CHAPTER 3

PRICING AND LOT-SIZING FOR SOME VEBLEN PRODUCTS WITH PRICE-DEPENDENT DEMAND

3.1 The Problem

Here, the inventory management problem of a single retailer is being considered who is interested in determining its pricing and inventory policies independent of its supplier. The retailer purchases the Veblen product at a unit cost of C_P from a supplier and sells it to its customers at a unit selling price of P.

The following assumptions detail the problem considered here.

(i) There is only one retailer, who purchases goods from a single supplier and optimizes its profit independently of the supplier.

(ii) Replenishment is instantaneous.

(iii) The demand is not random but a function of price, and is deterministic in nature.

(iv) Shortages are not allowed.

Irrespective of the nature of variation in demand with respect to price, the retailer's replenishment cycle will be as shown in Fig. 3.1.

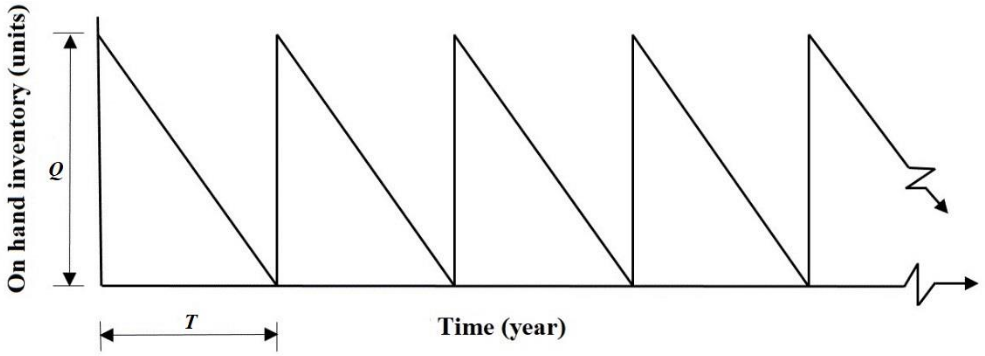

Fig. 3.1 Retailer’s replenishment cycle

In the scenario of demand being price-sensitive, the retailer has the scope for manipulating demand by changing the selling price. This change has to support his logical desire for maximizing the profit that can be determined and expressed as detailed below.

Annual revenue $= PD$ (3.1)

Annual ordering cost $= A/T$ (3.2)

Annual purchasing cost $= C_P D$ (3.3)

Lot size (Q) $= DT$ (3.4)

Annual holding cost $= (h_r C_P\, Q)/2 = (h_r C_P\, DT)/2$ (3.5)

For determining total annual profit, annual ordering cost, annual purchasing cost and annual holding cost from annual revenue are to be deducted. Thus,

$$T_P = PD - A/T - C_P D - (h_r\, C_P\, DT)/2 \quad (3.6)$$

or, $$T_P = Pf(P) - A/T - C_P f(P) - h_r\, C_p\, f(P)T/2 \quad (3.7)$$

Where $D = f(P)$.

If $f(P)$ is continuous over P, differential calculus can be used over the relationship (3.7) to determine the optimal value of P as P^* and the corresponding value of D^* as $f(P^*)$. In cases, when $f(P)$ is discontinuous (Den-Boer and Keskin 2018), available search algorithms including evolutionary approaches can be used. Another possible approach could be to pose this problem as a mathematical programming model is presented below.

Maximize $T_P = PD - A/T - C_P D - (h_r\, C_P\, DT)/2$

Subject to

$D = f(P)$

$Q = DT$

$D, Q, P, T \geq 0$

3.2 Price-demand Relationships and Profit

The possible nature of demand in the case of Veblen products has been given by Leibenstein (1950) but without any expression or data supporting the variation in demand with respect to price. Fig. 3.2, Fig. 3.3 and Fig. 3.4 with values of price and demand can be taken as representative relationships suggested by him. Fig. 3.2 depicts the pure

