CHAPTER 4

DETERMINING OPTIMAL PROMOTIONAL EXPENSE, SALES PRICE AND LOT-SIZE FOR SOME VEBLEN PRODUCTS WITH PRICE-DEPENDENT DEMAND

4.1 The Problem

The demand for products, in general, is positively influenced by promotional expenses. Research suggests that, in case of the price-dependent demand, the promotional effort may provide an opportunity to charge more of selling price without sacrificing the demand to be experienced without any promotion. Considering this aspect, this chapter focuses on determining the optimal promotional expenses as well as optimal sales price and lot-sizing policies for some Veblen products. Two different relationships between the promotional expenses and the addition in sales price have been considered, one for the niche market and the other for the mass market.

The following assumptions detail the problem considered here.

(i) There is only one retailer, who purchases goods from a single supplier and optimizes its profit independently of the supplier.

(ii) Replenishment is instantaneous.

(iii) The demand is not random but a function of price, and is deterministic in nature.

(iv) Shortages are not allowed.

(v) Promotional or advertising expenses are incurred not to influence the demand. It has a positive impact on the sales price to the customers.

The retailer purchases the product at a unit cost price of C_P from a supplier. It can sell it to its customers at a unit selling price P when it is not making any expense on product-promotion. It is the price which the retailer can minimally realise from its customers. At this selling price, a company will have an annual demand for the product as *D(P)*. The product promotion with advertising expenses (*B*) is assumed only to influence the selling price in an upward direction and not the demand as *D(P)*. Thus an expense of *B* will help in realising an additional amount of $\rho(B)$ over *P*. As a result, each unit demanded of the total demand as *D(P)* will now have a selling price as $\{P + \rho(B)\}$. Price-demand relationships considered are due to Leibenstein (1950) and as discussed in Chapter 3 and shown in Fig. 3.2 to Fig. 3.4.

Besides the price, the demand can also be influenced positively by resorting to various promotional schemes. Thus, for the same selling price, different levels of annual demand can be witnessed depending upon the amount of the advertisement expense incurred. In this case, the demand is not only sensitive to the selling price only but also to promotional expenses. In the present problem, the effect of promotional expenses is however assumed to be only in terms of the additional realizable selling price from the sale of a product and with no effect on the annual demand. It is believed that more expenses will facilitate the possibility of charging more of the sales price. Of course, the marginal utility of promotional expenses need not be the same at every level of the expenses. Kotler (2013) mentioned that this relationship is mostly concave, but can be *S*-shaped. According to him, advertising intensity helps but increase in the sales volume eventually flattens out. It has also been mentioned that nichers strategise to offer to fully meet the needs of a certain group of customers and command a premium price in the process. Niche market would not generally demand high promotional expenses.

Giri and Sharma (2014) worked to decide a manufacturer's pricing strategy for advertising cost dependent demand. They showed that the increase in unit retail price increases at a decreasing rate with the increase in the advertisement expense. Using the graph reported by them, the relationship between the additional sales price and the advertisement expenses has been worked out in the present work and is found to be expressed either as $\rho = 30\left(1 - e^{-\frac{B}{13000}}\right)$ with R^2 value of 0.91 or $\rho = 0.009184189B^{0.8}$ with R^2 value of 0.86. Based on these, general expressions, the relationships between advertisement expense and the corresponding additional realisable selling price are shown in Fig. 4.1, with type 1 describing the relationship as $\rho = \alpha\left(1 - e^{-\frac{B}{\gamma}}\right)$ and type 2 as $\rho = \beta B^{\theta}$. Fig. 4.1 depicts the same. The relationships shown in this figure need not be applicable in general for all types of products. However, they would represent a class of product types where the increase in the marginal utility (in terms of an enhanced selling price) decreases with the increase in the promotional expenses. In the case of a niche market, the mass of the population is not addressed. Therefore, a little expense will yield high utility value and saturation point will be achieved quite early as opposed to what will be witnessed in the case of a mass market. Since the marginal utility of the advertisement expense goes down with the increase in the promotional expenses, it may not make sense to incur more of promotional cost beyond a point to have a comparatively smaller gain on the revenue side and finally in total profit.

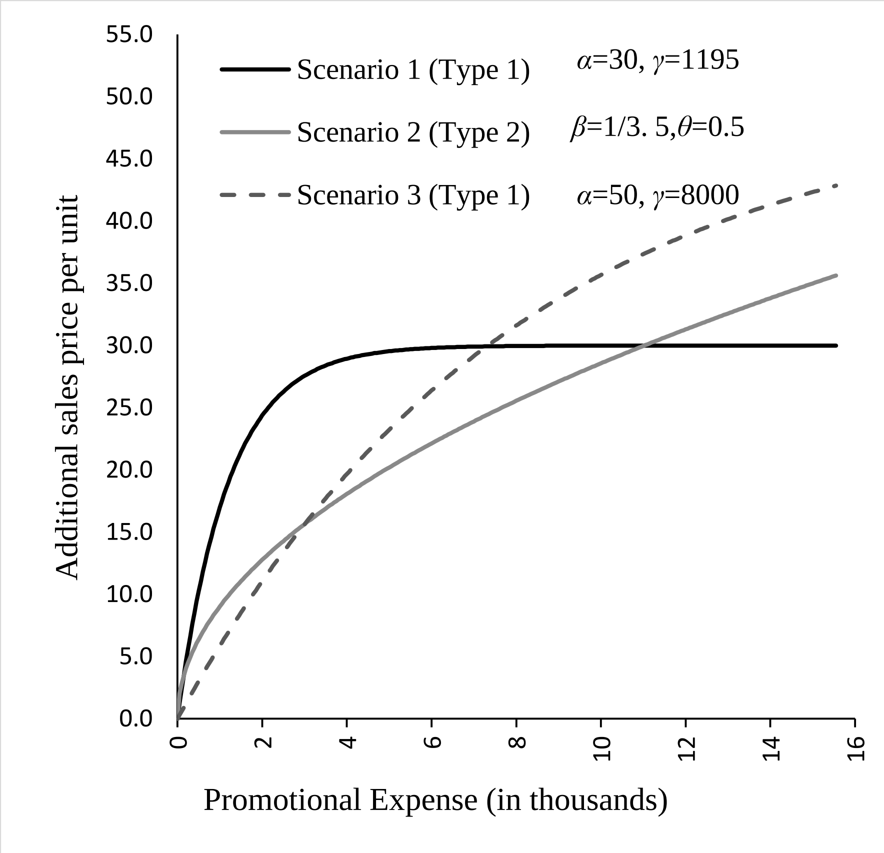

Fig. 4.1 Relationship between promotional expenses and additional sales price

Scenario 1 (Type 1) is taken as the representative relationship for the niche market and Scenario 3 (Type 2) for the mass market. In Scenario 1, the demand saturation will be achieved much earlier than the same under scenario 3. Scenario 2 (Type 2) relationship will be describing the mass market situation. In this case, the market can be influenced further by having an increased level of promotional expense. This situation would be representative of those organisations who have deep pockets and can afford to have a big budget for product promotion.

