Finite Deformation Analysis of Crack Tip Fields in Plastically Compressible Hardening-Softening-Hardening Solids

Thesis Submitted in partial fulfillment
for The Award of The Degree

Doctor of Philosophy

By

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It is certified that the work contained in the thesis titled **"Finite Deformation Analysis of Crack Tip Fields in Plastically Compressible Hardening-Softening-Hardening Solids"** by **Shushant Singh** has been carried out under our supervision and that this work has not been submitted elsewhere for a degree.

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I, **Shushant Singh**, certify that the work embodied in this thesis is my own bonafide work and carried out by me under the supervisions of **Dr. Debashis Khan** and **Prof. S.K. Panda** from July 2013 to July 2018, at the Department of Mechanical Engineering, Indian Institute of Technology (BHU) Varanasi. The matter embodied in this thesis has not been submitted for the award of any other degree/diploma. I declare that I have faithfully acknowledged and given credits to the research workers wherever their works have been cited in my work in this thesis. I further declare that I have not wilfully copied any other's work, paragraphs, text, data, results, etc., reported in journals, books, magazines, reports, dissertations, theses, etc., or available at websites and have not included them in this thesis and have not cited as my own work.

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DEDICATION

This thesis is dedicated to

Lord ***Sri Krishna***

ॐ पूर्णमदः पूर्णमिदं पूर्णात्पुर्णमुदच्यते|

पूर्णस्य पूर्णमादाय पूर्णमेवावशिष्यते ||

Sri Isopanishad

The Personality of Godhead is perfect and complete. And because He is completely perfect, all emanations from Him, such as this phenomenal world, are perfectly equipped as a complete whole. Whatever is produced of the complete whole is also complete by itself. And because He is the Complete Whole, even though so many complete units emanate from Him, He remains the complete balance.

ACKNOWLEDGEMENTS

At the outset, let me pay my obeisance to Lord Krishna for catapulting me to this level of academic pursuit. Thereafter, let me express my sincere gratitude to my respected supervisors - Dr. Debashis Khan and Prof. S.K. Panda for their matured guidance, invaluable encouragement, moral support, friendship, providing enough space to work over the past five years. I owe them much for all of their superior guidance over the years.

I wish to acknowledge my sincere gratefulness to Prof. S.K. Sinha, Head, Department of Mechanical Engineering, Indian Institute of Technology (BHU), Varanasi, the previous heads Prof. A.K. Jha, Prof. A.K. Agrawal, and Prof. V.P. Singh for their kind gesture and extending all sorts of facilities in the department to pursue this kind of research work.

I would like to express my grateful thanks to my RPEC members Prof. Rajesh Kumar and Prof. P. Maiti for being part of my Ph.D. Committee and for being kind enough to make evaluation of my work-progress, extend meaningful comments and fruitful suggestions from time to time.

I am deeply indebted to Prof. Sandeep Kumar, for his help, spiritual values, precious advice and for the patience shown in our long and useful discussions. I would like to extend special thanks Prof. A.P. Harsha for his invaluable support, insightful conversations and continuous encouragement. My sincere thanks also go to Ministry of Human Resource Development, Govt. of India, for providing me financial assistance during my research tenure.

I am also thankful to Convener DPGC, Prof. Rajesh Kumar, previous Conveners Prof. A.P. Harsha, Prof. K.S. Tripathi, Prof. Sandeep Kumar and all respected DPGC members for their kind support.

I express my sincere thanks to the technical and office staff, especially Mr. S.P. Singh, Mr. Dinesh Kumar, Mr. J.K. Sinha, Mr. Akash Mishra, who helped me in various ways during this research work. I would like to thank all other people who have provided a truly conductive environment to successfully carry out this research. I shall be failing in my duty if I do not put on record my sincere appreciation and thankfulness to my seniors, lab mates and friends; Mr. Ambuj Sharma, Mr. Bharat S. Patel, Mr. Khemraj Sahu, Mr. Avinash Ravi Raja, Mr. Pushkar Jha, Mr. Rajeev Nayan Gupta, Mr. Manvandra K. Singh, Mr Parshant Kumar, Mr. Ankit Sharma, Mr. Sunil Kumar, Mr. Harish Babu, Mr. Gaurav Gugliyani, Mr. Arun Kumar, Mr. Ashish Srivastava, Mr. Ravindra Prasad, Mr. Vivek Gupta, Mrs. Prerna Mishra, Mr.Ashish Pareta. Mr. Hemant Nutiyal, Mr. Sooraj rawat, Mr. Homendra Kumar, Mr. Hemant Chaudhary, Mr. Amod Kashyap, Mr. Yash Mittal. My special thanks go to Mrs. Amrita Khan, for her help, spiritual values, and for providing me a family like atmosphere.

