Wu (2000b) tackles the shear lag problems for complex plated structures with more general and complex geometries, including core walls with openings, multi-cell box girders and box girders with curved flanges.

Chiewanichakorn et al. (2004) described a new effective flange width definition and the associated finite element modeling scheme for a steel–concrete composite bridge girder. Nonlinear finite element analysis has been employed to investigate and develop a more versatile effective flange width definition for simple-span bridges.The parametric studies were conducted to investigate the influence of some key geometrical parameters on the shear lag effect for these types of structure [Lee and Wu 2000b; Gara et al 2011]. Gara et al (2011) proposed a simplified method of analysis for the design of twin-girder and single box steel- concrete composite bridge decks. The proposed approach is capable of handling different loading conditions, such as constant uniformly distributed loads, envelopes of transverse actions due to traffic loads, support settlements, and concrete shrinkage. Abbu et al. (2013) stated that difficulty in the analysis and design of composite box girder bridges could be handled by the use of the digital computers in the design. The method was also capable of dealing with different material properties, relationships between structural components, boundary conditions, as well as statically or dynamically applied loads. The linear and nonlinear structural response of such bridges can be predicted with good accuracy using this method.

Zou et al. (2011) provided an analytical shear lag model for effective flange width for orthotropic bridge decks, applicable to various materials including Fiber-Reinforced Polymer (FRP) and concrete decks. To verify this solution, a Finite Element (FE) parametric study was conducted on simply-supported FRP deck-on-steel girder bridges. The results from the shear lag model correlate well with the FE results. It was also illustrated that the shear lag model, with the introduction of a reduction factor, can be applied to predict effective flange width for FRP deck-on-steel girder bridges with partial composite action, by favorable comparisons between the analytical and testing results for a T-beam section cut from a one-third scaled bridge model, which consists of an FRP sandwich deck attached to steel girders by mechanical connectors.

Shang-min and Shui (2014) established a three-dimensional finite element (FE) model, using the ANSYS FE software, and a corresponding FE model of composite girder with cogurated steel webs to study the effect of shear lag on a composite girder with steel truss webs, performed comparative analysis. The shear lag effect and distribution of shear lag coefficients along the cross section of the composite girder with steel truss webs are investigated under concentrated and uniform loads. The results show that (1) under concentrated and uniform loads, the shear lag effect was evident on the top and bottom plates of the composite girder with steel truss webs; (2) for the composite girder with steel truss webs, the shear lag effect is more pronounced under concentrated loads than under uniform loads; (3) under the same load, the normal stresses of the top and bottom plates of the composite girder with steel truss webs are less than those of composite girder with cogurated steel webs; and (4) the shear lag coefficient distributions of the top and bottom plates along the cross section are almost similar for the two types of composite girders.

Zhu et al. (2015) proposed a simplified analysis method based on effective width to account for the shear-lag effect of composite decks, which would greatly assist in design analysis as it was easy to use in a general beam element model. In this paper, the static tests were carried out on a composite twin I-girder deck and a composite box girder deck specimen. These two specimens are subjected to vertically flexural and axially compressive loads in the tests. An elaborate element model was later built and utilized to analyze the shear-lag effect of composite decks. Finally, based on a general beam element model, the simplified analysis method of a composite continuous I-girder deck for the design process was performed taking into consideration three load cases, which include gravity load, traffic load envelope and prestressing load.

2.3.4 Cable Stayed Bridges

Svensson et al. (1986) design and proposed construction method for the cable-stayed steel bridge by using relatively new steel composite girder. In this concept, the girder comprises a grid of welded steel plate girders and was topped by a concrete deck from precast slabs. The concrete towers with diamond-shaped box girder legs are connected to the girder by parallel wire or strand cables. DIN Code 1075 was applied to calculate the effective girder widths in bending (shear lag). The effective width (shear lag) has quite an influence on the size of moments because of the location of main girders which are at the very outside of the cross-section. Luo et al. (2002b) established governing differential equations for the shear lag in cable-stayed bridges under combined axial and lateral loads using the principle of minimum potential energy. The displacement patterns of the finite segment applied to calculate the shear lag effect in cable-stayed bridges. A model test of a box girder was conducted. A case study was reported by Feng and Hong (2005) on the shear lag effect of the Lanzhou Xiaoxihu Yellow River's cable-stayed bridge. width to span ratio of cable-stayed bridges results in significant shear lag effect to cause nonuniform stress distribution along the flanges of the beam of the bridge. A 3D finite element analysis of the model of the bridge was developed and finite element analysis (FEA) was done to obtain the theoretical results. To evaluate the theoretical results, a scaled model was made to conduct a static test in the laboratory.