Veblen effect in which the demand initially increases with the price and, after a point, it starts declining due to non-affordability and less utility value perceived by the customers. Fig. 3.3 depicts the combined normal demand and Veblen demand. Initially, the curve follows the law of demand, and this law is violated at a high price beyond which the demand starts increasing with the price. With further increase in the price, the demand declines and ultimately reaches zero showing that no consumer is interested in buying the goods at such a very high price. Fig. 3.4 represents a relationship similar to that depicted in Fig. 3.3, the only difference being the demand to be zero for some price range. These relationships were not substantiated by Leibenstein (1950) either specifying any data or specific products, but as strong possibilities. The scope of the present chapter is not to address the case of any specific Veblen product, but to observe as what happens when the Veblen effect is present. With this in view, the essential form and feature of price-demand relationships depicted by Fig. 3.2 to Fig. 3.4 have been captured by a cubic function as $D = f(P) = -aP^3 + bP^2 - cP + d$. This relationship is also based on our findings from the relationship between the number of subscribers and the price of the broadband schemes (Fig. 1.2) offered by an Indian telecom industry. Data considered on price-demand relationships for the purpose of analysis are provided in Table 3.1.

Depending upon the case and the product, the values of the constants in this function may change or the relationship can be expressed all together by a different function itself. In this case, T_P, defined either by equation (3.6) or (3.7), will be a continuous function and it can be maximized using the classical optimization approach. With the assumed cubic demand function, let us take a numerical example with data as A = 1000 per order, C_P = 100 per unit and h_r = 0.1 per unit item value per year. Now the optimum annual profit, lot size, unit selling price and cycle time for the three price-demand relationships (Fig. 3.2, Fig. 3.3 and Fig. 3.4) are determined and the respective results are shown in Table 3.2. Fig. 3.5, Fig. 3.6 and Fig. 3.7 show the profit along with respective price-demand relationships shown in Fig. 3.2, Fig. 3.3 and Fig. 3.4. These figures also show the total profit for each level of the selling price.

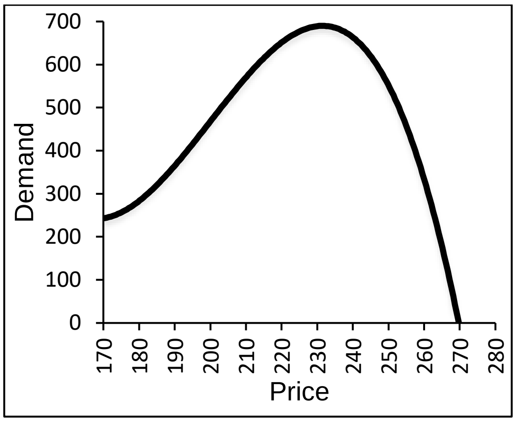

Fig. 3.2 Demand with pure Veblen effect

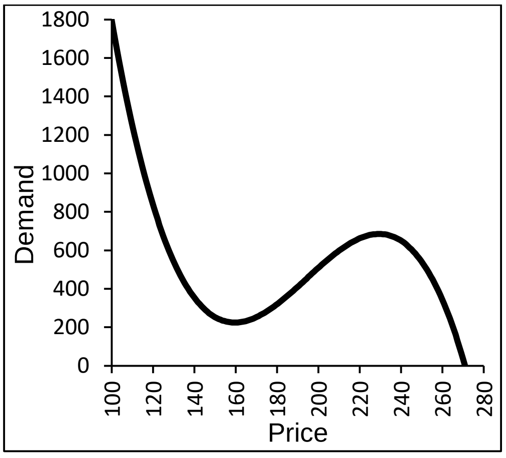

Fig. 3.3 Demand following the law of demand and Veblen effect

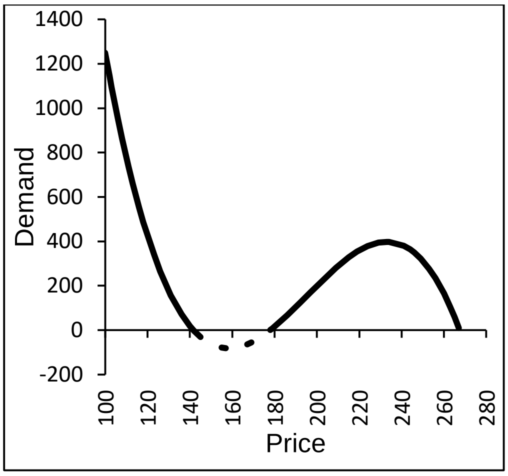

Fig. 3.4 Demand following the law of demand and Veblen effect with no demand for the certain price range