Because of the demand for the product being static, continuous and deterministic, it would be prudent to follow inventory cycle as shown in Chapter 3, Fig. 3.1.

Using the notations already defined, cost elements and revenue, etc. can be worked out as detailed below.

Annual Revenue: $R \qquad = \{P + \rho(B)\}D(P) \qquad (4.1)$

Cycle time: $T = Q/D(P)$ (4.2)

Annual ordering cost: $O = A/T$ (4.3)

Annual purchasing cost: $C = C_P D(P)$ (4.4)

Annual holding cost: $H = \frac{h_r C_P\, Q}{2}$ (4.5)

Thus, using the expression given in (4.1) to (4.5), the annual profit (T_P) can be expressed as given below, while also taking the promotional expense (B) as a cost.

$$T_P = \{P + \rho(B)\}D(P) - \frac{AD(P)}{Q} - C_P D(P) - \frac{h_r C_P\, Q}{2} - B \qquad (4.6)$$

In case the product being considered is discrete in nature, Q has to be an integer. Hence analytical calculus cannot be used to find the optimal profit and values of the other decision variables. Even when the product is continuous, a closed-form solution is not possible because of the mathematical expressions representing the relationships among the decision variables. In view of this, the problem of determining optimal inventory and promotional policies is expressed in the form of a mathematical model given below that can be used to solve the problem using commercial optimisation software package such as LINGO (Schrage, 2002).

Maximize T_P

subject to

$$T_P = \{P + \rho(B)\}D(P) - \frac{AD(P)}{Q} - C_P D(P) - \frac{(h_r\, C_P\, Q)}{2} - B$$

$$T = Q/D(P)$$

$$T_P, T, B \geq 0$$

$Q \geq 0$ and integer

The above mathematical model is a non-linear integer programming problem with discrete solution space, and so it is difficult to solve computationally. Therefore, in order to solve the problem in a computationally efficient manner, heuristics based on evolutionary approaches, such as Genetic Algorithm (GA) or Teaching-Learning-Based Optimisation (TLBO) can be used. GA has been used to optimise inventory problem (Pasandideh, 2011; Mousavi et al., 2019; Saif-Eddine et al., 2019) while TLBO for production planning problem (Kadambur and Kotecha, 2015).

4.2 Promotional Expense and the Operating Parameters

For a better understanding of the problem and also for illustrating the use of the proposed model, a numerical example problem has been taken with data: A = 1000 per order, C_P = 100 per unit and h_r = 0.1 per monetary unit of stock per year. Data on considered price-demand relationships are given in Table 3.1. Mathematical expressions to describe the relationship between the additional sales price and advertisement expense are given in Table 4.1.

Table 4.1 Mathematical expressions taken to describe the relationship between ρ and B

Type of Market	Representative relationship	Scenario	Mathematical expression for ρ
Niche	Type 1	1	$\rho = 30(1 - e^{-\frac{B}{1195}})$
Mass	Type 2	2	$\rho = \frac{1}{3.5}B^{0.5}$
	Type 1	3	$\rho = 50(1 - e^{-\frac{B}{8000}})$

The problem, with the above input, is solved with the help of LINGO 17.0 software using the framework of the proposed mathematical model. Optimal selling price, cycle time, lot-size, maximum annual profit and advertisement expenses for all the three demand scenarios for the niche market (type 1 curve) and two mass markets (type 1 and type 2) are respectively shown in Tables 4.2, 4.3 and 4.4.

Table 4.2 Optimal results in case of a niche market (Scenario 1, Type 1)

Demand scenario	*P*	*B*	*ρ*	*(P+ρ)* Selling Price	*D*	*T*	*Q*	T_P
I	237.15	3388	28.24	265.38	678.48	0.54	368	105138
II	236.09	3376	28.22	264.31	671.50	0.55	366	103294
III	237.48	2735	26.96	264.44	392.82	0.71	280	59057

Table 4.3 Optimal results in the case of mass market (Scenario 2, Type 2)

Demand scenario	*P*	*B*	*ρ*	*(P+ρ)* Selling Price	*D*	*T*	*Q*	T_P
I	237.16	9393	27.69	264.85	678.41	0.54	368	98761
II	236.12	9199	27.40	263.52	671.37	0.55	366	96922
III	237.77	3137	16.00	253.77	392.09	0.71	280	54355

Table 4.4 Optimal results in case of mass market (Scenario 3, Type 1)

Demand scenario	*P*	*B*	*ρ*	*(P+ρ)* Selling Price	*D*	*T*	*Q*	T_P
I	236.86	11571	38.23	275.09	679.65	0.54	369	103739
II	235.76	11491	38.11	273.87	672.84	0.55	367	101826
III	237.41	7189	29.64	267.06	392.98	0.71	280	55658

From the results shown in Tables 4.2 to 4.4 for the two markets (shown in Fig. 4.1), it can be observed that the optimal advertisement expenses are much less for the niche market as compared to those for the two mass markets. It is so because the marginal utility for additional expenses on the promotion becomes very low in the niche market as compared to that for the mass market beyond an expense level. Even though price sensitivities with respect to promotional expense are different for the two markets, the optimal price and demand are in close proximity. Of course, the annual profits came out to be different because of the change in the optimal expense on the product promotions.

This kind of result cannot be generalized. However, the results obtained further strengthen the belief of working in the niche market if one has to spend little on product promotion.