A finite element formulation was presented for the analysis of composite decks accounting for partial interaction theory and shear-lag effects [Gara et al 2008]. For these particular bridge solutions, stress concentrations induced in the slab by the application of concentrated forces, i.e. due to the anchorage of prestressing cables or stays, or due to the presence of web members in arch bridges. The ease of use of the proposed deck finite element was outlined considering two case studies for which the calculated results have been compared against those obtained using a more refined model implemented using shell elements in a commercial finite element software.

2.3.5 Prestressed Concrete Bridges

Bazant and Kim (1989) investigated the limits on long-time deflections and internal forces of prestressed concrete segmental box girder bridges. For the sake of simplicity the box-girder bridge analyzed as a beam by neglecting the shear lag. Chang (1992) has been recognized shear-lag effect in a thin-walled box girder and introduced in the technical literature, and reported that the discussion of the shear-lag effect caused by the combination of prestress force and self-weight of a box girder were limited. The shear-lag effect was treated by the principle of superposition by considering that the configuration of a prestressed tendon takes the form of a broken straight line. Two equal spans of a continuous box girder of constant depth with a shear-lag effect were used as an illustrative example.

The time-dependent effects of creep and shrinkage of concrete and relaxation of prestressing tendons on stresses and deflections in segmentally erected, cable-stayed, concrete bridges stress have been investigated considering redistribution necessitated by shear lag [Cluleyt and Shepherd 1996]. A three-dimensional finite element code was developed to analyze these effects. Chang (2004) provided derivation and computed formulas for the shear lag coefficient in a simply supported prestressed concrete box girder under dead load. In the case of prestressed tendons having parabolic configurations, formulas to compute the shear lag effect are also developed. Conclusions are drawn that the shear lag effect caused by dead load and prestress force is equivalent to dead load acting alone, provided that the prestressed tendon is set up with a parabolic profile. Shushkewich (2006) presented a special computer program STRUTBOX for the transverse analysis of strutted box girder bridges, and particularly for bridges designed and constructed using the strutted box widening method adopting deck prestressing and other reinforcing. The program was based on folded plate method which indicated the severity of shear lag effects. Zhou (2011) presented a finite-element method based on the variational principle to analyze the effect of prestressing on shear lag in box girders. It was concluded that the shear lag effect in box girders under prestressing was more apparent than that under uniformly distributed loads or vertical concentrated loads. The values and distributions of shear lag coefficients are related to the anchorage locations of prestressing and the distributions of internal forces along the girder under the combined uniformly distributed load and prestressing. A finite element was used to verify the result. A case study was reported by Bažant et al. (2012 I) and wrote that the segmental prestressed concrete box girder of Koror-Babeldaob (KB) Bridge in Palau, which had a record span of 241 m, presents a striking paradigm of serviceability loss because of excessive multi-decade deflections. It is calculated that, compared with the classical theory of bending, all the shear lags combined increased the elastic downward deflection because of self-weight and the elastic upward deflection because of initial prestress. The shear lag effect with the associated creep-induced stress redistributions within the cross sections necessitates 3D simulations. It cannot be realistically captured by the classical concept of the effective width of the top slab (which was actually used in the design of the KB Bridge). The different ways of occurrences of shear lag were discussed.

Xiao (2014) presented three-dimensional finite element analysis for a three-span continuous PC box girder bridge with corrugated steel webs and the corresponding conventional box girder bridge with concrete webs. The results showed that at the sections in the negative bending moment near the intermediate piers, the shear lag effect in the bridge with corrugated steel webs is more obvious than that in the bridge with concrete webs; and the corresponding effective flange width coefficient in the bridge with corrugated steel webs was even smaller than 0.9, so the shear lag effect at these sections should be considered in the design of this type of bridges and at mid-span the shear lag coefficient found close to one.

