Chapter 4

Evolution of weak shock waves in non-ideal magnetogasdynamics

4.1 Introduction

A large number of physical phenomena taking place in the nature are described by means of mathematical models represented by hyperbolic system of partial differential equations see (Whitham (1974), Anile et al. (1993), Sharma and Venkatraman (2012)). In non-linear systems, study of discontinuity waves i.e. shock waves, acceleration waves, weekly non-linear waves are of great importance due to its applications in Gasdynamics. As discontinuities in the form of shock waves are natural phenomenon in various astrophysical situations e.g. supernova explosions, stellar winds, photo ionized gas, collision between high velocity clumps of interstellar gas, collision of two or more galaxies etc. and non-idealness of the gas with Magnetohydrodynamics applies to many conducting fluids and plasma flows encountered in nature as well as in industrial applications. In past, several approaches have been used to investigate the asymptotic properties of weakly non-linear waves and propagation of waves in different material media, governed by quasilinear hyperbolic system of equations (Sharma (2010), Hunter (1995)). A remarkable attention on small amplitude non linear progressive waves has been drawn by Choquet-Bruhat (1969) in which they have considered a shockless solution of system of hyperbolic partial differential equations that depends on a single phase function. Authors such as Germain (1971), Fusco (1982), Fusco and Engelbrecht

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(1984) and Sharma et al. (1987) used the perturbation technique to analyze the non linear wave propagation in various material media. Hunter and Keller (1983) presented the Ray method, to determine a small amplitude high frequency wave solution of system of quasilinear hyperbolic partial differential equations.

If temperature of the gas is very high and density is too low, the assumption that the gas is ideal is no longer valid; therefore the alternative to the ideal gas is a simplified van der Waals model. The study of shock related phenomena through a non-ideal gas is of great technical interest in many industrial applications such as chemical processes, nuclear reactions and aerospace engineering and science etc. In recent years several studies have been performed related to the problem of strong shock by using the modified van der Waals gas (Wu and Robert (1996), Pandey and Sharma (2007)). The study of shock related phenomena in van der Waals fluids is more complex than the ideal gas fluid. For physical meaning of van der Waals gas and its influence on motion of waves, see (Thompson (1971), Cramer and Sen (1987)). Zhao et al. (2011) has studied a complete classification of shock waves and shock splitting phenomena together with their admissibility in van der Waals fluids. Further, the theory of progressive wave is used to study the finite and moderately small amplitude disturbances in non ideal gas modelled by van der Waals equation of state (Ambika et al. (2014)). Singh et al. (2011) studied the problem of propagation of acceleration waves along characteristics by using the characteristics front under the assumption that the intermolecular force between the particles of the gas is absent.

The motion of inviscid, infinite electrically conducting, van der Waals fluids in the presence of a magnetic field is responsible for a number of outstanding phenomena such as in astrophysics, high speed flow and plasma physics (Robert (1967), Pai (1992)). A number of studies have been done by considering simplified models such as covolume magnetogasdynamics (Singh et al. (2015, 2011, 2014)). A symmetric perturbation scheme is used to study the propagation of weak shock waves in non-uniform radiative magnetogasdynamics by Singh et al. (2010). Further evolution of weak discontinuities in radiative magnetogasdynamics has been studied by Singh et al. (2011). Also Jena et al. (2013) have studied the existence and interaction of acceleration wave with a characteristic shock in transient pinched plasma.

In this chapter an asymptotic approach is used to study the propagation of weakly non linear waves in a non-ideal gas with infinite electrical conductivity modelled by van der Waals equation of state permeated by a transverse magnetic field. An evolution equation, characterizing the wave process in a high frequency domain is derived. Also the growth and decay behaviour of disturbances in the form of sawtooth profile in planar and cylindrically symmetric flows are discussed. Further the influence of the van der Waals gas parameters and magnetic field on the wave profiles is evolved. Also a remarkable difference in length of sawtooth profile and velocity profile of sawtooth wave in planar and nonplanar flows have been studied.

4.2 Governing equations

We consider a general class of real gases whose equation of state is given by

$$\left(p + a/V^2\right)\left(V - b\right) = RT\,, \tag{4.2.1}$$

where p is the pressure, V is the volume, R is the universal gas constant, T is the absolute temperature. Here the constant a denotes the amount of intermolecular force of attraction between the particles, and b denotes the neglected volume which is associated with the volume of the gas. It is well known that the gases behave like real gases at low temperature and high pressure. For given equation of state (4.2.1), the internal energy E, in view of $R=(\gamma-1)C_V$ where C_V is the specific heat at constant volume, can be written as

$$E=\frac{\left(p+a\rho^2\right)(1-b\rho)-a(\gamma-1)\rho^2}{(\gamma-1)\rho}, \tag{4.2.2}$$

where ρ is the density of the gas and γ is the adiabatic index. Note that, if we put the constant a and b equals zero, then the equation of state (4.2.1) for real gases turns to the equation of state for ideal gas equation.