To visualize the impact of promotional expenses on profitability etc., this example problem was once again solved without considering any aspect of promotion. The solution is shown in Table 4.5. Looking into Tables 4.2, 4.3, 4.4 and 4.5, it can be noticed that the promotional effect has a little impact on the overall sales price and on the inventory policy. However, it positively affects the annual demand as well. For example, for demand scenario III, the respective values of the sales price, the annual demand, and the lot-size as 238.27, 390.72 and 280 (Table 4.5) have changed to 264.44, 392.82 and 280 (Table 4.2), and to 253.77, 392.09 and 280 (Table 4.3) and to 267.06, 392.98 and 280 (Table 4.4). The impact on the selling price, the revenue and the annual profit for the two markets can be better visualised by Tables 4.6, 4.7 and 4.8, and also by Figures 4.2, 4.3 and 4.4 that use the data from Table 4.2 to 4.4. It can be noticed from Tables 4.6, 4.7 and 4.8 (and also from Figures 4.2, 4.3 and 4.4) that the increase in the sales price results into a slightly higher percentage increase in the revenue due to the increase in the annual demand as well. In all the cases, it was found that the product promotion has helped in increasing the profit. This finding is very useful and can be employed by the industries to increase profitability by incurring promotional expenses to its optimal level. In the case of the niche market (Table 4.6), it is observed that an increase in the sales price results in a much higher increase in the profit for all the demand scenarios. But for the mass markets (Tables 4.7 and 4.8), this was found not to be true. However, it cannot be concluded from the above observations that the percentage increase in the profit will always be more than the corresponding percentage increase in the sales price in the case of the niche market only. This can be easily understood from the results given in Tables

4.6 and 4.8, tables that are for the similar nature of the gains in the sales price with the promotional expenses (Type 1).

Table 4.5 Optimal results without the effect of promotion

Demand scenario	***P*** Selling Price	***T***	***D***	***Q***	T_P
I	238.15	0.55	673.89	367	89426
II	237.26	0.55	666.20	365	87788
III	238.27	0.72	390.72	280	51228

Table 4.6 Impact of promotional effect in case of the niche market (Scenario 1, Type 1)

Demand scenario	**Increase (due to the promotional effect) in**					
	Selling price		**Revenue**		**Profit**	
	Value	**%**	**Value**	**%**	**Value**	**%**
I	27.23	11.44	19571	12.19	15712	17.57
II	27.05	11.40	19426	12.29	15505	17.66
III	26.17	10.98	10781	11.58	7829	15.28

Table 4.7 Impact of promotional effect in case of mass market (Scenario 2, Type 2)

Demand Scenario	**Increase (due to the promotional effect) in**					
	Selling price		**Revenue**		**Profit**	
	Value	**%**	**Value**	**%**	**Value**	**%**
I	26.70	11.21	19191	11.96	9335	10.44
II	26.26	11.07	18864	11.94	9133	10.40
III	15.51	6.51	6406	6.88	3127	6.10

Table 4.8 Impact of promotional effect in case of mass market (Scenario 3, Type 1)

Demand Scenario	**Increase (due to the promotional effect) in**					
	Selling price		**Revenue**		**Profit**	
	Value	**%**	**Value**	**%**	**Value**	**%**
I	36.94	15.51	26475	16.50	14313	16.01
II	36.61	15.43	26212	16.58	14038	15.99
III	28.79	12.08	11853	12.73	4430	8.65

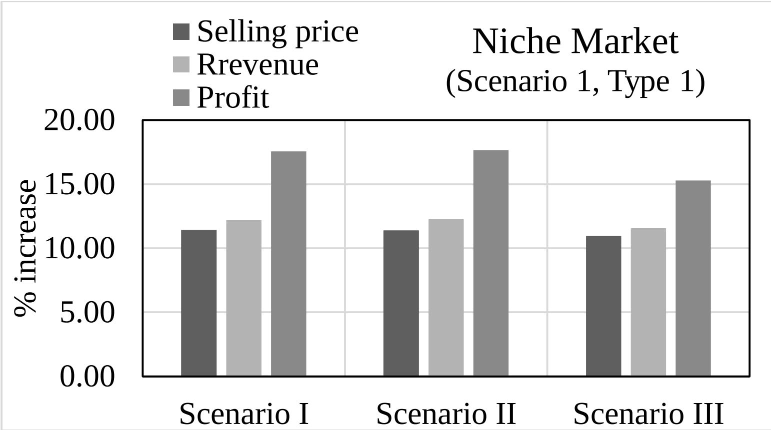

Fig. 4.2 Impact of promotion in case of the niche market (Scenario 1, Type 1)

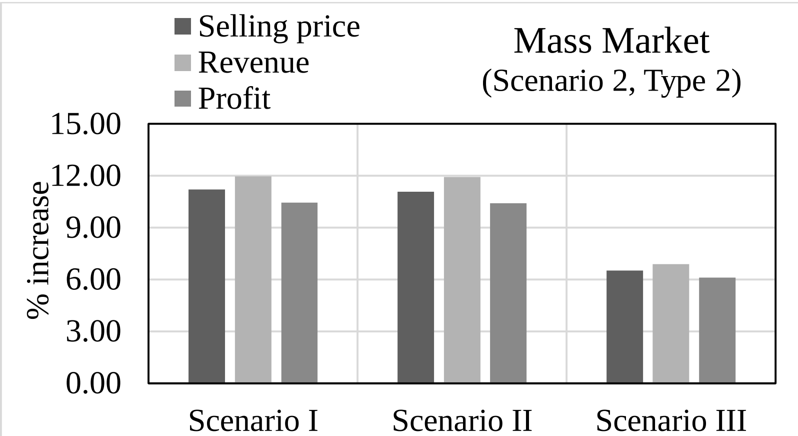

Fig. 4.3 Impact of promotion in case of a mass market (Scenario 2, Type 2)

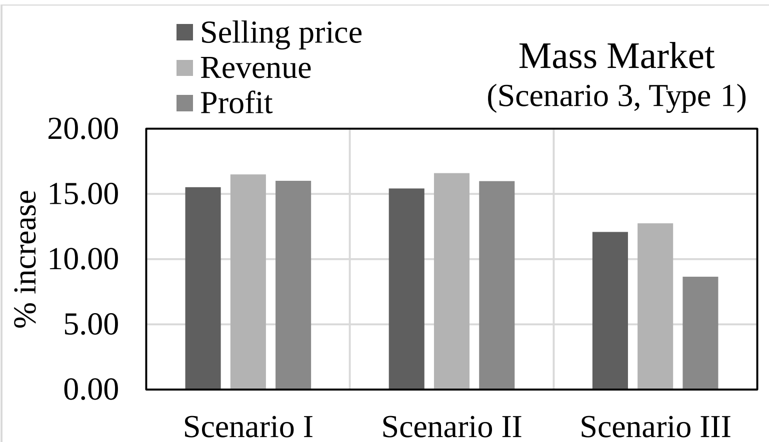

Fig. 4.4 Impact of promotion in case of a mass market (Scenario 3, Type 1)

4.3 Sensitivity Analysis

Sensitivity analyses with respect to input parameters h_r, A, and C_P have been carried out to check the robustness of optimal decision with respect to variation in these. For the analyses, the demand curve shown in Fig. 3.2 has been considered. Input parameter values have been varied on either side of their original values taken in Section 4.2.