2.3.6 Girder Bridges

Hasebe et al. (1985) reported the problems of effective width and the shear lag phenomenon of the curved girder bridges. The theory used in the analysis was the refined beam theory by Kano, et al. (1982), and can be used to treat the effects of shear deformation and shear lag of thin-walled curved members. Mari and Valde (2000) tested a 1:2 scale model of a two-span continuous bridge in order to study its behavior during the construction process and under permanent loads. Time-dependent concrete properties, as well as support reactions, deflections, and strains in concrete and steel were measured and compared with a numerical model. Some difficulties observed in accuracy probably due shear lag effects in the tensile zone. Shear lag combined with creep effects have a big influence. A further analysis needed to better quantify the contribution of creep, shrinkage, cracking, and shear lag effects in structural stiffness of top slab. Dezi et al. (2006) showed the shear-lag effect in slabs of twin-girder steel–concrete composite decks due to the main prestressing behaviors. A beam model was presented taking into account the loss of planarity for the slab cross section, the flexibility of the shear connection, and the time-dependent behavior of the concrete. A linear elastic behavior was considered for the steel beams and shear connection while Viscoelastic behavior was assumed for the concrete slab. A finite-element modeling scheme was briefly discussed, and the model is successfully verified with experimental results by Aref et al (2007)**.** The proposed definition was developed for negative moment regions. Numerical results also indicate that the effective slab width criteria in the current AASHTO-LRFD Specifications were typically conservative for larger girder spacing. Chen et al. (2007) proposed simpler and more versatile design criteria for computing the effective width in steel-concrete composite bridges. A parametric study was conducted in simple-span and multiple-span continuous bridges, based on finite-element analysis of bridges selected by a statistical method. A time-dependent finite-element analysis of a two I-girder composite bridge with a concrete slab was presented [Okui and Nagai 2007]. The analysis also included the shear-lag effect of the concrete slab on the time-dependent behavior of two I-girder bridges. It was shown that the shear-lag effect becomes significant at the edge of the cracking region and at the bridge ends.

Gara et al. (2009) presented a beam finite element for the long-term analysis of steel–concrete composite decks taking into account the shear lag in the slab and the partial shear interaction at the slab-girder interface. Using the displacement approach, beam kinematics was developed from the Newmark model for composite beams with the partial shear connection; warping of the slab cross section is caught with the product of an established function which describes the warping shape, and an intensity function that measures the warping magnitude along the beam axis. Time-dependent behavior is considered an integral type out Viscoelastic creep law for the concrete.

2.4 SHEAR LAG IN FRAMED TUBE STRUCTURES

Framed tube structures anonymously pronounce as tubular structures. Framed tube structures consisted single tube or multiple-tube. The multiple-tube structures generally referred as a tube in the tube structures. The tubular structures also have core/shear wall. The various literature attributing shear lag effect on such tubular structures are being reported here in the following sections.

2.4.1 Tubular Structures (Single Tube)

Chang and Foutch (1984) analyzed tube frame structures subjected to static or dynamic loads by the finite element method. The model considered the shear-lag effect as well as shear and flexural deformation. The model continuum represented by a set of first-order differential equations. It was found resulting deflection, frequencies and mode shapes predicted by the continuous model shows an excellent comparison to the finite element results. Using finite element method, Coull and Chee (1984) also investigated the interaction of slab which was connected by moment resisting joints to the core and peripheral frames in tubular structures. The relative influences of a range of geometrical and stiffness parameters were evaluated and found the column to slab flexural stiffness ratio had a significant effect on the rotational stiffness and effective width of the slab. The effective slab width increased with both relative core width and core depth. The slab reactions induced by coupling action have the effect of increasing the loads in the interior columns and reducing the loads in the corner columns in the external framed tube, thereby reducing the shear lag effect in the basic framed tube action.

Notch (1984a, b) worked on to develop a lateral load resistant system for tall and slender buildings. A tree beam continuum element was developed to discretize the structures. The cross arm introduced a point of reverse bending in the wind girders greatly augmenting their strength and stiffness. An assured reduction in doubler plate efficiency due to shear lag was incorporated into the design. It was also attempted to minimize shear lag concentration on the use of outrigger trusses which used to link the trussed core with the perimeter frames. The non-homogeneity of tubular structures due to opening was considered first time by Spires and Arora (1990) to analyze and design. The effects of shear lag and story shear were considered. A tubular structure was optimized as to minimize the shear lag. If column spacing increases or column stiffness decreases, the frames parallel to the lateral load start to behave as conventional rigid frames and story shear interaction occurs. As spandrel beam stiffness in these parallel frames decreases, shear lag increases, resulting in disproportionate stresses in the corner columns. Connor and Pouangare (1991) presented a very simple model for the analysis and design of framed-tube structures subjected to lateral loads. The structure is modeled as a series of stringers and shear panels. Using these concepts, a reasonable first estimate for the member sizes can be made. The model can be used directly for the analysis of structures that incorporate different materials and different properties along the height of the structure. A transfer matrix formulation has been developed.