The governing equations for one dimensional unsteady motion of a non-ideal gas with infinite electrical conductivity modelled by van der Waals equation of state in presence of transverse magnetic field may be written as (Wu and Robert (1996), Korobeinikov (1976))

$$\rho_t+u\rho_x+\rho u_x+\rho mu/x=0, \tag{4.2.3}$$

$$u_t+uu_x+\left(p_x+h_x\right)/\rho=0, \tag{4.2.4}$$

$$p_t+up_x+\rho c^2\left(u_x+mu/x\right)=0, \tag{4.2.5}$$

$$h_t+uh_x+2h\left(u_x+mu/x\right)=0, \tag{4.2.6}$$

where u is the fluid velocity, $h=\mu H^2/2$ is the magnetic pressure with H as the magnetic field strength, μ is the magnetic permeability, t is the time , x is the spatial coordinate and c is the speed of sound in real gases modelled by van der Waals equation of state and is given as

$$c=\left(\left(\gamma p+a\rho^2\left(\gamma-2+2b\rho\right)\right)/\rho(1-b\rho)\right)^{1/2}. \tag{4.2.7}$$

In system of equations (4.2.3)-(4.2.6) the letter subscript denotes partial differentiation unless stated otherwise. Also m takes value 0 for planar flow and 1 for cylindrically symmetric flow.

Equations (4.2.3)-(4.2.6) may be written in a matrix form as

$$U_t + AU_x + B = 0, \tag{4.2.8}$$

where

$$U = \begin{bmatrix} \rho \\ u \\ p \\ h \end{bmatrix}, \quad A = \begin{bmatrix} u & \rho & 0 & 0 \\ 0 & u & \rho^{-1} & \rho^{-1} \\ 0 & \rho c^2 & u & 0 \\ 0 & 2h & 0 & u \end{bmatrix} \text{ and } B = \begin{bmatrix} m\rho u/x \\ 0 \\ \rho c^2 mu/x \\ 2hmu/x \end{bmatrix}. \tag{4.2.9}$$

The system of equations (4.2.8) is hyperbolic in nature and the coefficient matrix A have eigenvalues $u-w,\ u, u$ and $u+w$. Here $w = \left(c^2 + e^2\right)^{1/2}$ is the magneto sonic speed, where $e = \left(2h/\rho\right)^{1/2}$ is the Alfvén speed and $c = \left[\left(\gamma p + a\rho^2\left(\gamma - 2 + 2b\rho\right)\right)/\rho\left(1 - b\rho\right)\right]^{1/2}$ is the speed of sound in real gases modelled by van der Waals gas. The left and right eigenvectors of A corresponding to the eigenvalue $u+w$ are

$$l = \left(0,\ \rho w, 1, 1\right), \quad r^T = \left(1,\ w/\rho,\ c^2,\ e^2\right), \tag{4.2.10}$$

where superscript represents transposition

4.3 Progressive wave solution

Equation (4.2.8) can be written as

$$U_t^i + A^{ij} U_x^i + B^i = 0, \qquad i, j = 1, 2, 3, 4 \tag{4.3.1}$$

where U^i, A^{ij} and B^i are the components of the column vector of U, A, B respectively.

Here we are looking for asymptotic solutions of equation (4.3.1), exhibiting the features of progressive waves. Let us consider the following asymptotic expansion

$$U^i\left(x,t\right) = U_0^i + \varepsilon U_1^i\left(x,t,\xi\right) + \varepsilon^2 U_2^i\left(x,t,\xi\right) + O\left(\varepsilon^3\right), \tag{4.3.2}$$

where U_0^i is known uniform solution of equation (4.3.1) such that $B^i\left(U_0\right)=0$, while all the terms associated with equation (4.3.2) are of progressive wave nature. The value of ε depends on the physical problem to be studied here. Suppose τ_{chr} is the characteristics time scale of the medium and τ_{at} is the attenuation time, then we define parameter $\varepsilon=\tau_{chr}/\tau_{at} << 1$. Here the variable ξ is a "fast variable" and defined as $\xi = f\left(x,t\right)/\varepsilon$, where $f\left(x,t\right)$ is a phase function which is to be determined latter. Note that the case $\varepsilon << 1$, which corresponds to the condition in which the characteristics frequency of the medium is much greater than the attenuation frequency of the signal, characterizing a high frequency propagation condition (Seymour and Varley (1970)).