4.3.1 *Sensitivity analysis with respect to h_r*

From the sensitivity results provided in Table 4.9, it is observed that there is a very small change in the optimal sales price with the change in h_r. If the change in the optimal price is insignificant, the corresponding change in demand will also be insignificant. The same can be observed in Table 4.9. On h_r being increased five times from 0.05 to 0.25, it is seen that the demand has marginally or insignificantly decreased from 678.58 to 678.27. Optimal advertisement expense also changes insignificantly and additional promotional price thus remains unaffected with the change in h_r. With the sales price and the annual demand remaining almost unaffected, the annual revenue will also be unaffected. With the revenue being practically constant and the overall cost to increase with the increase in h_r, the profit margin will come down. This can also be noticed from the results provided in Table 4.9.

From Table 4.9, it can be further noticed that the cycle time decreases with the increase in h_r as it happens in case of classical EOQ model. Since the demand is almost constant and the cycle time is decreasing with the increase in h_r, the optimal order quantity (Q) can be noticed to decrease naturally.

Thus, the present analysis shows that the change in h_r will cause a change in the lot-sizing policy and profit, but pricing and promotional expense policies would remain almost unaffected.

Table 4.9. The result of sensitivity analysis with respect to h_r

h_r	*P*	*B*	*ρ*	*(P+ρ)*	*T*	*D*	*Q*	T_P
0.05	237.12	3388.18	28.24	265.36	0.77	678.58	521	106217
0.06	237.13	3388.14	28.24	265.37	0.70	678.55	476	105968
0.07	237.13	3388.10	28.24	265.37	0.65	678.53	440	105740
0.08	237.14	3388.06	28.24	265.38	0.61	678.51	412	105527
0.09	237.14	3388.03	28.24	265.38	0.57	678.49	388	105327
0.10	237.15	3388.00	28.24	265.38	0.54	678.48	368	105138
0.11	237.15	3387.97	28.24	265.39	0.52	678.46	351	104958
0.12	237.15	3387.94	28.24	265.39	0.50	678.44	336	104787
0.13	237.16	3387.92	28.24	265.40	0.48	678.43	323	104622
0.14	237.16	3387.89	28.24	265.40	0.46	678.41	311	104463
0.15	237.16	3387.86	28.24	265.40	0.44	678.40	301	104310
0.25	237.19	3387.64	28.24	265.43	0.34	678.27	233	102998

4.3.2 *Sensitivity analysis with respect to A*

The results of the sensitivity analysis with respect to the ordering cost (*A*) are shown in Table 4.10. In this case also, the change in the optimal sales price is insignificant with the increase in *A*. Similar to h_r, the change in the demand remains almost constant with respect to change in *A*. Even if *A* is increased sizably from 500 to 2500, the demand value decreases slightly from 678.58 to 678.27. Similar behaviour was also observed while analysing the sensitivity with respect to h_r. The optimal advertisement expense and the additional sales price remain practically unaffected with the change in *A*. With the overall sales price and the annual demand being practically constant, the revenue will also become constant but the profit will come down due to the increase in the ordering cost. The same can also be witnessed from Table 4.10.

Optimal values of the cycle time (*T*) and the lot-size quantity (*Q*) both increase with the increase in *A*. This behaviour is also seen in classical EOQ model where the order-size increases with the increase in the ordering cost. Thus this characteristic is opposite to the observation made while carrying out sensitivity analyses with respect to h_r.

In short, a change in A will affect the optimal lot-sizing policy and the profit while the optimal pricing policy and promotional expense would remain almost unaffected.

Table 4.10. The result of sensitivity analysis with respect to A

A	***P***	***B***	***ρ***	***(P+ρ)***	***T***	***D***	***Q***	***T_P***
500	237.12	3388.18	28.24	265.36	0.38	678.58	260	106217
600	237.13	3388.14	28.24	265.37	0.42	678.55	285	105968
700	237.13	3388.10	28.24	265.37	0.45	678.53	308	105740
800	237.14	3388.06	28.24	265.38	0.49	678.51	329	105527
900	237.14	3388.03	28.24	265.38	0.52	678.49	349	105327
1000	237.15	3388.00	28.24	265.38	0.54	678.48	368	105138
1100	237.15	3387.97	28.24	265.39	0.57	678.46	386	104958
1200	237.15	3387.94	28.24	265.39	0.59	678.44	404	104787
1300	237.16	3387.92	28.24	265.40	0.62	678.43	420	104622
1400	237.16	3387.89	28.24	265.40	0.64	678.41	436	104463
1500	237.16	3387.86	28.24	265.40	0.66	678.40	451	104310
2500	237.19	3387.64	28.24	265.43	0.86	678.27	582	102998

4.3.3 *Sensitivity analysis with respect to C_P*

The outcome of sensitivity analysis with respect to the unit purchasing cost (C_P) is shown in Table 4.11. In the above sensitivity analyses with respect to h_r and A, the change in optimal sales price was observed to be insignificant. But the change is clearly visible with respect to C_P. With the unit cost (C_P) increasing from 50 to 250 (by 5 times), the selling price increases from 235.92 to 253.03. The reason for this change is as follows. An increase in C_P will naturally try to push up the sales price. If the selling price is allowed to increase in the same order, the demand will go down drastically because of the price-demand relationship considered and will cause revenue to shrink. Therefore, the model will not allow such thing to happen and will allow for only a small increase in the sales price while making the best use of the price-demand relationship in order to have the maximum possible profit. This profit can be seen to come down with the fall in revenue because of the increase in the unit purchase cost.

Besides, the change in the demand is also significant with respect to the change in C_P. The reason for this change is as follows. With the increase in C_P, the total cost increases and thus the profit decreases. In order to maximize the profit, the model tries to trade off the demand for the selling price. So, the selling price increases and the demand decreases, and the same is also visible in Table 4.11. The optimal advertisement expense is found to decrease slightly with respect to the increase in the unit purchasing cost. This is in contrast to the insignificant change noticed earlier either with respect to h_r or *A*.

In Table 4.11, the optimal of the cycle time (*T*) and order-quantity (*Q*) both can be observed to decrease with the increase in C_P.

Thus, with the change in C_P, optimal pricing and lot-sizing policies, as well as optimal profit and promotional expenses, are expected to get affected.