I have no words to express my indebtedness and gratitude to my parents, Mrs. Asha and Mr. Satendra Kumar for showering their affectionate love and all possible support during my research-endeavour. They have been a guiding force all through my life, and I could always try to measure up to their expectations. I further express my sincere thanks to my elder brothers, Mr. Sachin Singh and Dr. Prashant Singh for their constant motivation and support during all these periods and all through my life. Words at my command are not sufficient enough to express my feelings especially for my Sister Dolly Singh and for my sweet nieces Ananya, Yashi, Kashvi, nephew Subh their smiling face always rejuvenates me.

I would fail in my duty if I fail to express my deep sense of gratefulness and thankfulness to my love Chetna, for being a constant source of love, energy and ecstasy to make each day of my life brimmed with a wonderfully pleasant experience.

To all these people and to those unmentioned, my heartfelt thanks.

Shushant Singh

ABSTRACT

Considering the potential applications of relatively new materials like toughened structural polymers, metallic foams, plastics, transformation toughened ceramics, vertically aligned carbon nanotubes (VACNTs) etc and their limited exploration till now, it appears that there is a need to investigate more about the behavior of such materials under a wide range of loadings. Even though the effect of plastic dilatancy is neglected in classical plasticity theory, the above materials exhibit plastic volume changes and/or pressure-sensitive flow strength. In a recent study, it has been observed that the deformation of the entangled arrays of carbon nanotubes or VACNTs follow elastic-viscoplastic constitutive relation which incorporates plastic compressibility, plastic non-normality and a hardening-softening-hardening type hardness function. These VACNTs have prospective uses in a variety of applications like viscoelastic energy absorption, compliant thermal interfaces, biomimetic dry adhesives etc and hence it is useful to develop a predictive framework for the mechanical behavior of VACNTs under a wide range of loadings. In this thesis work, finite element finite deformation quasistatic mode I plane strain small scale yielding analysis of crack tip blunting and near crack tip fields was carried out for plastically compressible solids exhibiting a variety of uniaxial stress – strain responses. In particular solids with hardening-softening-hardening responses as can occur for foams and VACNTs have been considered. The novelty of this model includes unique characteristics as mentioned earlier like the hardening-softening-hardening material response, strain rate-dependence, and plastically compressible solids with plastic non-normality.

As for localization studies it needs a finite strain description, using FORTRAN a finite element finite deformation code has been developed in this work for the simulation

purpose. A convected coordinate Lagrangian formulation of the field equations was used. Quasistatic deformation conditions have been assumed and the equilibrium equations were expressed through the virtual work principle. The plane strain calculations were carried out for a semicircular region with a blunt notch. Quadrilateral elements each comprised of four crossed constant strain triangular elements have been used for mesh generation. Such elements with a proper aspect ratio and orientation are extensively used to replicate localized deformation pattern at finite strains. The initial part of the investigation refers to crack tip blunting and field quantities analyses under monotonic load while in the next part studies were conducted for fatigue loading to find the key results of crack tip blunting and fields. Even though most of the results presented are for plastic normality condition, however for comparison purpose some results have also been illustrated for constitutive relations exhibit plastic non-normality. While to simulate fatigue crack growth by means of finite elements, different techniques have been proposed over the years, however, in this work the crack growth modelling strategy employed was crack tip blunting/ resharpening mechanism where it is assumed that the crack tip blunts during the maximum load and resharpening of the crack tip takes place under minimum load. The simulations attempt to explain some of the salient features, like crack tip opening displacement, crack tip advancement, plastic zone shape and size, equivalent plastic strain distribution, equivalent stress, and distribution of hydrostatic stress at near crack region. The influences of plastic compressibility, material softening, cyclic stress intensity factor range, load ratio, number of fatigue load cycles on the near tip deformation and stress-strain fields were studied.

Numerical results obtained from the quasistatic mode I plane strain analysis demonstrate that plastic compressibility is found to give an increased crack opening displacement for a given value of the applied loading. The plastic zone shape and size are found to depend

on the plastic compressibility. Even though, material softening does not have a significant effect on the plastic zone size and shape, however, the near crack tip stress and deformation fields depend sensitively on whether or not material softening occurs. The combination of softening or softening-hardening material response and plastic compressibility leads to major deviation in the near crack tip stress and deformation fields from those that prevail for a hardening material. Plastic compressibility coupled with a softening or softening hardening material response leads to localized deformation in front of the initial crack tip, which in turn affects the shape of the blunted crack tip. The present numerical calculations show that the convergence of the cyclic trajectories of CTOD to stable self similar loops and plastic crack growth depend significantly on cyclic stress intensity factor range, load ratio, number of fatigue load cycles.

Keywords: Finite Deformation, Mode I crack, Plasticity, Compressible Solids, Material Softening, Monotonic Loading, Fatigue Loading, Finite Element Method.