Table 3.1 Demand scenarios

Demand scenario	**D(P)**	**Price range**	**Nature of demand**
I	$(-0.003539P^3 + 2.1215P^2 - 413.3P + 26580)$	(170 - 270)	Fig. 3.2
II	$(-0.002703P^3 + 1.577P^2 - 296.8P + 18413)$	(100 - 270)	Fig. 3.3
III	$(-0.0023P^3 + 1.35P^2 - 254.5P + 15500)$	(100 - 270)	Fig. 3.4

Table 3.2 Optimal results for all the three demand scenarios

Demand scenario	P^*	T^*	D^*	Q^*	T_P^*	**Nearby peak demand**
I	238.15	0.54	673.89	367.12	89426	690
II	237.26	0.55	666.20	365.02	87788	685
III	238.27	0.72	390.72	279.54	51228	398

In all three cases, the optimal selling price (P^*) corresponding to the maximum profit was found to be more than the price corresponding to the nearest maximum demand (P_m) with $P_m < P^*$. In the case of the demand-price relationship of Fig. 3.5, the profit function is uni-modal and is observed to follow the demand pattern with variation in the sales price, but with a lag. This characteristic is not observed for price-demand relationships shown in Fig. 3.6 and Fig. 3.7. Here, the profit initially goes up with the increase in the price and then starts decreasing for a while with further increase. Afterwards, it is following the demand pattern with a lag as was observed in Fig. 3.5. Even though a common pattern was observed regarding optimal sales price in all the three scenarios of price-demand relationship, but a general statement cannot be made for an occurrence of such a pattern as presented and discussed below.

Let the price-demand relationship shown in Fig. 3.5 is modified to what is shown in Fig. 3.8. The extended tail (W-X) on the higher price side is indicative of comparatively low sensitivity of the price on the demand. This kind of situation can be encountered for those luxury goods which are meant for rich to very rich people since such persons will generally have a deep pocket and will be ready to pay even very high price for the products. Even though the pattern of demand-price relationships shown in Fig. 3.5 and Fig. 3.8 are similar, the profit curve patterns are different. The profit curve, instead of being uni-modal, has become bi-modal. However, the maximum profit is still achieved on the high price side demand as with the earlier case.

The pattern of price-demand relationships shown in Fig. 3.6 and Fig. 3.7 are similar with the only difference of no demand for some price range as shown in Fig. 3.7. For the purpose of further analysis, a demand scenario, in between the two scenarios of Fig. 3.6 and Fig. 3.7, is taken and is as shown in Fig. 3.9. The demand with the increase in the sales price ultimately reduces to zero and then increases with the increase in the price up to some extent and, after this, it starts reducing to finally fall to zero. Optimal profit function for this demand pattern is also shown along with (Fig. 3.9). In this case also, the profit function was found to be bi-modal with optimal demand being a little less compared to the nearest maximum demand on the high price side. The pattern of demand–price relationship shown in Fig. 3.9 is further modified to a pattern (shown in Fig. 3.10) with the tail (W-X) and the front (U-V) further elongated along the two axes. From the profit function, shown in Fig. 3.10, it can be noticed that the optimal demand is again on the high price side.