Table 4.11 The result of sensitivity analysis with respect to C_p

C_P	*P*	*B*	*ρ*	*(P+ρ)*	*T*	*D*	*Q*	T_P
50	235.92	3395.92	28.25	264.17	0.77	682.99	523	140269
60	236.13	3394.76	28.25	264.37	0.70	682.32	477	133193
70	236.35	3393.42	28.25	264.59	0.65	681.56	441	126145
80	236.59	3391.88	28.24	264.83	0.61	680.68	413	119120
90	236.85	3390.09	28.24	265.10	0.57	679.66	389	112118
100	237.15	3388.00	28.24	265.38	0.54	678.48	368	105138
110	237.47	3385.54	28.24	265.71	0.52	677.08	351	98180
120	237.83	3382.61	28.23	266.06	0.50	675.42	336	91246
130	238.24	3379.10	28.23	266.46	0.48	673.44	322	84337
140	238.70	3374.84	28.22	266.92	0.46	671.04	310	77457
150	239.22	3369.61	28.21	267.43	0.45	668.12	298	70608
250	253.03	3018.58	27.60	280.63	0.40	498.05	200	7247

4.4 Heuristic Approach

A heuristic approach is a problem-solving technique designed to solve a complex problem which is either difficult to solve with the help of classical approach or too much time-consuming. When the classic methods fail to find the exact solution, a heuristic is helpful in finding the approximate solution for the problem. A meta-heuristic is a problem independent general framework that provides a set of rules to develop heuristic. A few examples of meta-heuristics are Tabu Search, Simulated Annealing, Genetic Algorithm, TLBO, PSO, Ant Colony Optimization, etc. Meta-heuristics tend to find the best feasible solution out of all possible solutions to the problem. Heuristics being proposed here use the framework of two meta-heuristics and are detailed in the following sub-sections.

4.4.1 *Genetic Algorithm*

To solve the problem in a computationally efficient manner, a Genetic Algorithm (GA) framework due to Holland (1975) is being proposed. GA is a heuristic approach to solve optimization problems. It is an evolutionary approach based on the process of natural selection. The evolution starts with the randomly generated initial population. GA requires a genetic representation of solution and a fitness function. The fitness of every member in the population is evaluated and fit individuals are selected for modification of genes through crossover and mutation. The process is iterated until the stopping criteria are met and finally the best fit solution is obtained. The framework of the proposed GA is described in the form of a flowchart shown in Fig. 4.5.

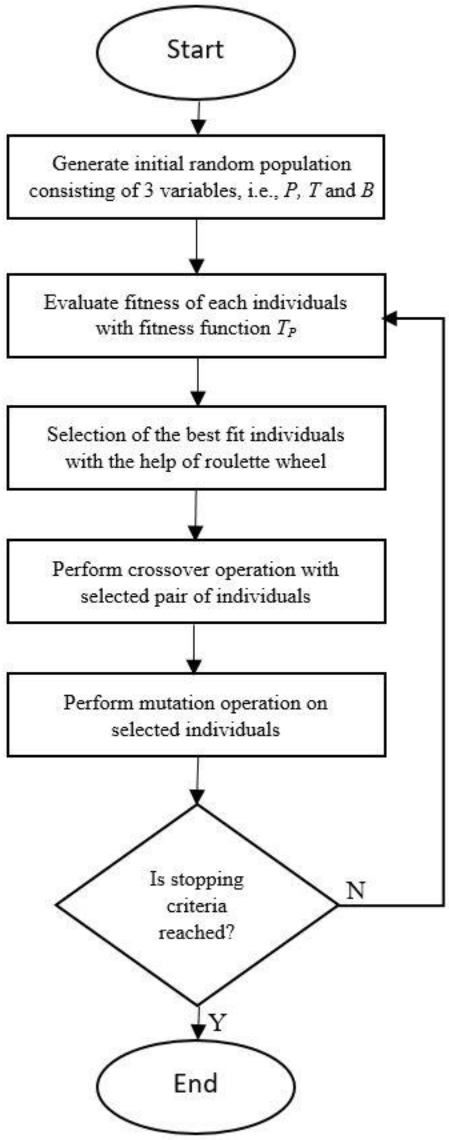

Fig. 4.5 Flowchart for Genetic algorithm

a) *Initialization*: A chromosome is taken to consist of three genes, i.e., the selling price, the cycle time and the advertisement expense and is represented structurally as given below.

P	*T*	*B*

The initial random population will comprise of such chromosomes. To demonstrate the effectiveness of the use of proposed GA in solving the illustrative example of Section 4.2, the population size is taken as 100. The range for the price is taken from 100 to 270, for the cycle time from 0 to 1 and for the advertisement expense from 0 to 15000.

b) *Fitness Evaluation*: Fitness of each individual is evaluated based on a fitness function. Here fitness function is taken as the profit function (T_P) provided by equation (4.6).

c) *Selection*: Fit chromosomes are selected using the roulette wheel method to form a new population.

d) *Crossover*: Few pairs of chromosomes are taken for the crossover of genes and they are replaced by their descendants.

   In the proposed GA framework, a single-point crossover is performed with a crossover rate of 0.2. This means that only 20% of the fit chromosomes will participate in the crossover process.

e) *Mutation*: The genes of the few selected chromosomes are mutated and the value of genes are changed.

   The mutation rate is taken as 0.1.

f) The steps from (b) to (e) are iterated until the stopping criteria are met.

   Here the number of generations as 100 is taken as a stopping criterion.

Proposed GA was implemented using MATLAB software (Version 2017) on a PC embedded with i7 processor (7th generation) and 8 GB RAM. The graph between the number of generations and total profit (objective function) obtained after running GA algorithm on MATLAB is shown in Fig. 4.6 Fig. 4.7 and Fig 4.8 for the three demand scenario. Results for illustrative examples taken in Section 4.2 for the niche market are shown in Table 4.12. Looking at this table and Table 4.2, the closeness between the values of the objective function and the decision variables can easily be noticed. The results of closeness analyses are shown in Table 4.14. Looking at the percentage gap in the values of the decision variables and the objective function, it is observed that the gaps are almost negligible and the results obtained from GA are in very close proximity to the optimal solution obtained analytically. However, the CPU time requirement for GA approach is more than the time required by LINGO in resulting the optimal solution.