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LIST OF ABBREVIATIONS AND SYMBOLS

FEM	Finite Element Method
BEM	Boundary Element Method
LEFM	Elastic Fracture Mechanic
EPFM	Elastic Plastic Fracture Mechanics
CTOD	Crack Tip Opening Displacement
VACNT	Vertically Aligned Carbon Nano Tube
PFM	Probabilistic fracture mechanics
SSY	Small Scale Yielding
HRR	Hutchinson, Rice and Rosengren
X-FEM	Extended Finite Element Method
CZM	Cohesive Zone Model
FDM	Finite Difference Method
σ_{ij}	Stress Tensor
$\sigma_{\mathrm{xx}}, \sigma_{\mathrm{yy}}, \sigma_{\mathrm{zz}}$	Stress Components in x, y and z direction
ν	Poisson ratio
μ	Shear modulus
K	Stress Intensity Factor
r_{t}	Radius
$u_{\mathrm{xx}}, u_{\mathrm{yy}}, u_{\mathrm{zz}}$	Displacement in x, y and z direction
K_{IC}	Mode I Critical Stress Intensity Factor
U	Potential Energy
V	Volume of the Body
W	Strain Energy Density
S_{t}	Part of the body Subjected to Traction
E	Young’s Modulus
G	Energy Release Rate

Γ	Contour Enclosing the Crack Tip
n	Unit Outward Normal on Γ
$\bar{T}$	Surface Traction
T_{i}	Traction Vector
r_{y}	Irwin Plastic Zone Correction
d	Crack Tip Opening Displacement
a	Half Crack Length
J_{app}	Applied J integral
F_{ij}	Deformation Gradient
e_{ij}	Eulerian Strain Tensor
D_{ij}	Deformation Tensor
L_{ij}	Velocity Gradient
W_{ij}	Velocity tensor or spin tensor
ρ	Density
$\hat{\tau}_{\mathrm{ij}}$	Jaumann rate of Kirchhoff stress
$\dot{\varepsilon}_{\mathrm{p}}$	Plastic Strain Rate
σ_{e}	Effective Equivalent Stress
δ_{ij}	Kronecker Delta
p_{ij}	Deviatoric Kirchhoff Plastic Stress Tensor
L_{ijkl}	Tensor of elastic Moduli
$\dot{\varepsilon}_0$	Reference Strain Rate
m	Rate Hardening Exponent
σ_0	Reference Stress
α_{p}	Plastic Compressibility

Symbols not listed here, are defined as they appear in text.

LIST OF PUBLICATIONS

Journal Publications:

1. Khan, D., **Singh, S.**, Needleman, A. (2017). “Finite deformation analysis of crack tip fields in plastically compressible hardening-softening hardening solids”. **Acta Mechanica Sinica**. Vol 33 (1) : pp. 148-158.
2. **Singh, S.** and Khan, D. (2018) **“**Quasi-statically Growing Crack Tip Fields in Plastically Compressible Hardening and Hardening-Softening-Hardening Solids”. **International Journal of Structural Integrity.** Vol 9 (4) : pp. 1-17
3. **Singh, S.** and Khan, D. (2018) “On Fatigue Crack Growth in Plastically Compressible Hardening and Hardening-Softening-Hardening Solids using Crack-Tip Blunting”. **International Journal of Fracture.** Vol 213 (2) : pp. 139-155
4. **Singh, S**. and Khan, D., (2018) “Finite deformation analysis of crack-tip fields in plastically compressible hardening solid under cyclic loading”. **Procedia Engineering (Accepted).**

International Conference Papers:

1. **Singh, S**., Khan, D., Panda, S. K., (2014) Crack Tip Fields in Compressible Hardening Solids , Proceedings of the International Conference on Multifunctional Materials, Structures and Applications (ICMMSA), December 22-24, 2014, MNNIT, Allahabad, India. pp. 89-92
2. **Singh, S**., Khan, D., Panda, S. K., (2014) Crack Tip Blunting in Plastically Compressible Hardening-Softening-Hardening Solids , Proceedings of the

International Conference on Theoretical, Applied, Computational and Experimental Mechanics (ICTACEM), December 29-31, 2014, IIT, Kharagpur, India. ICTACEM/033

3. **Singh, S**., Khan, D., Panda, S. K., (2015) Effect of *T*- Stress on Crack Tip Fields in Plastically Compressible rate dependent elastic-viscoplastic solids, Proceedings of the International Conference on multifunctional materials for future applications (ICMFA), October 27-29, 2015, IIT (BHU), Varanasi, India. pp.76-77
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5. **Singh, S**., Khan, D., (2017) Finite deformation analysis of crack-tip fields in plastically compressible hardening solid under cyclic loading, Proceedings of the International Conference on Vibration Problems (ICOVP), November 29-02 December, 2017, IIT, Guwahati, India. ICOVP/223
6. **Singh, S**., Khan, D., (2017) On Fatigue crack growth in plastically compressible hardening solids by crack tip blunting, Proceedings of the International Conference on Theoretical, Applied, Computational and Experimental Mechanics (ICTACEM), December 28-30, 2017, IIT, Kharagpur, India. ICTACEM/462

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