So far, the maximum profit was found to be on the high price side after the nearest peak demand. Further experimentation and investigation were carried out to prove that the possibility of this behaviour cannot be generalized. For this purpose, the demand-price curve of Fig. 3.9 was modified by extending the tail (U-V) on the low price side to have a demand-price relationship as shown in Fig. 3.11. This will be representative of very high price demand sensitivity on the low price side and can be seen particularly in the case of the high-cost items or luxury goods meant for the lower middle class. From this figure, it can be seen that the profit function is again bi-modal with the maximum profit now occurring on low price side (U-V) instead of on the high price side of local peak demand (W-X) (Fig. 3.5 to Fig. 3.10).

From the above, it can be said that the optimal profit to the retailer can be anywhere from low price side to high price side and the same will depend upon the demand-price relationship. From the profit functions shown in Fig. 3.5 to Fig. 3.11, it can be further observed that the maximum profit is always achieved on that part (U-V or W-X) of the price-demand curve where the demand decreases with the increase in the price. In other words, the optimal profit lies at one of those parts of the price-demand curve that represents the law of demand. Thus those parts of the demand curve can be neglected when the demand increases with the increase in the selling price. This finding leads to important learning for choosing the relevant part of the demand-price relationship for Veblen products while determining optimal pricing and inventory policies for them.

From the above elaborations, it is clear that no generalization can be made for the pattern of profit curve. The maximum profit can be on the low price side as well as on the high price side wherever the law of demand is operative.

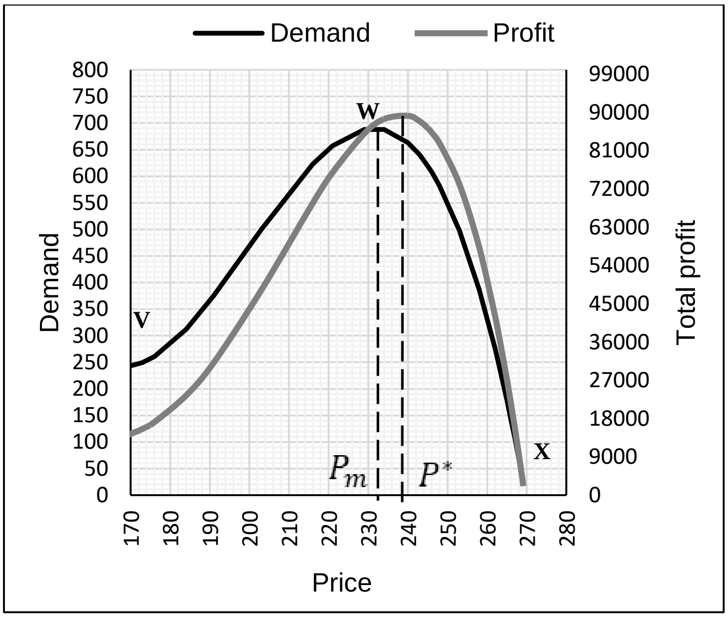

Fig. 3.5 Variation in total profit for demand under pure Veblen effect

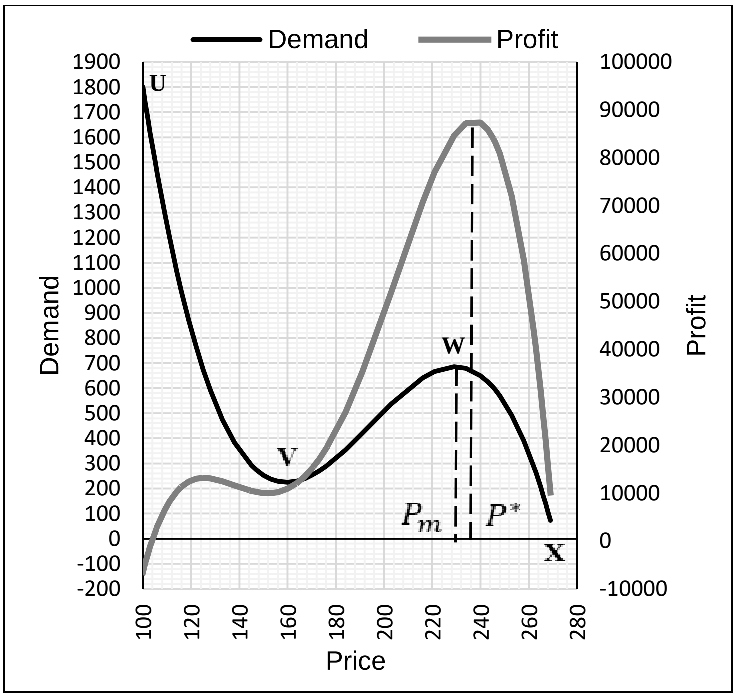

Fig. 3.6 Variation in total profit for demand following the law of demand and Veblen effect