Kristek and Bauer (1993) presented harmonic analysis that enabled the quick and convenient evaluation of stress distributions induced by lateral loads. The method idealizes the front of the building as a system of axial load carrying column members and shear carrying segments. The segment represents the flexural properties of horizontal connecting beams distributed, or smeared, over the story heights. The various stress distributions were discussed; including those corresponding to the negative shear lag. Takabatake et al. (1993a, b) presented an extended rod theory including the bending, transverse shear deformation, shear lag and torsion. It was dealt with closed-form approximate solutions for free and forced vibrations of a doubly symmetric tube structure to use in preliminary stages of the design. The cross sectional stiffness was presented in terms of extended Dirac function. The deflection, shear lag, and torsional angle of the variable tube structures with braces were given in closed-form solutions. The natural frequencies for a uniform tube structure were presented in closed form in the second part of the study. Natural frequencies for variable tube structures were formulated by means of the Galerkin method and in the closed-form approximate solutions. Second, a dynamic analysis for a variable tube structure was proposed in closed-form approximate solution. The proposed solutions can be applied to a tube structure with the discontinuous variation of stiffness due to frame members and bracings.

A simple hand-calculation method was proposed by Kwan (1994) for approximate analysis of framed tube structures with the shear lag effects taken into account. In the proposed method, independent distributions of axial displacements were used for the web and flange panels and thus the shear lag in each panel was individually allowed for. Closed-form solutions are obtained, from which the effects of various parameters on the overall structural behavior can be readily evaluated.

Singh and Nagpal (1994) presented a comprehensive study for the explanation of the origin of negative shear lag. The present paper separated column axial-force distribution in a story into two modes; one contributing to the positive shear lag and the other to the negative shear lag. The net shear lag effect can be calculated by superimposing these two different shear lag effects. An analogy between shear lag behavior of a uniform box girder and uniform framed-tube building was established by interpreting structural parameters occurring in the closed-form solution of a box girder in terms of those of the framed-tube building. Haji-Kazemi And Company (2002) analyzed tubular structures by means of stress equilibrium and compatibility conditions. The power series was used in solutions. The analogy between the shear lag behavior of a cantilever box representing a uniform framed-tube building for exact analysis of stress and displacement components of perimeter columns were presented. Rahgozar et al. (2010) developed a model that consist their stress relations and minimizing the total potential energy of the structure with respect to the lateral deflection, rotation of the plane section, and unknown coefficients of shear lag. The optimum location of a belt truss reinforcing system on tall buildings was generated. The effect of belt truss and shear core on framed tube was modeled as a concentrated moment applied at belt truss location. This moment acts in a direction opposite to rotation created by lateral loads. The axial deformation functions for flange and web of the frames were considered to be cubic and quadratic functions respectively. The shear lag was evaluated by estimating the stress of frame element (Mahjoub et al. 2011) in the tubular buildings. The proposed method assumed tube frames as a web and flange panels. Their corresponding deformation functions are written for their stress relations and principle of minimum energy was applied for their analytical solutions for lateral and vertical displacement of the structure.

2.4.2 Tubular Structures (Multiple Tubes)

Chang (1985) analyzed tube-in-tube structures using a continuum approach in which the two beams were individually modeled by a tube beam that accounts for flexural deformation, shear deformation, and shear-lag effect. A rigid system was assumed in realistic way when there were many stories because the floor slab provides a rigid connection between the two systems. The beams were assumed to have equal lateral deflections. An efficient method was presented for determining the global deflection behavior of tube-in-tube structures by using minimum potential energy principle and finite element methods. A numerical modeling technique for estimating the shear-lag behavior of framed-tube systems with multiple internal tubes was developed by Lee et al. (2000, 2001). An orthotropic box beam analogy approach was developed in which each tube was individually modeled by a box beam that accounts for the flexural and shear deformations, as well as the shear lag effects. The method idealized the tube(s)-in-tube structure as a system of equivalent multiple tubes, each composed of four equivalent orthotropic plates capable of carrying loads and shear forces. The numerical analysis developed was based on the minimum potential energy principle in conjunction with the variational approach. In the second part of the study the assumptions in relation to the patterns of strain distributions in external and internal tubes; the structural analysis was reduced to the mere solution of a single second-order linear differential equation. The proposed method, which was intended to be used as a tool for preliminary design purposes, can be applied for the analysis of framed-tube structures with single and multiple internal tubes as well as those without internal tubes.