Introducing the Taylor's series expansion of A^{ij} and B^i in the neighborhood of the known uniform solution U_0^i and using equation (4.3.2), we have

$$A^{ij}=A_0^{ij}+\varepsilon\left(\partial A^{ij}/\partial U^k\right)_0 U_1^k+O\left(\varepsilon^2\right), \tag{4.3.3}$$

$$B^i=B_0^i+\varepsilon\left(\partial B^i/\partial U^k\right)_0 U_1^k+O\left(\varepsilon^2\right). \tag{4.3.4}$$

Now using equations (4.3.2)-(4.3.4) in equation (4.3.1) and collecting the coefficients of ε^0 and ε^1, we have the following equations

$$\left(A_0^{ij}-\lambda\delta_j^i\right)\partial U_1^j/\partial\xi=0, \tag{4.3.5}$$

$$\begin{aligned}&\left(A_0^{ij}-\lambda\delta_j^i\right)\partial U_2^j/\partial\xi+\left(\left(\partial U_1^i/\partial t\right)+A_0^{ij}\left(\partial U_1^j/\partial x\right)\right)f^{-1}(x)\\&+U_1^k\left(\partial A^{ij}/\partial U^k\right)_0\partial U_1^j/\partial\xi+f^{-1}(x)U_1^k\left(\partial B^i/\partial U^k\right)_0=0\end{aligned}, \tag{4.3.6}$$

where $\lambda=-f_t/f_x$, δ_j^i is the Krönecker delta and the subscript 0 means the quantity involved is evaluated at uniform state U_0. Equation (4.3.5) gives the characteristics polynomial $\lambda^2\left(\lambda^2-w_0^2\right)=0$, providing nonzero eigenvalues $\pm w_0$ of A_0. Considering the velocity $\lambda = w_0$ the corresponding left and right eigenvectors of A_0 are given by equation (4.2.10) with subscript 0. In view of equation (4.3.5) we observe that $\partial U^1/\partial\xi$ is collinear to r_0 and therefore U_1 may be written as

$$U_1(x,t,\xi) = \alpha(x,t,\xi) r_0 + \Omega(x,t). \tag{4.3.7}$$

Equation (4.3.7) represents the solution of equation (4.3.5). Here $\alpha(x,t,\xi)$ is the amplitude factor which is to be determined latter and the Ω^i (the components of the column vector Ω) are integration constants which are not of progressive wave nature and therefore can be omitted. The phase function $f(x,t)$ may be found as

$$f_t + w_0 f_x = 0. \tag{4.3.8}$$

If $f(x,0) = x - x_0$, then equation (4.3.8) gives

$$f(x,t) = x - x_0 - w_0 t. \tag{4.3.9}$$

Further multiplying equation (4.3.6) by l_0^i, and using equation (4.3.9) in the resulting equation we get, the following evolution equation for α

$$\frac{\partial \alpha}{\partial \tau} + \chi_0 \alpha \frac{\partial \alpha}{\partial \xi} + \psi_0 \alpha = 0, \tag{4.3.10}$$

where $\partial/\partial\tau = \partial/\partial t + w_0 \partial/\partial x$ is the ray derivative taken along the ray direction and the value of χ_0, ψ_0 is given as

$$\chi_0 = \left(\partial(u+w)/\partial U^k\right)_0 r_0^k = \frac{(\gamma+1)c_0^2 + 3e_0^2(1-b\rho_0) + 2a\rho_0(\gamma - 2 + 3b\rho_0)}{2w_0\rho_0(1-b\rho_0)} > 0, \tag{4.3.11}$$

$$\psi_0 = \frac{l_0^j r_0^k}{l_0^i r_0^i}\left(\partial B^i/\partial U^k\right)_0 = \frac{m w_0}{2x}. \tag{4.3.12}$$

Here the dimension of ψ_0^{-1} is time and may be taken as having attenuation time τ_{at} characterizing the medium. Equation (4.3.10) is the first order hyperbolic partial differential equation so one can find its characteristics curve as

$$\xi = \begin{cases} \xi_0 + \tau\chi_0\phi(x_0,\xi_0), & \text{for } m=0 \\ \xi_0 + 2\chi_0(x_0/w_0)\phi(x_0,\xi_0)\left\{\left(1+(\tau w_0/x_0)\right)^{1/2}-1\right\}, & \text{for } m=1 \end{cases}, \tag{4.3.13}$$

where $\phi(x_0,\xi_0)=\alpha|_{\tau=0}$, $\xi_0=\left(f|_{\tau=0}\right)/\varepsilon$ and $x_0=x|_{\tau=0}$.

The existence of an envelope of the characteristic curves which is given by equation (4.3.13) gives us the evidence of the formation of a shock. It is evident that the shock is formed for $\tau>0$ only by those characteristics for which $\partial\phi/\partial\xi_0<0$. Also the shock formation time (τ_{shf}) for planar ($m=0$), and cylindrical ($m=1$) compressive waves turns out to be

$$\tau_{shf} = \begin{cases} \min\left(\chi_0\left|\partial\phi/\partial\xi_0\right|\right)^{-1}, & \text{for } m=0 \\ \min\left[x/w_0\left\{\left(1+w_0/\left(2x_0\chi_0\left|\partial\phi/\partial\xi_0\right|\right)\right)^2-1\right\}\right], & \text{for } m=1 \end{cases}, \tag{4.3.14}$$

where the minimum is evaluated over an appropriate range of the quantity x_0, ξ_0.

