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ABSTRACT

This Thesis introduces an analytical technique for the design of Fractional order controller for Non-Monotonic system, Time delay systems and the process plants. The design is carried out in terms of frequency domain specifications i.e., phase margin, gain cross over frequency, robustness constraint, sensitivity function and complementary sensitivity function. Fractional order controllers are inherited with the dynamical behavior represented by differential equations with derivatives whose order is of non-integer. This thesis addresses such type of controllers particularly for single input single output systems. The aim of fractional order controllers is to enhance the control quality of given single input single output systems.

The first part of the thesis demonstrates the design and analysis of the existing fractional order controllers such as Fractional order Lead-Lag controller and three generations of CRONE controllers with the help of Illustrative examples.

The first objective of the thesis has proposed an improved technique for the analytical design of the fractional order Proportional Integral (FOPI) controller is proposed for non-monotonic phase systems. The controller design is carried out in terms of gain crossover frequency and phase margin specification in frequency domain. To justify the superiority of the proposed FOPI controller, an integer order proportional integral and derivative controller (IOPID) is designed for same set of tuning constraints. From the simulation results, it is found that FOPI controller provides minimum phase margin in the desired bandwidth, which assures better response of the system to step input. The proposed method redefines the concept of phase margin for non-monotonically decreasing phase systems. Introduction of the fractional power parameters offers some advantage to ensure robust performance. For comparison, the proposed non-monotonic method and the conventional monotonic method are chosen for the same set of imposed tuning constraints for both FOPI and IOPID controllers.

The second objective of the thesis is to design A fractional-order [proportional derivative] (FO-[PD]) controller is proposed for non monotonic phase process. Emphasizing mainly

on the open loop frequency response parameters of the control systems, a practical and systematic tuning procedure has been developed for the proposed FO-[PD] controller synthesis. The superiority of the proposed controller technique (Non-Monotonic compensation) is shown by comparing with the controller which is tuned by conventional technique (Conventional Monotonic compensation) with the same number of design parameters and the same specifications. Side-to-side fair comparisons of the three controllers (i.e., FO-PD (Non-Monotonic compensation) and FO-PD (Conventional Monotonic compensation) via both simulation and experimental tests have revealed some interesting facts: 1) FO-PD (Non-Monotonic compensation) controller achieve the desired gain cross over frequency and phase margin, moreover it assures the flat-phase around the gain cross over frequency which satisfies the iso-damping property of the system. 2) FO-PD (Conventional Monotonic compensation) controller designed may not always be stabilizing to achieve flat-phase specification.

The third objective of the thesis is devoted to design of Fractional Order PID controller, in which the order of the integral is denoted by λ and order of the derivative is by μ with the values of non integer. The objective is to take the advantage of the additional parameters and fulfill the additional specifications of the control systems. Further Fractional Order PID assures the robustness performances while system faces the gain type uncertainty and external disturbances. Here an analytical tuning technique is used for tuning of Fractional Order PID controller by which we can fulfill five different design specifications such as gain crossover frequency, phase margin, iso-damping property, sensitivity function and complementary sensitivity function of the control system. Simulation results prove that the frequency domain specifications are achieves precisely.

The fourth objective of the thesis is to design a fractional order phase shaper by using the idea of “flat phase”, i.e., the phase derivative with respect to the frequency is zero at a frequency (Gain cross-over frequency) so that the closed-loop system is robust to gain variations and the step responses exhibit an iso-damping property i.e., maintains constant overshoot with change in gain. This implies enhanced parametric robustness to variation in gain. The third objective is to propose a phase shaping idea to make the slope of the open loop phase of the system is zero with respect to frequency. The phase shaper, based

on the idea of FOC (Fractional Order Calculus), is actually a fractional order integrator or differentiator. Here to design the phase shaper the bode integral formula has been used. The phase shaper is employed with the plant controlled by the classical PID controller. Simulation examples illustrate the effectiveness and the simplicity of the proposed method with an iso-damping property.