2.4.3 Shear/Core Wall

Gupta (1984) studied reinforced concrete shear walls as the primary lateral load resisting system. These walls having low height to length ratio, often less than unity exhibited marked shear lag phenomenon. The walls behaved in three parts i.e. the symmetric flange action, anti symmetric web shear and web bending. An appropriate stiffness equation has been derived and solved for any non-liner cross sections. It was concluded that in short buildings shear lag plays a very important role. The formulation like beam which either ignores shear lag or includes, lead to an erroneous result. Kwan (1996) recognized shear lag effect in shear/core walls first time and stated that most existing theories neglect shear lag in the webs and flange, although they are acceptable for bridge decks that normally have flanges wider than webs, they may not be applicable to shear/core walls whose webs can be much wider than flanges. A parametric study using finite-element analysis was carried out. The results indicate that the shape of the longitudinal stress distribution in an individual web or flange panel is quite independent of the dimensions of the other panels. Vaez (2014) investigated the effect of core on shear lag phenomenon in tubular structures. Three different tubular structure models including model without core, model with central core and model with central core but eliminated in last 15 stories have been analyzed. A shear lag index was defined for evaluating these models. From examination of the results, the effective influence of core for improving the behavior of framed-tube structures has been concluded.

2.5 T AND I-SECTION BEAMS

Song and Scordelis (1990a) have developed harmonic shear-lag analysis using plane stress elasticity for the stresses in flanges of simple or continuous beams with I, T and box cross sections. Since harmonic analysis was used, convergence of the solution and the peak stresses in the flanges was studied in detail. The shear-lag effect upon the longitudinal distribution of the stress resultants of a continuous beam with wide flanges was studied. In the second part of the study Song and Scordelis (1990b) have been developed a simplified empirical formulas and some diagrams for the determination of the shear lag effect in simple beams under various loading with I and box cross sections based on an empirical matching of analytical results for the stresses in wide flanges given by Song 1984, using the plane stress elasticity. Whereas Qin et al. (2015) developed an analytical method for the shear lag problem of T-beams. The flange slab of a T-beam is simplified into a plane stress plate. A system of Hamilton dual equations of the flange slab using symplectic elasticity theory can be obtained. The main cause of the positive shear lag and negative shear lag phenomenon were studied. Theoretical results indicated that the method was superior for analyzing the shear lag phenomenon. The analytical solution has been validated by comparison with the beam solution and FEM solution of a cantilever T-beam subjected to a uniform load.

2.6 OTHER STRUCTURES

2.6.1 Thin Walled Structures

A spline finite member element method for thin walled structures was developed by Wang and co worker (1997, 1999 and 2000). A model outlined and extended to cover the dynamics of thin walled members with open or closed cross section, making use of Hamilton's principle. Based on the displacement variational principle, a systematic method has been developed for vibration analysis of thin walled members with arbitrary cross section called the spline finite member element method Wang (1997). The analysis takes in to account the effect of shearing strains of the middle surface of walls on the vibration which reflected the shear lag phenomenon. The spatial buckling analysis of thin-walled eccentric compressive members with arbitrary cross sections, considering warping was performed by Wang and Li (1999a). A transformed B3 spline function was used to simulate the longitudinal warping displacement field along the cross section of a thin-walled member. The method differs from Vlasov's classical theory of spatial buckling in that it takes into account the effects of shearing strains of the middle surface of walls on the buckling, which reflect the shear lag phenomenon. A displacement variational principle for buckling analysis of thin-walled eccentric compressive members with arbitrary cross section considering shear lag was applied (Wang and Li 1999b). Compared with the results from classical theory, the numerical results proposed demonstrate the versatility and accuracy of the proposed method. In Part 2 of the study, the adaptive refinement procedure will be extended to the shear lag analysis. A transformed B3-spline function presented to simulate the longitudinal warping displacement yield along the cross section of the thin-walled member [Wang and Li 2000]. The analysis takes into account the effect of shearing strains of the middle surface of walls on the buckling, which reflects the shear lag phenomenon. The fast convergence predicts the reliability of the results.

2.6.2 Composite Structures

Herrmann et al. (1984) presented composite characterization and accompanying finite element analysis for layered systems. It is concluded that the shear lag edge effect mode was a stress state that produces severe local deformations in material layers relative to the average displacement of the system. The primary dependent variables for the composite theory have been selected such that the highest derivatives appearing in the strain energy function were first order, thus requiring only continuity of the finite element approximations. Ricles and Popov (1989) investigated the behavior of composite floor beams in eccentrically braced steel frames subjected to severe cyclic loading experimentally. Web buckling in the composite links was adequately controlled using bare-steel-link design criteria to assure cyclic ductility. The test results indicated that the floor slab by itself does not provide adequate lateral bracing at the ends of the link. The shear lag in the slab's longitudinal strain was evident from measured values of floor slab strains. The mixed variational method proposed was simple and can be applied to the shear lag analysis in practical designs.