4.4 Acceleration waves

The above analysis may be used to study the acceleration waves for the system of equations (4.2.3)-(4.2.6). Let us assume that, the curve $f(x,t)=0$ represents the acceleration front and across such a front the velocity is continuous but its first order and higher order derivatives undergo finite jump discontinuities. In the neighborhood of the front, the velocity u may be represented by the following expansion

$$u=\varepsilon u_1(x,t,\xi)+O\left(\varepsilon^2\right), \tag{4.4.1}$$

where $u_1 = 0$ for $\xi < 0$ and $u_1 = O(\xi)$ for $\xi > 0$. Since u_1 is an element of the column vector U_1 given by equation (4.3.7), therefore we have (Germain (1971))

$$\alpha(x,t,\xi) = \begin{cases} 0, & \text{if } \xi < 0 \\ \xi\beta(x,t) + O(\xi^2), & \text{if } \xi > 0 \end{cases}, \tag{4.4.2}$$

with $\beta = (\rho_0/w_0)\sigma$, where $\sigma = [\partial u/\partial x]$ denotes the jump in velocity gradient across the acceleration front.

Using equation (4.4.2) in equation (4.3.10) and then evaluating the resulting equation at the front $\xi = 0$, we get a Bernoulli type equation

$$\frac{d\sigma}{dt} + \psi_0\sigma + \Pi_0\sigma^2 = 0, \tag{4.4.3}$$

where

$$\Pi_0 = \frac{1}{2}\left\{(\gamma+1)(1+b\rho_0) + 3\Lambda + \frac{2a\rho_0}{c_0^2}(\gamma - 2 + 3b\rho_0)(1+b\rho_0)\right\}(1+\Lambda)^{-1}, \tag{4.4.4}$$

with $\Lambda = e_0^2/c_0^2$ as the Alfvén number , $\psi_0 = mw_0/2x$, and the derivative d/dt of any quantity, which is assumed to be expressed on the front $f(x,t) = 0$, is same as the ordinary time derivative of the quantity. On solving equation (4.4.3), we get

$$\sigma = \begin{cases} \dfrac{\sigma_0}{1+\sigma_0\Pi_0 t}, & \text{for } m = 0 \\ \dfrac{\sigma_0\left(1+w_0t/x_0\right)^{-1/2}}{1+\left(2\Pi_0x_0\sigma_0/w_0\right)\left\{\left(1+w_0t/x_0\right)^{1/2}-1\right\}}, & \text{for } m = 1 \end{cases}, \tag{4.4.5}$$

where σ_0 is the value of σ evaluated at $t = 0$.

4.5 Weak shock waves

From the previous analysis it is evident that a compressive pulse, however weak initially, always culminates into a shock wave after a finite time. The flow and field variables ahead and behind the shock denoted by the subscript 0 and 1 respectively and satisfy the following jump relations for the non-ideal magnetogasdynamics modelled by van der Waals gas equation (Korobeinikov (1976))

$$\rho_1 = \rho_0\left(1+\delta\right), u_1 = \delta G/\left(1+\delta\right),\ h_1 = h_0\left(1+\delta\right)^2,$$

$$p_1 = p_0 + \rho_0 \delta G^2\left(1+\delta\right)^2 - h_0\delta\left(2+\delta\right), \tag{4.5.1}$$

where δ is the shock strength parameter and is defined as $\delta = \left(\rho_1 - \rho_0\right)/\rho_0$, and G is the shock speed. The parameter δ and G are related by the following expression

$$G^2 = \frac{2\left(1+\delta\right)\begin{bmatrix} c_0^2 + a\delta\rho_0\left(\gamma - 2 + b\left(\delta + \rho_0 + \gamma\rho_0\right)\right) + \\ e_0^2\left\{\left(1-b\rho_0\delta\right)\left(1+\delta/2\right) - \left(\left(\gamma-1\right)/2\right)\delta\left(1+b\rho_0\right)\right\} \end{bmatrix}}{2\left(1-b\rho_0\delta\right) - \delta\left(\gamma-1\right)\left(1+b\rho_0\right)} \tag{4.5.2}$$

For a weak shock $\delta << 1$, from equation (4.5.1) and (4.5.2), we have the first approximation as

$$\rho_1 = \rho_0\left(1+\delta\right),\ u_1 = w_0\delta,\ p_1 = p_0\left(1+\delta\gamma\left(1+b\rho_0\right)\right) + \delta a\rho_0^2\left(\gamma\left(1+b\rho_0\right) - 2\right), \tag{4.5.3}$$

and

$$G = w_0\left(1+\Pi_0\delta/2\right). \tag{4.5.4}$$

The conditions derived in equation (4.5.3) are used in further analysis.