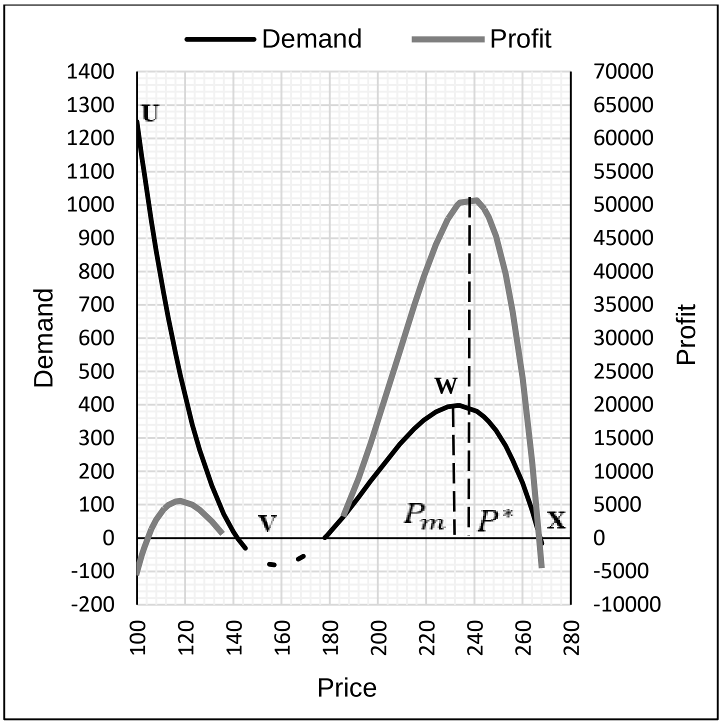

Fig. 3.7 Variation in total profit for demand following the law of demand and Veblen effect with no demand for the certain price range

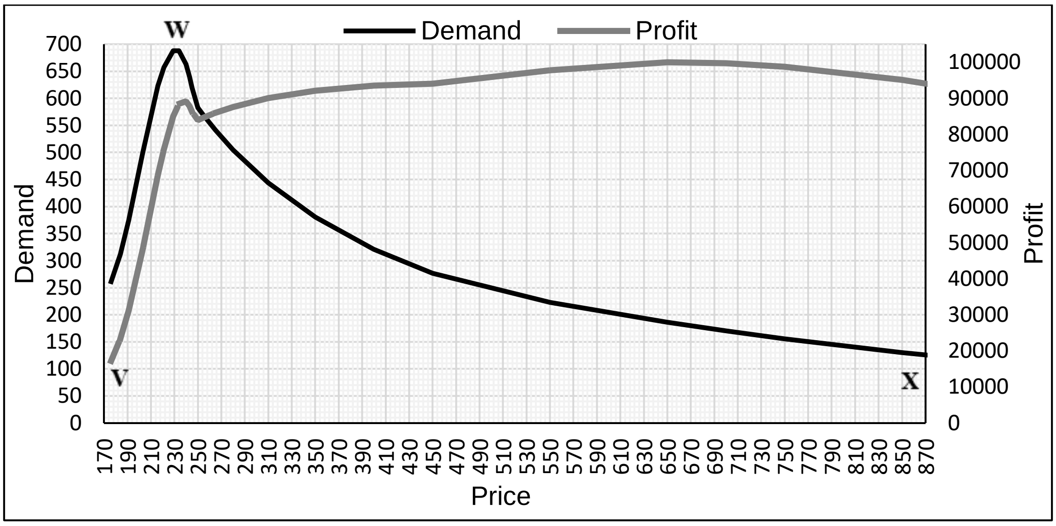

Fig. 3.8 Variation in total profit for demand under Veblen effect (low demand price sensitivity on the high price side)