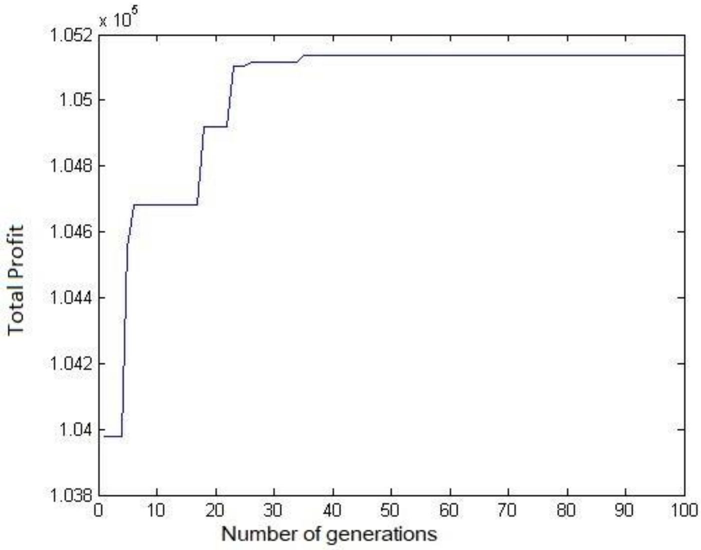

Fig. 4.6 Total profit at different number of generation of GA for demand scenario I

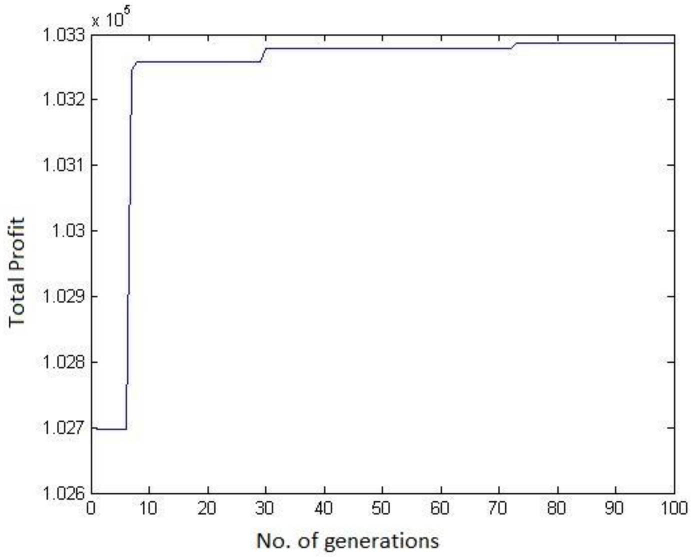

Fig. 4.7 Total profit at different number of generation of GA for demand scenario II

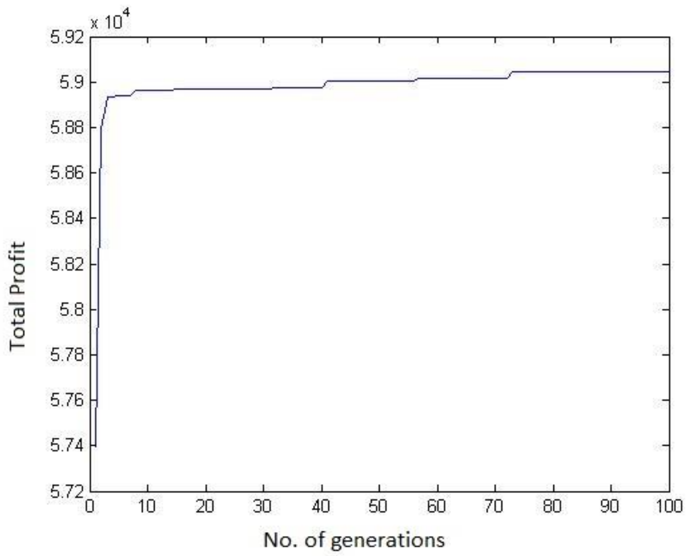

Fig. 4.8 Total profit at different number of generation of GA for demand scenario III

Table 4.12 Results obtained from the application of Genetic Algorithm

Demand Scenario	*P* ($)	*T* (year)	*B* ($)	T_P ($)	CPU time to obtain the best value (second)	Iteration no.
I	237.24	0.54	3375	105121	0.10	85
II	236.24	0.55	3356	103289	0.09	78
III	237.58	0.71	2709	59046	0.08	71

4.4.2 *Teaching-Learning-Based Optimization Algorithm*

The basic framework of this approach was developed by Rao et al. (2011). It uses the concept of improving the knowledge of the students within the classroom by the teacher first, and then by the mutual interaction among the students. So, there are two phases in this algorithm: (i) the teacher phase, and (ii) the student phase. The knowledge imparted by the teacher to the students falls under the teacher's phase and the improvement in the knowledge of students by their mutual interaction under the student's phase. The flowchart for the algorithm is shown in Fig. 4.9. The steps of TLBO algorithm are as follows:

Step1. Initialization

The variables selling price (P), cycle time (T) and advertisement expense (B) act as the subjects and the annual profit (T_P) acts as total marks.

A classroom of a fixed number of students is chosen. These students will have the subject marks generated randomly in a range considering the practical aspects of the problem.

Step 2. Teacher's phase

Sum of marks for all the subjects for each student is determined. A student with the best total marks value is chosen to be at par with the teacher. Thus, the teacher's knowledge in various subjects is taken to be the same as the subjects' marks of the best student. Next, the process of learning by the students from the teacher is initiated. Under this phase, the mean value of the total marks of the whole class shifts towards the teacher.

Step 3. Student's phase

In this phase, two students are randomly picked up for sharing the knowledge among themselves. This exercise is carried out for some specified number of pairs of the students. If the modified solution is better than the original one, then it is accepted. Otherwise, it is rejected and the original solution is restored.

Step 4.Temination

The teacher's and student's phases are repeated until reasonable improvement has been encountered.

The algorithm has been coded on MATLAB software (Version 17). The classroom size and the total number of iterations both were taken as 100. The graph between total profit and number of generation obtained after running the TLBO algorithm on MATLAB is shown in Fig. 4.10, Fig. 4.11 and Fig. 4.12. Results, from the use of TLBO, for the illustrative examples taken in Section 4.2 for the niche market are in presented in Table 4.13. Looking at this table and Table 4.2, it can be noticed that TLBO provided the same solutions what were obtained from LINGO. However, the CPU time requirement for TLBO approach is more than the time required by LINGO in resulting in the optimal solution.