Dezi and Tarantino (1993) performed viscoelastic analysis of composite steel-concrete continuous beams with flexible shear connectors. The method proposed evaluates the stress redistributions which occur with time as a result of creep and shrinkage of the concrete part. The effects produced both by geometric and static actions are considered. An elastic law for the steel part and an integral-type creep law for the concrete part are used. A numeric algorithm is proposed performing two standard discretizations in time and along the beam axis. A more realistic model presented in this paper which also take into account the effects of shear lag a nonlinear relationship for the connectors, the effects of drying on both shrinkage and creep, and cracking.

Nairn (1997) introduced the minimum assumptions required to derive the most commonly used shear-lag equations from the exact equations of elasticity for axisymmetric stress states in transversely isotropic materials. These assumptions can now be checked to study the accuracy of shear-lag analysis on any problem. It was concluded the shear-lag method does a much worse job of predicting shear stresses and energy release rates. Furthermore, the shear-lag method does not work for low fiber volume fractions. Kim *et al.* (2010) proposed a stress function-based analysis to provide a simple and efficient approximation method of three-dimensional (3D) state of stress that exists near the free edge of bonded composite patches. In order to apply plane strain assumption in a composite patch, a linear superposition of sliced section from a bonded patch was used. In addition, to describe the load transfer mechanism from the substrate to the composite patch, a simple shear lag model was introduced. The 3D stress behavior at the free edge of the composite patch was modeled by Lekhnitskii stress functions, and the governing equations of the given composite patch were obtained by applying the principle of complementary virtual work.

2.6.3 Multi-Cellular Structures

Evans and Shanmugam (1984) presented a simplified method of analysis to predict the linear and nonlinear behavior of multi-cellular structures and to evaluate the ultimate load of such structures. The continuous plated structure was idealized as a grillage and the effects of shear lag are taken into account by using empirical coefficients. Results are compared to those obtained from a finite element analysis for a number of cellular structures to confirm the accuracy of the proposed approach in the elastic range. The Balendra and Shanmugan (1985) verifIed the grillage idealization for dynamic analysis of multi-cellular structures by experimental study carried out. Perspex material was used to construct two models of same size, one with no web openings and the other with openings. The natural frequencies and the corresponding mode shapes are determined for two different sets of boundary conditions, namely all four sides simply supported, two opposite sides simply supported, and the remaining sides free. The experimental results are found to compare well with the theoretical results using grillage idealization and finite element methods.

2.6.4 Laminated Plate Structures

Harmon and Zhangyuan (1989) studied modified layered element capable of predicting shear failures of plates and shells with different shear reinforcement ratios and support conditions with reasonable accuracy. Shear lag through the depth of a plate or shell section cannot be predicted because of the restriction that normals to the middle surface remain straight. The analysis maintains the straightness of normals to the middle surface where the test specimens may exhibit a certain amount of shear lag. A final reason was that the analysis applies all loads and support restraints to the middle surface rather than to opposing sides of the plate, which may be significant in a relatively thick plate. Zheng et al. (2001) developed a design method for pavements in the Taklamakan desert through the investigation of test roads and theoretical analysis. The function of geotextile in pavement structures was analyzed by the shear lag model, with a result that the shear bearing capacity of geotextile was proportional to its tensile modulus and the second derivative of its tensile deformation. Combining the layer-peeling method, a bearing plate method, and modulus back calculation, the modulus of resilience of each layer in the pavement structure was determined. Leung and Yang (2006) kept the basic assumption of the shear lag in all the analysis.The equations are derived and solved to obtain the distribution of longitudinal and shear stresses along the FRP plate. The energy balance equation was solved in analyzing the debonding behavior.

Ryvkin and Abodi (2007) obtained the stress field in a periodically layered composite with an embedded crack oriented in the normal direction to the layering and subjected to a tensile far-field loading based on the continuum equations of elasticity. The analysis was based on the combination of the representative cell method and the higher-order theory. The representative cell method was employed for the construction of Green's functions for the displacements jumps along the crack line. The problem of the infinite domain was reduced, in conjunction with the discrete Fourier transform; to a finite domain (representative cell) on which the Bornvon Karman type boundary conditions were applied. Comparisons with the predictions obtained from the shear lag theory were presented. Jiang and Peters (2008) derived a shear-lag model for unidirectional multilayered structures whose constituents vary throughout the cross section through the extension of an existing optimal shear-lag model suitable for two-dimensional planar structures. Solution algorithms for a variety of boundary conditions were discussed. Numerical predictions for a single-fiber composite and a unidirectional laminated composite were presented.