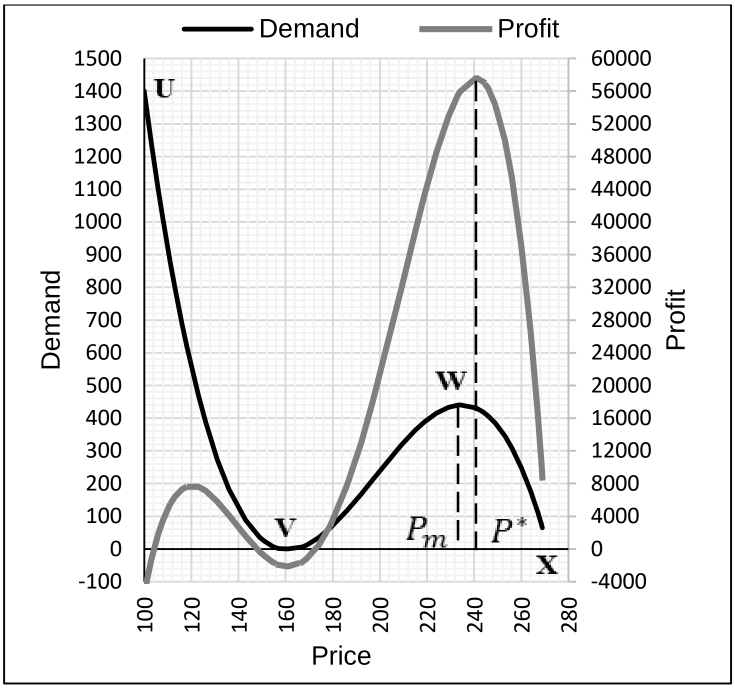

Fig. 3.9 Variation in total profit for demand following the law of demand and Veblen effect

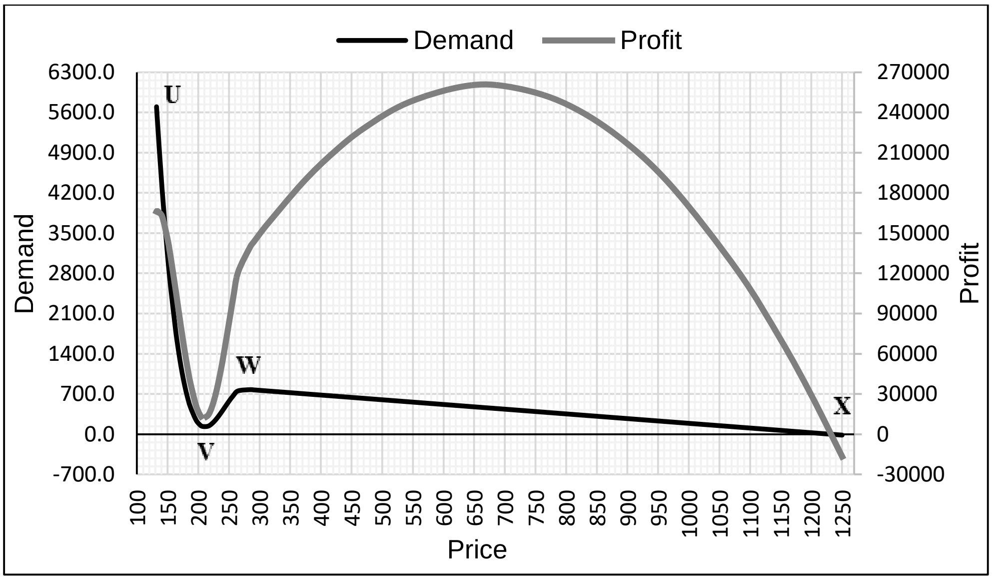

Fig. 3.10 Variation in total profit for demand under the law of demand and Veblen effect (low demand price sensitivity on the high price side)