4.6 Decay of sawtooth profile

The shock wave, after travelling a long distance becomes sufficiently weak so that one can apply the relations of weak shock waves obtained in equation (4.5.3) and (4.5.4). Here, we assume a weak shock wave at the beginning and study the propagation of disturbances given in the form of sawtooth profile. The left end of the shock profile located initially at x_0 travels with the speed w_0 of the undisturbed fluid, while the shock at the right located initially at x_{s0} moves faster. Let L_0 is the initial length of the sawtooth profile. Suppressing the subscript 1 notation, let us denote by u and w the state at the rear side of the shock, which at any time t is located at $x_s(t) = x_0 + w_o t + L(t)$, where $L(t)$ is the length of the sawtooth profile at any time t. Then

$$G = \frac{dx_s}{dt} = w_0 + \frac{dL}{dt}. \tag{4.6.1}$$

Further, from second part of the equation (4.5.3) and equation (4.5.4) we get

$$G = w_0 + \frac{u\Pi_0}{2}. \tag{4.6.2}$$

The velocity of fluid in the sawtooth profile with constant $\partial u/\partial x$ may be described as

$$u = \sigma L(t), \tag{4.6.3}$$

where $\sigma = (\partial u/\partial x)_{x - x_0 = w_0 t}$ is the slope of the sawtooth profile at any time t, and is given by equation (4.4.5).

Using equation (4.6.3) in (4.6.2) and on comparing to equation (4.6.1), we get

$$\frac{dL}{dt} = \frac{\sigma L(t)\Pi_0}{2}. \tag{4.6.4}$$

Let σ_0, L_0 and G_0 are the values of σ, L and G at time $t = 0$ respectively. Then equation (4.6.2) and (4.6.3) evaluated at $t = 0$, gives the following relation connecting the values σ_0, L_0 and G_0.

$$\sigma_0 = \frac{2(G_0 - w_0)}{L_0\Pi_0}. \tag{4.6.5}$$

Using equation (4.4.5) in equation (4.6.4), we get the length of sawtooth profile as

$$\frac{L}{L_0} = \begin{cases} (1 + \sigma_0\Pi_0 t)^{1/2}, & \text{for } m = 0 \\ \left\{1 + (2\sigma_0 x_0 \Pi_0/w_0)\left((1 + w_0 t/x_0)^{1/2} - 1\right)\right\}^{1/2}, & \text{for } m = 1 \end{cases}. \tag{4.6.6}$$

In view of equation (4.4.5) and (4.6.6), equation (4.6.3) gives the velocity of sawtooth profile as

$$\frac{u}{u_0}=\begin{cases}\left(1+\Pi_0 t\right)^{-1/2}, & \text{for } m=0\\ \left(1+w_0 t/x_0\right)^{-1/2}\left\{1+\left(2\sigma_0 x_0\Pi_0/w_0\right)\left(\left(1+w_0 t/x_0\right)^{1/2}-1\right)\right\}^{-1/2}, & \text{for } m=1\end{cases}, \qquad (4.6.7)$$

where u_0 is the value of u evaluated at time $t=0$.

4.7 Results and discussion

In case of $\bar{a}=0.0$, $\bar{b}=0.0$ and vanishing magnetic field the results obtained in equations (4.6.6) and (4.6.7) reduces to the ordinary gasdynamics results given in (Zieřep (1978)). The length and velocity profiles of sawtooth wave are plotted in figs. (1)-(8) for planar and cylindrically symmetric flows. The three sets of values of van der Waals parameter appearing in numerical computations are taken as (i) $\bar{a}=0.0, 0.4, 0.8$ (ii) $\bar{b}=0.0,\ 0.3,\ 0.6$. Figs. (1) and (2) represent the variation of length of sawtooth profile in a non-ideal gas without magnetic field for planar and nonplanar flows respectively. From the figures we observe that if we increase the value of $\bar{b}$ (keeping $\bar{a}$ fixed) the length of sawtooth profile increases which results in an early decay of shock while an increasing value of $\bar{a}$ (keeping $\bar{b}$ fixed) causes to decrease the length of sawtooth profile as a result the decay of shock will be delayed. Also the length of sawtooth profile increases faster in case of planar flows as compared to the nonplanar flows. Figs. (3) and (4) represents the variation of length of sawtooth profile in planar and nonplanar flows in presence of magnetic field. From these figures we observe that the behaviour of the profiles remains same as in the absence of magnetic field. Figs. (5) and (6) represents the velocity profile of sawtooth waves in absence of magnetic field for planar and nonplanar flows. From these figures we observe that an increase in the value of $\bar{b}$ (keeping $\bar{a}$ fixed) causes to decrease the velocity of wave, while an opposite trend is noted in case of increasing the value of van der Waals parameter $\bar{a}$ (keeping $\bar{b}$ fixed). Similar results are obtained in presence of magnetic field with non-idealness effect which is evident from figs. (7) and (8).