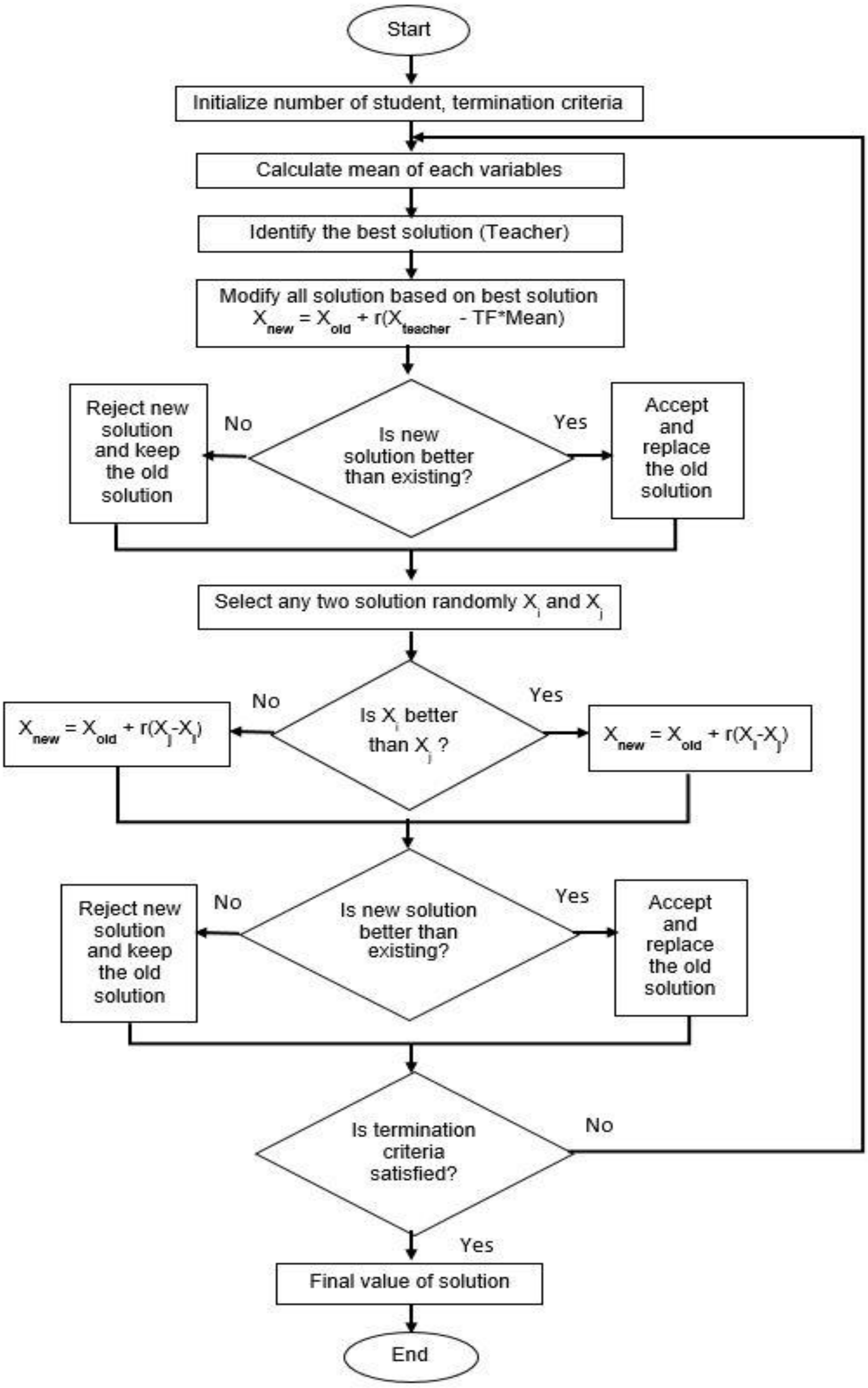

Fig. 4.9 Flowchart for TLBO algorithm

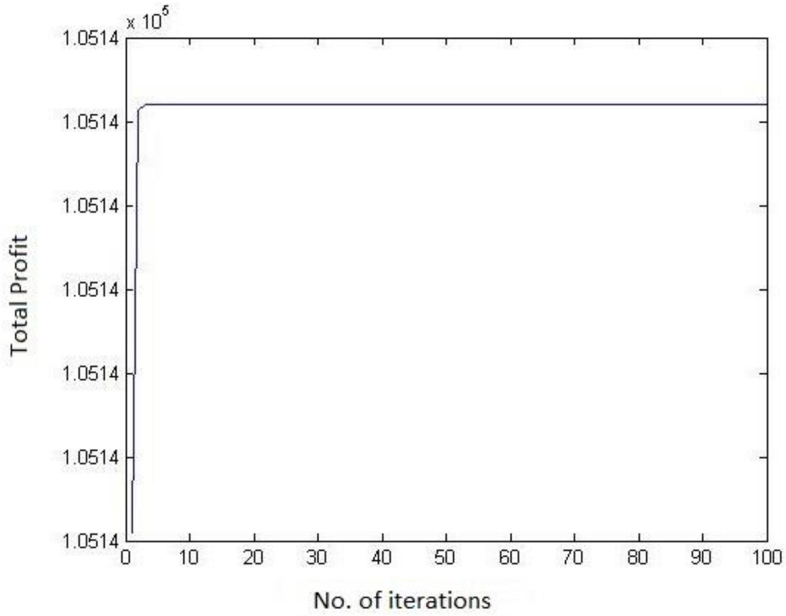

Fig. 4.10 Total profit at different number of iteration of TLBO for demand scenario I

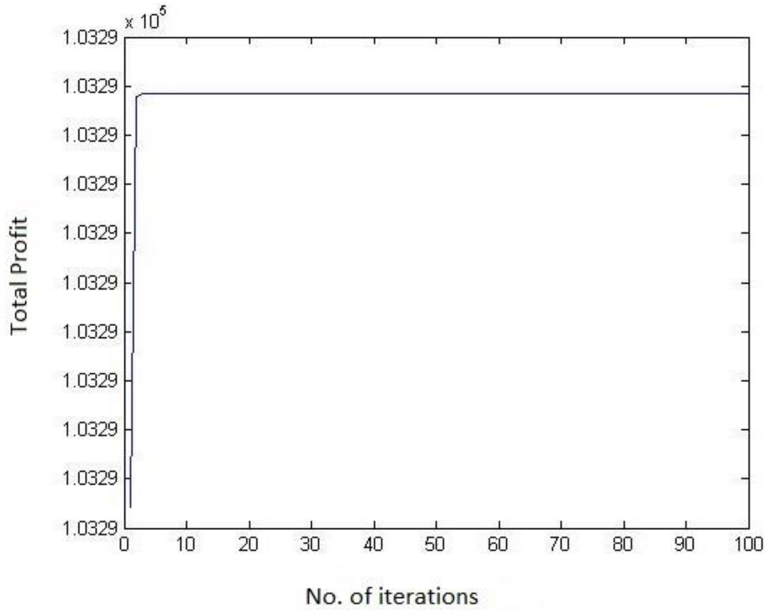

Fig. 4.11 Total profit at different number of iteration of TLBO for demand scenario II

