

# **Finite Deformation Analysis of Crack Tip Fields in Plastically Compressible Hardening-Softening- Hardening Solids**

**Thesis Submitted in partial fulfillment  
for The Award of The Degree  
Doctor of Philosophy**

**By**

**SHUSHANT SINGH**

Supervisor

**Dr. Debashis Khan**

Associate Professor

Co-Supervisor

**Prof. S. K. Panda**

Professor



DEPARTMENT OF MECHANICAL ENGINEERING  
INDIAN INSTITUTE OF TECHNOLOGY  
(BANARAS HINDU UNIVERSITY)  
VARANASI – 221 005

**Roll No. 13131011**

**2018**

## Summary and Conclusion

---

### 7.1 Introduction

The thesis comprises constitutive relations of the plastically compressible rate dependent elastic-viscoplastic solids with hardening and hardening-softening-hardening kinds of flow rule and various numerical investigations which essentially deal with phenomena or problems within the framework of nonlinear fracture mechanics. These constitutive relations are applicable for characterizing relatively new materials like toughened structural polymers, metallic foams, plastics, transformation toughened ceramics etc. Contrary to the analysis of hardening materials, the constitutive equations of these newer materials contain material softening. The physical mechanism responsible for triggering localization often is a consequence of this softening process. To the best of the author's knowledge there is no such study exists in open literature so far which explores the combined effects of pressure sensitive yielding and material softening in the hardness function on the near crack tip finite deformation and stress fields.

### 7.2 Numerical Investigations

In the present study finite element calculations of crack tip fields have been carried out for solids characterized by an elastic-viscoplastic constitutive relation with trilinear hardening-softening-hardening and hardening solids with plastic normality as well as non-normality flow rule under monotonic and cyclic loadings. Results were obtained for plastically compressible and for comparison purposes some results were also obtained for plastically incompressible state. In the calculations here, the material parameters were taken to be spatially uniform. A number of interesting observations emerged from a careful study of the

results may be useful as guidelines for future research. The following conclusions have been drawn.

- Plastic compressibility is found to give an increased crack opening displacement for a given value of the applied loading.
- Plastic compressibility coupled with a softening or softening-hardening material response leads to more or less abrupt jumps in the CTOD curve.
- The plastic zone shape and size are found to depend on the plastic compressibility.
- The near crack tip stress and deformation fields depend sensitively on whether or not material softening occurs.
- The combination of softening or softening - hardening material response and plastic compressibility leads to major deviations in the near crack tip stress and deformation fields from those that prevail for a hardening material.
- Plastic compressibility results in reduced near crack tip stress levels. In particular, in a reduced hydrostatic stress level. This has implications for various failure mechanisms such as micro-cracking and porosity evolution.
- Plastic compressibility coupled with a softening or softening - hardening material response leads to localized deformation in front of the initial crack tip. This, in turn, affects the shape of the blunted crack tip, and can give rise to a somewhat polygonal crack tip shape in monotonic loading.
- Cyclic CTOD trajectories appear to be dependent on  $K_{max}$ ,  $K_{min}$ ,  $R$ ,  $\alpha$  and  $N$ . When the material is plastically compressible and  $\Delta K$  is rising, CTOD increases significantly and convergence of the CTOD - trajectories to the steady-state and

self-similar loops is delayed (with  $R = 0$ ). However, for positive  $R$  values, the above convergence is very fast. Ratcheting of CTOD - trajectories diminishes with rising  $N$ .

- For both hardening and hardening-softening-hardening materials, yielding occurs during both loading and unloading phases and resharping of the crack tip during the unloading phase of the loading cycle is very significant.
- In fatigue loading and for the total number of load cycles considered here, most of the crack growth occurs in the first cycle, and then very slowly and finally becomes relatively constant. The crack tip advancement is more when the material is plastically incompressible; however, the growth is very less when the material is plastically compressible and in presence of plastic compressibility and material softening the crack growth is further reduced. Crack growth is also very much sensitive to the overload cycle
- Computed crack growth by cyclic plastic blunting-resharping agreed with the real-world trends, such as acceleration with  $\Delta K$  and therefore, the present fatigue crack growth data reveals that Paris power law relationship is also followed here.
- The introduction of plastic non-normality flow rule in the constitutive equation and its subsequent implementation in the numerical method makes the crack tip deformation as well as the field quantities somewhat different as compared to those results when the constitutive equation exhibits plastic normality.

### **7.3 Recommended Future Work**

The constitutive equations of the relatively new materials and their analyses described in this thesis are attractive due to its applicability in various types of geometries, material models

and applied loads. Only limited investigations carried out so far have been reported here and results presented should help those continuing research in fracture mechanics. By pursuing some of the recommendations listed below the future researchers will help towards solving a vast number of theoretical and practical engineering problems.

- Crack tip blunting and studies of the field quantities for the present constitutive equations with mode I loading has not been investigated experimentally so far. Extension of the present analysis to experimentation and testing is a potential field of future work.
- The constitutive formulation used in the current study is isotropic while the materials considered for analysis are expected to be anisotropic. Therefore, the present research needs to be extended by conducting more detailed numerical experiments in order to have better realistic predictive capabilities.
- Numerical computations carried out in this thesis work are for only plain strain i.e. two-dimensional problems. Extensions for three-dimensional applications are still needed. The extension should also include the effects due to various other loads like thermal strains, initial strains, body forces and dynamic loading etc.
- The results under fatigue loading suggest that modifying the code for many load cycles by introducing remeshing technique at several stages of the plastic deformation may be an interesting work.
- It is recognized that the present numerical calculations may be mesh sensitive when shear band propagation occurs. Studies with different mesh layouts and element technologies during shear band propagation could be a future work.

- Current developments in computational mechanics point towards cost effectiveness, especially for large problems. These range from hybrid formulation of the FEM to the replacement of FEM by that based on boundary element method (BEM). Therefore, work in this area is also suggested.