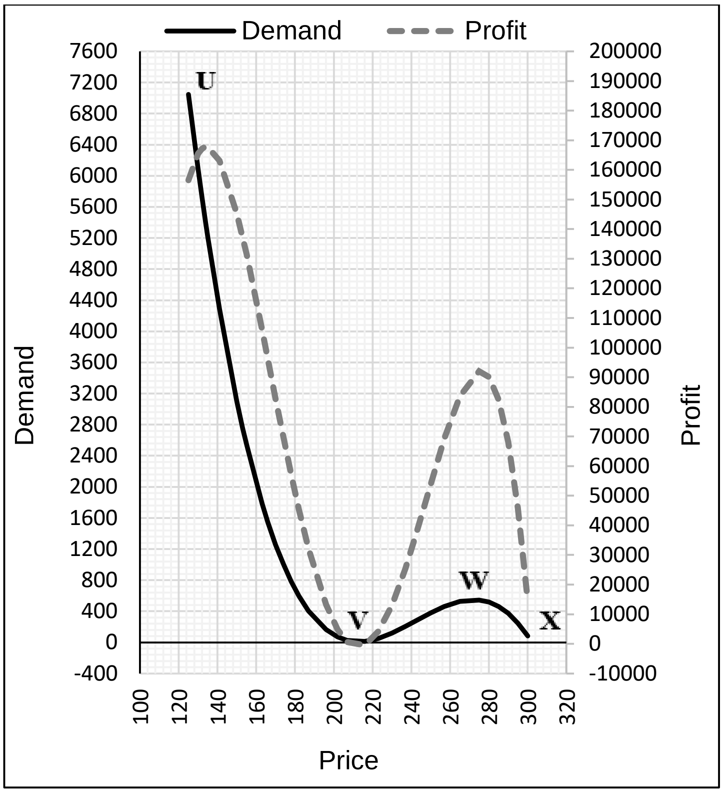

Fig. 3.11 Variation in total profit for demand under Veblen effect (high demand price sensitivity on the low price side)

3.3 Sensitivity Analysis

In order to evaluate the robustness of the solution, sensitivity analyses with respect to input parameters as h_r, A, and C_P have been carried out. Price-demand relationship considered for the analyses is the demand curve shown in Fig. 3.5. Parameter values have been varied in the range of -50% to +50% of their original value taken in Section 3.2.

3.3.1 *Sensitivity analysis with respect to h_r*

The result of sensitivity analysis with respect to h_r is shown in Table 3.3. It is observed that the change in the optimal price is very small and is insignificant with the change in h_r. Since the change in optimal price is insignificant, so the corresponding change in demand should also be insignificant and the same can be seen to happen in Table 3.3. From Table 3.3, it can be seen that the cycle time decreases with the increase in h_r as it happens in classical EOQ model. Since the demand is almost constant and the cycle time is decreasing with the increase in h_r, the optimal quantity should decrease with the increase in h_r and the same can be seen happening in Table 3.3. Since the selling price and the demand are constant, the revenue will remain constant. So the decrease in the profit, with the increase in h_r, would be only due to related costs going up. Thus, this analysis shows that the change in h_r will impact lot-sizing policy but with almost no effect on the optimal sales price to be charged by the retailer.