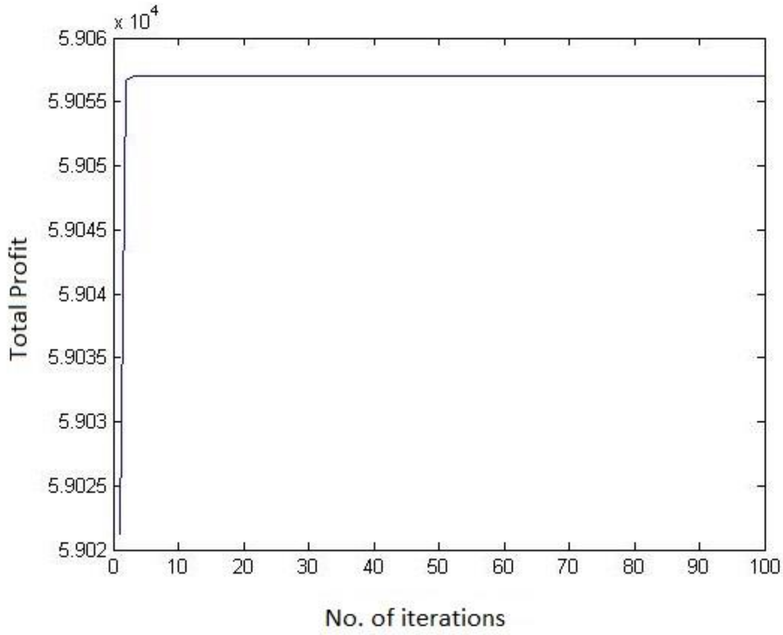

Fig. 4.12 Total profit at different number of iteration of TLBO for demand scenario III

Table 4.13 Results obtained from the application of TLBO algorithm

Demand Scenario	*P* ($)	*T* (year)	*B* ($)	T_P ($)	CPU Time to obtain the best value (second)	Iteration no.
I	237.15	0.54	3388	105138	0.48	39
II	236.09	0.55	3376	103294	0.35	30
III	237.48	0.71	2735	59057	0.43	36

4.4.3 *Performance evaluation*

Comparative performance of LINGO, GA and TLBO can be seen from Table 4.14 based on the illustrative example problems described in Section 3 for the niche market. The comparison is presented in terms of the values of the objective function and the decision variables, and also in terms of the CPU time requirement. Results obtained from GA and TLBO have hardly any difference compared to the results from LINGO. For a fair comparison, the paired *t*-test for equivalence using one-sided test was conducted using NCSS 2019 statistical software. The inferential statistical tests result in the following:

1. TLBO method and LINGO software are equivalent to each other as the two yield the same results. GA is found to yield inferior solutions compared to that from the application of TLBO.
2. CPU time of TLBO is much higher than that for GA.

The above analysis suggests that for solving the present problem it is advisable to make use of any optimisation software, such as LINGO, instead of going for evolutionary approaches. The reason is that the problem has only fewer decision variables and its number is not going to increase in any manner unless the present problem is extended to a multi-level supply chain problem or so.

Table 4.14 Performance evaluation of GA, TLBO and LINGO

				Demand Scenario		
				I	**II**	**III**
Decision Variable	***P***	Value ($)	LINGO (A)	237.15	236.09	237.48
			GA (B)	237.24	236.24	237.58
			TLBO (C)	237.15	236.09	237.48
		% Difference	(1-A/B)*100	0.04	0.06	0.04
			(1-A/C)*100	0.00	0.00	0.00
	T	Value (year)	LINGO (D)	0.5429	0.5457	0.7135
			GA (E)	0.5427	0.5457	0.7146
			TLBO (F)	0.5429	0.5457	0.7136
		% Difference	(1-D/E)*100	-0.05	-0.01	0.15
			(1-D/F)*100	0.00	0.00	0.00
	B	Value ($)	LINGO (G)	3388	3376	2735
			GA (H)	3375	3356	2709
			TLBO (I)	3388	3376	2735
		% Difference	(1-G/H)*100	-0.38	-0.58	-0.95
			(1-G/I)*100	0.00	0.00	0.00
Objective Function	T_P	Value ($)	LINGO (J)	105138	103294	59057
			GA (K)	105121	103289	59046
			TLBO (L)	105138	103294	59057
		% Difference	(1-J/K)*100	-0.02	0.00	-0.02
			(1-J/L)*100	0.00	0.00	0.00
CPU Time		Value (sec)	LINGO (M)	0.07	0.07	0.07
			GA (N)	0.10	0.09	0.08
			TLBO (O)	0.48	0.35	0.43
		% Difference	(1-M/N)*100	29.48	24.30	15.18
			(1-M/O)*100	85.31	80.26	83.74

4.5 Summary with Managerial Implication

In this chapter, the promotional expense has been considered to yield an opportunity for charging additional sales price without sacrificing the demand. Product promotion is another important aspect of marketing to maximize the profitability of the firm. The sales price is positively affected by incurring expenses on advertisement or promotion. The product promotion may help in enhancing the demand or provide leverage to charge higher sales price or both. The promotional strategy for the niche market and the mass market is different and so the promotional budget. In the case of a niche market, the best marginal revenue or utility is achieved at a low promotional expense compared to the mass market. This chapter considers the issue of determination of optimal sale price, lot-size and promotional expense in case of Veblen products. The backward bending curve for the price-demand relationship, suggested by Leibenstein (1950), were considered for analyses for the niche as well as for the mass markets. Using numerical examples, it is shown that a firm can earn an additional profit by incurring promotional expenses on the product. Sensitivity analyses show that the lot-size and the cycle time, both are affected by the change in the holding cost rate, the ordering cost and the unit cost price. The pricing or promotional decisions remained unaffected with the change in the holding cost rate, or the ordering cost, and the demand practically remained constant. However, the change in the unit cost impacted the annual demand significantly apart from some impact on the sales price and promotional budget decisions. The proposed mathematical model is helpful to the retailer in deciding its optimal sales price and lot-size. The marketing manager will come to know of the optimal budget size for making expenses towards promotion of their Veblen products. Through product promotion, they can increase the perceived value of the product and charge higher price. In case of niche market, with small promotional expense they can reap huge profit.