Shi (2004) is presented a numerical formulation of the mixed-mode fracture in concrete based on the extended fictitious crack model using maximum principal stress criterion. The main feature of this study was that normal and tangential tractions are applied directly to the crack surface, following specific tension-softening and shear-transfer laws. The first was the single-notched shear beam test by Arrea and Ingraffea (1982), and the second was the scale-model test of gravity dams by Carpinteri et al (1992). Reducing the shear strength to null the mode-I condition was then obtained, and the structural response becomes much more brittle. Imposing a larger shear lag in a practically identical response curve of the mode-I condition was obtained up to a point beyond the peak load. As the shear transfer takes place in the post peak region, the rapid decrease of the load is interrupted and the loading rebounds slightly as the structural deformation progresses further before the final failure of the beam.

2.7 CONNECTIONS

Various connections related slandered steel sections incorporated with shear lag effect in transmission of axial stresses to the gusset plate or directly in to other sections. The IS 800 (2007) also takes care about shear lag in connection which is included in clauses: 1.3.88; 6.3.3 and 8.2.1.5. The literature related the shear lag phenomenon in connections has been reported as follow.

2.7.1 Steel Section Connections (Bolted Connections and Welded Connections)

(i) Bolted Connections: Kulak and Wu (1997) investigated angle sections connections in brief. On the basis of the test of 24 specimens of single and double angle tension members that use bolted end connections; it was concluded that inappropriate connection of both legs of angle sections reflected a shear lag in such arrangement. The experimental works were supplemented by analytical evaluations of the connections and design recommendations were made that differ in some significant ways from design rules currently used. Rogers and Hancock (2000) also overviewed the design equations that were used for the prediction of bolted-connection capacity. The nominal cross-section tension capacity of a member that was not subject to shear lag and fails by material yielding of the gross cross section was formulated. It was closely observed the failure modes of bolted connections exhibiting tearing of the sheet material may belong to the bearing mode of failure. Localized tearing may occur with extreme out of plane deformations of the sheet caused by either curling at the end of the specimen, or as a result of the increased deformation capacity intrinsic to mild sheet steels. Orbison et al. (2002) performs load test to analyze the shear lag in tension members consisting of single and double angles, single channels, and similar sections, frequently used for lateral bracing and as truss elements. Such members will normally have eccentric connections, and it is often permitted by current design specifications, to neglect this eccentricity in the design of the member. Rupture load capacity of the net section was observed to be significantly reduced with moderate connection eccentricity, and a net section efficiency factor was developed and proposed as a replacement for the current shear lag factor in determining the effective net area of a tension member.

Paula et al. (2008) carried out tests on cold-formed steel angles fastened with bolts and under tension. The shear-lag phenomenon reduces net-section capacity. This reduction was computed through the reduction coefficient which is a function of two parameters: length of the connection and distance of the shear plane to the centroid of the cross-section. Prabha et al. (2011) dealt with the shear lag phenomenon in cold formed angles under tension, which are connected on one leg. A new expression for shear lag factor which represents the net section reduction coefficient has been suggested based on the regression analysis of experimental results reported. The experimental test parameters considered are number of bolts, pitch and shear lag distances and ratio of connected leg length to unconnected leg length. Teh and Gilbert (2013a, b) examined the accuracy of equations specified by the North American and Australasian steel structures codes for determining the net section tension capacity of a cold formed steel angle brace bolted at one leg. The modification to the equation derived for channel braces bolted at the web. A design equation was proposed for determining the net section tension capacity of a cold-formed steel angle brace bolted at one leg. The proposed equation is demonstrated, through laboratory tests. In the second part, net section tension capacity of a channel brace was determined likewise as in the first part. It points out that there were three distinct factors affecting the net section efficiency of a cold-formed steel channel brace bolted at the web. These factors include (1) the in-plane shear lag associated with stress concentration around a bolt hole that is also present in flat sheets, (2) the out-of-plane shear lag that is also present in an I-section bolted at the flanges only, and (3) the bending moment arising from the connection eccentricity with respect to the neutral axis.

Teh and Yazici (2013) examined the ‘‘three factors’’ approach previously presented by the senior author for determining the net section efficiency of a bolted cold-formed steel open profile. One objective was to ascertain that the net section efficiency governed by three factors: in-plane shear lag; out-of-plane shear lag and the bending moment arising from the connection eccentricity with respect to the neutral axis. 55 single and back-to-back channel braces bolted at the web including those connected with one row of bolts perpendicular to the axial load were tested. The test results affirmed the three factors approach, and it was found that the back-to-back channel braces were affected by local bending even though the connection eccentricity was nominally zero.