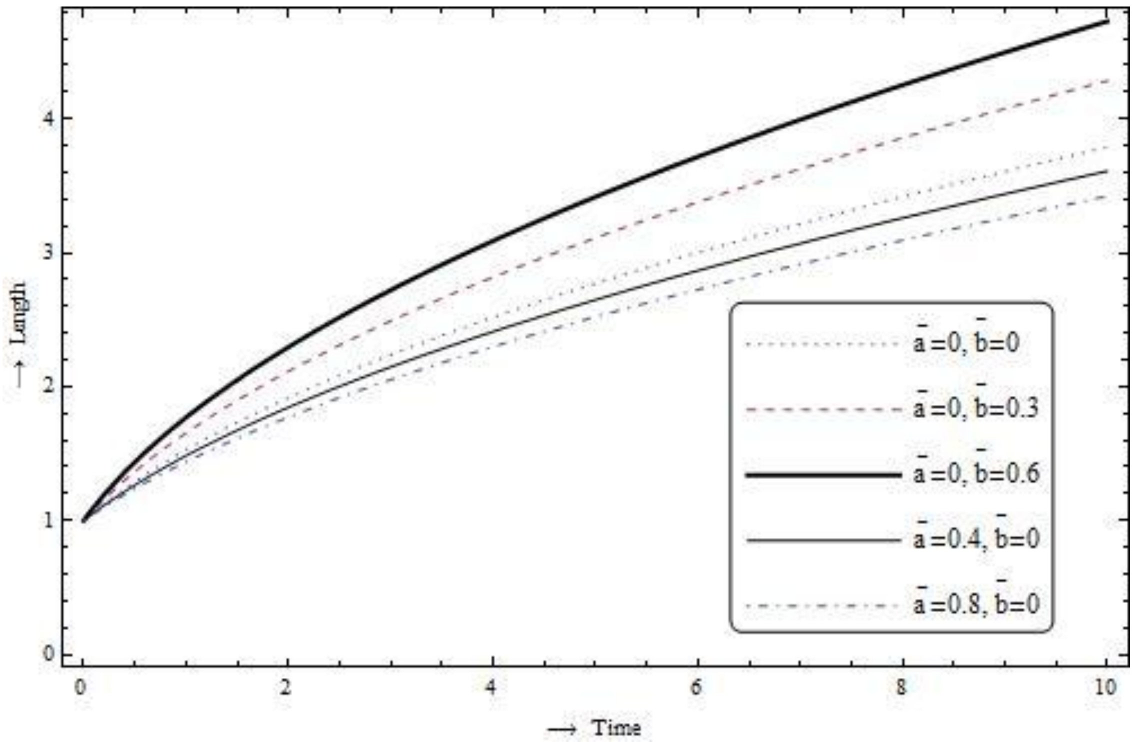

Fig.4.1 Variation of length of sawtooth profile in absence of magnetic field in planar case.