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LIST OF SYMBOLS AND ABBREVIATIONS

Symbols

a, b, c, d, e, f, g	Coefficients
C(s)	Controller Transfer function
I/P	Current to pressure converter
I/V	Current to voltage converter
θ/θ_d	Dead time
L	Delay time
$Q_v(s)$	Denominator s polynomial
$D_c(s)$	Denominator s polynomial of Process transfer function
$D_p(s)$	Denominator s polynomial of Process transfer function
τ_d	Derivative time
T_d, T_D	Derivative time
λ_i	Eigen values
y	Error in decibels
$\Gamma(x)$	Euler's Gamma function
η	Fractional power
β, μ	Fractional powers of differentiator
α, λ	Fractional powers of integrator
ω_c, ω_{cg}	Gain crossover frequency
h_s	Height of liquid level in the tank at steady state
τ_i	Integral Time
T_i, T_I	Integral time

J	ISE index
p_i	i^{th} pole
z_i	i^{th} zero
S	Laplace domain variable
S	Laplace operator
ω_b, ω_L	Lower frequency
$P_\mu(s)$	Numerator s polynomial
$N_c(s)$	Numerator s polynomial of Process transfer function
$N_p(s)$	Numerator s polynomial of Process transfer Function
$F_{\text{out},s}$	Outlet flow rate at steady state
$c(t)$	Output response
K_D and N_D	Parameters of rational function approximation of Fractional differentiator
∂	Partial differential operator
ω_p	Phase crossover frequency
ϕ_m	Phase margin
ϕ_u	Worst Case Phase margin
π	Pi
psi	Pounds per square inch
K	Process gain
$G_p(s)$	Process transfer function
K_c	Proportional Gain
T(s)	Sensitivity function

$r(t)$	Step input signal
K_p	System gain
τ	Time Constant
T	Time constant
ω_u	Worst Case Gain Cross over frequency
K_u	Ultimate gain
T_u	Ultimate period
ω_h, ω_H	Upper frequency
V/I	Voltage to current converter
z	Z domain variable

Abbreviations

ADC	Analog to digital converter
BFO	Bacterial foraging optimization
cm	Centimeter
CFE	Continued fraction expansions
CRONE	Control Robuste d'Ordre Non Entier
dec	Decade
dB	Decibels
DAC	Digital to analog converter
DCS	Distributed control systems
FIR	Finite Impulse Response
FOPDT	First Order plus dead time
FOTD	First order time delay

FC	Fractional Calculus
FOC	Fractional order control
F-MIGO	Fractional Ms constrained integral gain Optimization
FO-DOB	Fractional order disturbance observer
FO[PD]	Fractional order proportional derivative
FOPD	Fractional order proportional derivative
PD^β	Fractional order proportional derivative
FO[PI]	Fractional order proportional integral
FOPI	Fractional order proportional integral
$PI^\lambda, (PI)^\lambda$	Fractional Order Proportional Integral
$PD^\mu, (PD)^\mu$	Fractional Order Proportional derivative
FOPID	Fractional order proportional integral derivative
$PI^\lambda D^\mu$	Fractional Order Proportional integral Derivative
$PI^\alpha - PD^\beta$	Fractional order proportional integral Fractional order proportional derivative
FOS	Fractional order system
FOTF	Fractional order transfer function
FPP	Fractional power pole
FPZ	Fractional power zero
GA	Genetic Algorithm
IOPD	Integer order proportional derivative
IOPID	Integer order proportional integral

	derivative
IAE	Integral absolute error
ISE	Integral square error
ITAE	Integral time absolute error
IMC	Internal model control
LSE	Least Square error
LTI	Linear time invariant
MIMO	Multi input multi output
NIOPTD	Non integer order plus time delay
OFOPI	Optimum fractional proportional integral Controller
PSO	Particle swarm optimization
PC	Personal computer
Psi	Pounds per square inch
PHWR	Pressurized Heavy water Reactor
PLC	Programmable logic controller
P	Proportional
PD	Proportional Derivative
PI	Proportional integral
PID	Proportional integral Derivative
QFT	Quantitative feedback theory
rad	Radians
ROM	Run of mine
SOPDT	Second order plus time delay
Sec	Seconds

SISO	Single input single output
TID	Tilted proportional integral controller
UAV	Unmanned aerial vehicle
ZN	Ziegler Nichols