(ii) Welded Connections: Bauer and Benaddi (2003) described shear lag as a phenomenon that creates a loss in resistance in a tension member connected through only part of its cross-section. Parameters that influence the shear lag phenomenon are many and difficult to assess wiz: type and size of cross-section, type of connection, length of welds, length of member, joint eccentricities. Yield and ultimate loads were compared with the values calculated using the design guidelines of the Canadian Standard, which were reviewed in the article.

Easterling and Giroux (2003) examines shear lag in steel tension members. The purpose of this investigation was to review the shear lag provisions for welded tension members relative to those for bolted members, and to make recommendations for pertinent specification changes. The factors influencing the test efficiency of connections failing through a net section: the net section area, a geometrical efficiency factor, a bearing factor, a shear lag factor, and a ductility factor. Zhu et al. (2009) presented a study of the shear lag effects on the behavior and strength of welded steel single angle tension members. The test parameters included long and short leg connections, balanced and unbalanced weld arrangements and longitudinal fillet weld lengths. It can be observed from the test results that both the ultimate loads sustained by the short leg connected angles and the ductility of all the angle specimens were greater when the balanced weld arrangement was used in the connections than when the specimens were connected using the unbalanced welded arrangement. The test results were compared with finite element analyses. Fang et al. (2013) presented an experimental investigation about the shear lag effects of tension steel members with welded connections. Twelve on single angles and eight on single tees were tested. Various weld arrangements were considered as test parameters for the angle specimens who were connected by their short legs. The test results showed that, the test efficiency, which was defined as the ratio of the ultimate capacity to the calculated tensile capacity of the gross section, can be increased when a balanced weld arrangement instead of an unbalanced one was employed for the angle specimens. For the tee specimens, the test efficiency was not sensitive to the weld length. The design specifications for shear lag were examined, and the test-to-predicted ratios were presented.

2.7.2 Hollow Steel Section (HSS) Connections

The available literatures on HSS connection to the gusset plate demonstrate that most of the connections are welded. Some of the available literature related to the HSS section connections considering shear lag effect are as follow.

Cheng and Kulak (2003) performed an experiment and numerically analyzed the shear lag effect in round hollow structural section (HSS) tension members that were welded to gusset plates. The specimens failed by fracture of the tube somewhere between the two gusset plates. It was concluded that shear lag did not significantly affected the ultimate strength of the slotted tube connection, even with a small weld length. Also, restraint provided by the gusset plate at the slotted end effectively increased the load-carrying capacity of the tube as compared to the unrestrained portion of the member. Willibald et al. (2004) demonstrated by experiment and numerical analysis; that hollow structural sections (HSS) under tension loading with gusset plate connections can be susceptible to shear lag failure since only a part of the tube cross-section was connected to the plate. The shear lag reduced the fracture capacity of steel tension members if some, but not all, elements of the cross section transfer force at the connection [Dowswell 2005]. The AISC Load and resistance factor design specification for steel hollow structural sections has equations to account for shear lag in slotted HSS connections. This study compared the AISC equations to the available data from previous tests and finite element models to determine whether the equations were valid for rectangular HSS members. Linga et al. (2007) reported that shear lag failure was a premature tensile failure that may occur in gusset-plate welded connections in structural steel hollow sections (SSHS). A new procedure for determining the capacity of slotted end hollow structural section connections in tension which was shown to be a significant improvement over current international design provisions was presented by Saucedo and Packer (2009). To check more than one limit-state to determine the capacity of a slotted end HSS connection did not reflect the real behavior of such connections. An examination of the design methods found in current design provisions to prevent failures, against the data from a parametric FE analysis of slotted end CHS connections, has revealed the inaccuracy of the models used to account for shear lag. The analysis undertaken provided a clear explanation of the behavior of these connection types, where a gradual increase in the connection efficiency takes place, combined with a transition through several failure mechanisms

2.7.3 Grouting and Filled Steel Columns

Moon II et al. (2002) tested ductility of connections in grouting. The failures of connections stemmed from a loss of grout confinement followed by the crushing of the bottom face sheet of the composite due to localized bearing of the shear studs. A failure of grouting observed due to the severe shear lag, which overstressed the MMC FRP deck.

Ricles et al. (2004) reported that composite concrete filled steel tube column-wide flange beam moment connections during testing developed significant beam yielding outside the connection, as well as in the flanges of the extended-tees during the inelastic displacement cycles. Shear yielding initiated in the steel tube within the panel zone. Strain gage readings revealed that a shear lag phenomenon developed across the beam's tension flange within the connection, where the longitudinal strain was largest in the flanges of the extended tees.