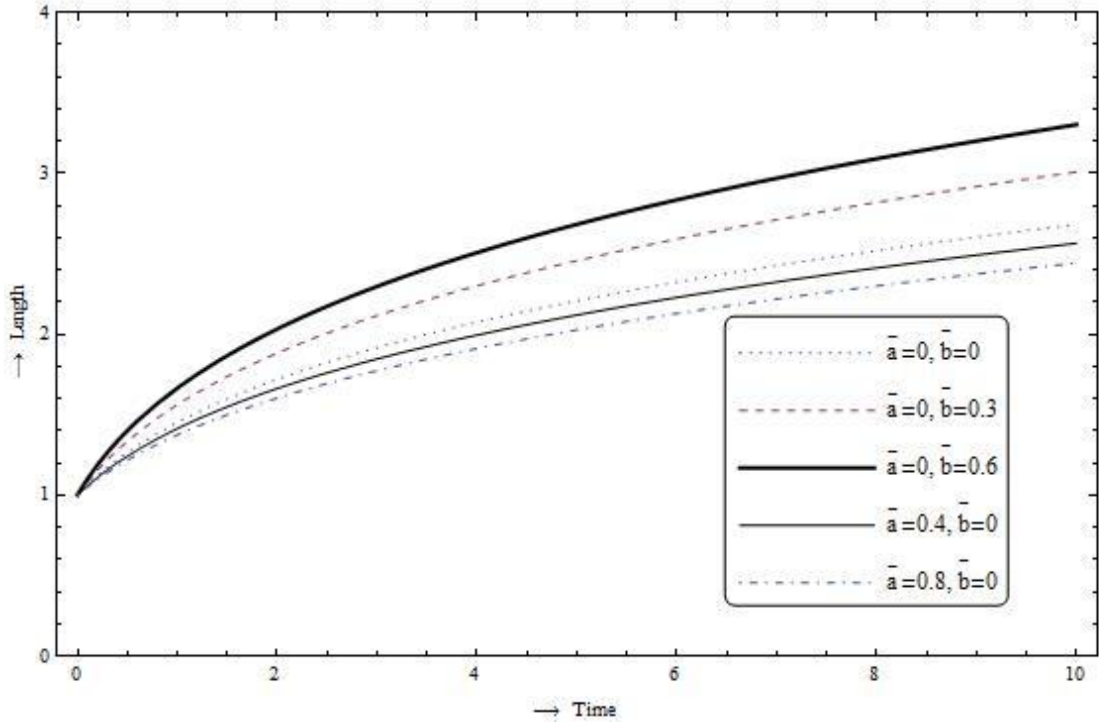

Fig.4.2 Variation of length of sawtooth profile in absence of magnetic field in cylindrical case.

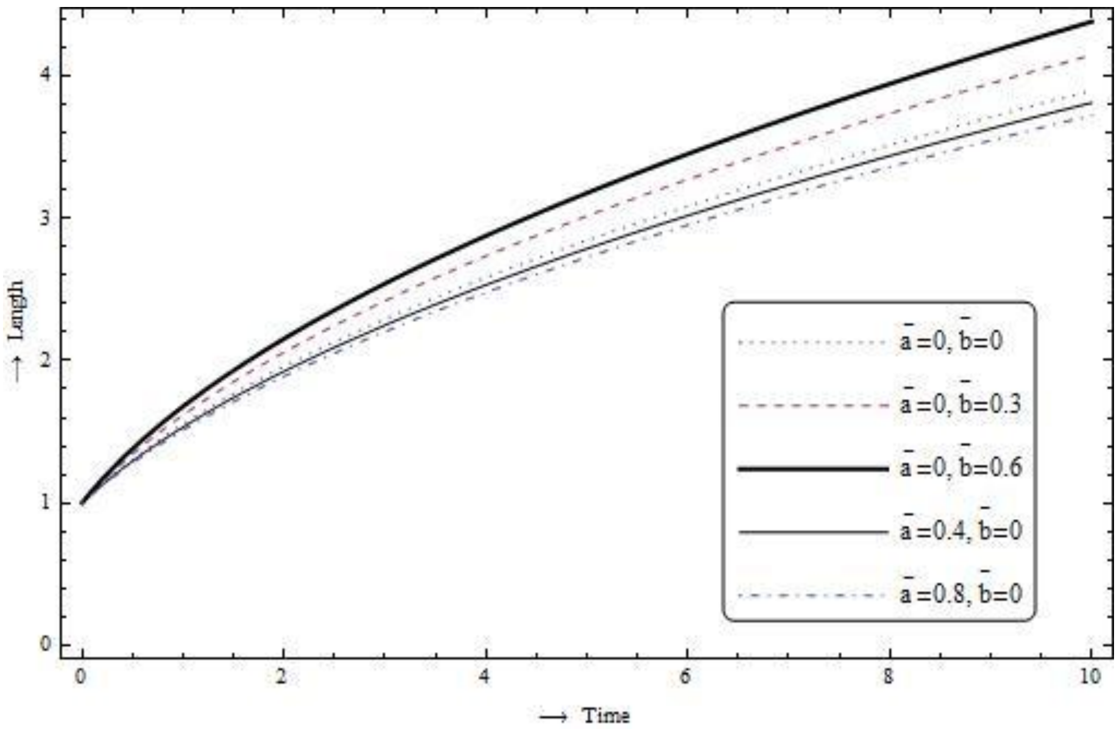

Fig.4.3 Variation of length of sawtooth profile in presence of magnetic field in planar flows.