Table 3.3 Sensitivity analysis with respect to h_r

h_r	P	T	Q	D	T_P
0.05	238.12	0.77	519	674.06	90502
0.06	238.12	0.70	474	674.02	90254
0.07	238.13	0.65	439	673.98	90026
0.08	238.14	0.61	410	673.95	89814
0.09	238.14	0.57	387	673.92	89615
0.10	238.15	0.54	367	673.89	89426
0.11	238.15	0.52	350	673.87	89247
0.12	238.16	0.50	335	673.84	89076
0.13	238.16	0.48	322	673.81	88912
0.14	238.17	0.46	310	673.79	88754
0.15	238.17	0.45	300	673.77	88601

3.3.2 *Sensitivity analysis with respect to A*

Results of sensitivity analysis with respect to ordering cost (*A*) are provided in Table 3.4. Here also, the change in the optimal price with the increase in *A* is found to be insignificant. The demand also remains almost constant with respect to the change in *A*. It can be seen that cycle time (*T*) and optimal order quantity (*Q*) both increase with the increase in *A*. This is also found to happen in classical EOQ model where lot-size increases with the increase in ordering cost. Thus, like h_r, with the change in *A*, the lot-sizing policy gets affected while the optimal price remains almost unaffected.

Table 3.4 Sensitivity analysis with respect to A

A	P	T	Q	D	T_P
500	238.12	0.39	260	674.06	90502
600	238.12	0.42	284	674.02	90254
700	238.13	0.46	307	673.98	90026
800	238.14	0.49	328	673.95	89814
900	238.14	0.52	348	673.92	89615
1000	238.15	0.54	367	673.89	89426
1100	238.15	0.57	385	673.87	89247
1200	238.16	0.60	402	673.84	89076
1300	238.16	0.62	419	673.82	88912
1400	238.17	0.64	434	673.79	88754
1500	238.17	0.67	450	673.77	88601

3.3.3 *Sensitivity analysis with respect to C_P*

Results of sensitivity analysis with respect to unit purchasing cost (C_P) are shown in Table 3.5. Since the increase in C_P would cause the total cost to move up, the profit will naturally go down for the retailer and the same can be observed in this table. It can also be observed that the cycle time and optimal lot-size both decrease with the increase in C_P. In case of the sensitivity analyses with respect to h_r and A, the change in optimal price and demand were found to be insignificant. But the change in the optimal price and demand with respect to C_P is clearly visible. The reason for this change is as follows. The increase in C_P will cause marginal profit to decrease. In order to maximize the profit, an effort will be made to push up the selling price and this would cause demand to decrease in a price-sensitive environment. Thus, the increase in C_P is found to affect the inventory policy and to cause a negative impact on the profitability of the retailer.

Table 3.5 Sensitivity analysis with respect to C_P

C_P	P	T	Q	D	T_P
50	236.53	0.77	522	680.89	124398
60	236.79	0.70	476	679.90	117345
70	237.08	0.65	440	678.75	110323
80	237.40	0.61	412	677.40	103329
90	237.75	0.57	388	675.80	96364
100	238.15	0.54	367	673.89	89426
110	238.60	0.52	349	671.59	82519
120	239.10	0.50	334	668.79	75646
130	239.68	0.48	320	665.33	68812
140	240.34	0.46	307	661.01	62022
150	241.10	0.45	296	655.55	55288

3.4 Summary with Managerial Implication

In this chapter, an inventory management problem of a retailer has been considered who faces a price-sensitive demand for a product under Veblen effect. Various variations of this demand, as mentioned by Leibenstein (1950), are taken for analyses. The problem of determining optimal order size and sales price for the product has been mathematically formulated for the objective of maximization of profit for the retailer. Taking the numerical examples, it has been shown that the optimal inventory management policy will be pushing the sales price of the product either on the low side or towards the high side depending upon the price-demand relationship. Optimal pricing on the low side can be encountered for those luxury products that are consumed by middle-class people who have bulk demand with high price-sensitivity. For rich people, it is expected to occur on the high price side. The analysis carried out shows that the optimal demand for the maximum profit to the retailer occurs on the part of the price-demand curve that is representative of the Law of Demand. In view of this, it is suggested to work with only these parts of the demand curve in order to determine optimal inventory and pricing policies. Carrying cost rate or ordering cost play important role in deciding inventory policies even for Veblen products, but do not impact the pricing policy. The unit cost of the item, on the other hand, impacts both inventory management and pricing policies. The retailers can use the proposed model for determining optimal sales price and inventory policy for the luxury products.