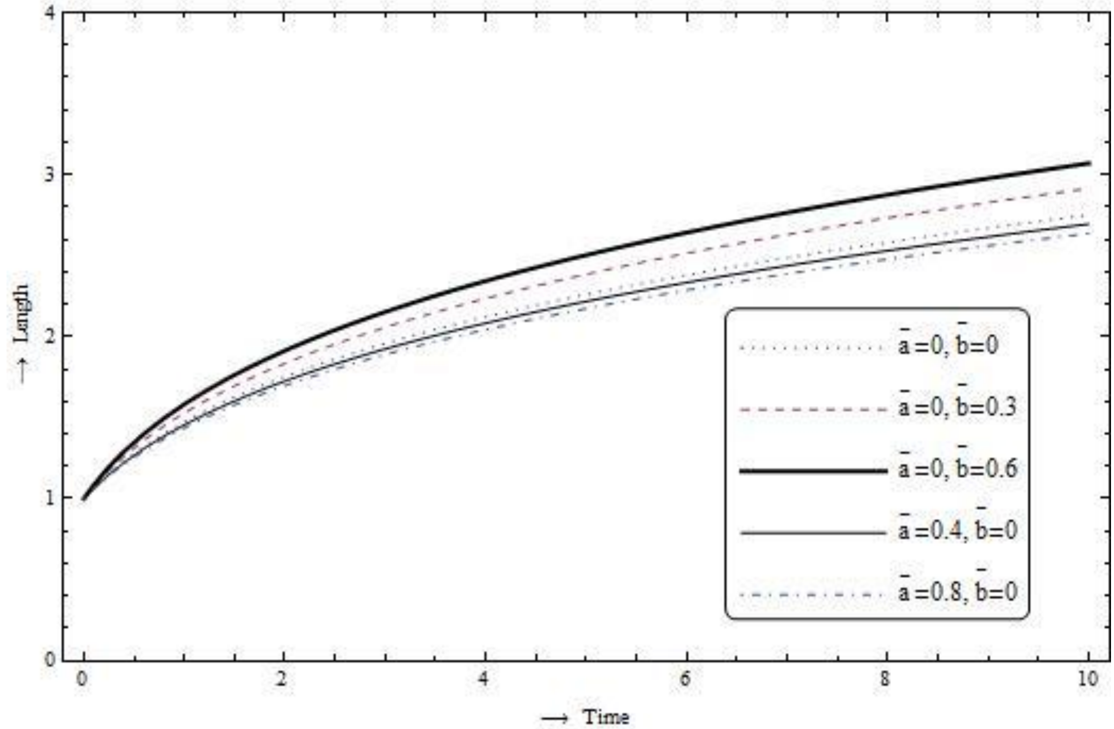

Fig.4.4 Variation of length of sawtooth profile in presence of magnetic field in cylindrical flows.

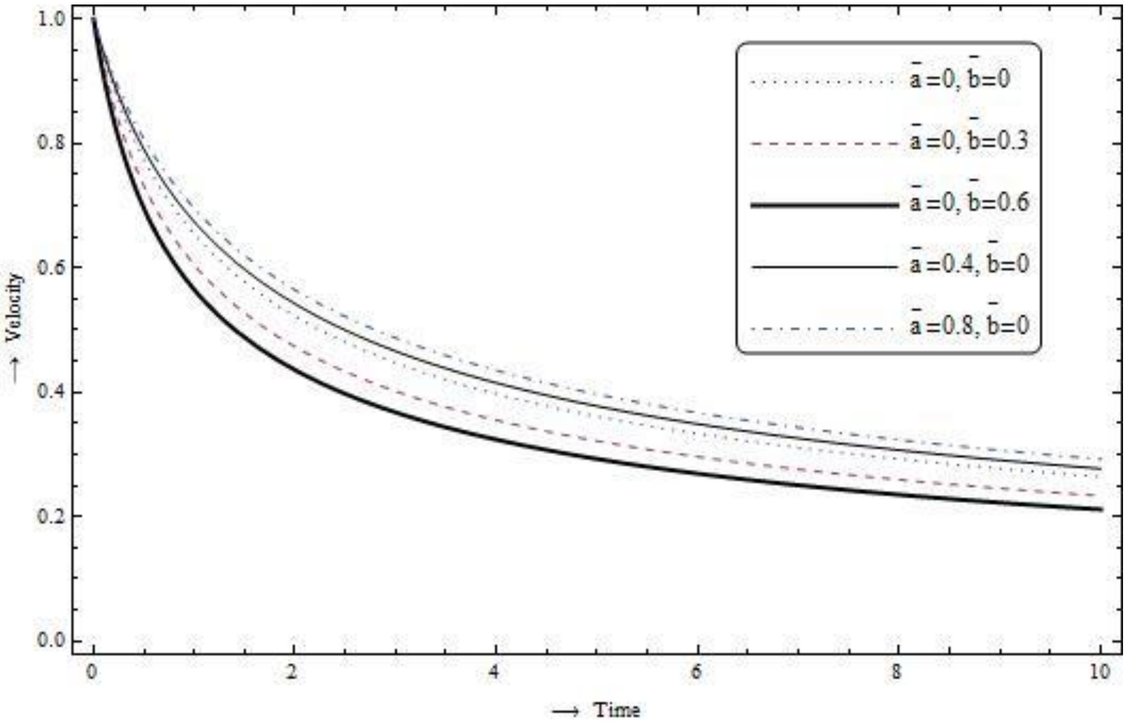

Fig.4.5 Variation of velocity of sawtooth profile in absence of magnetic field in planar flows.

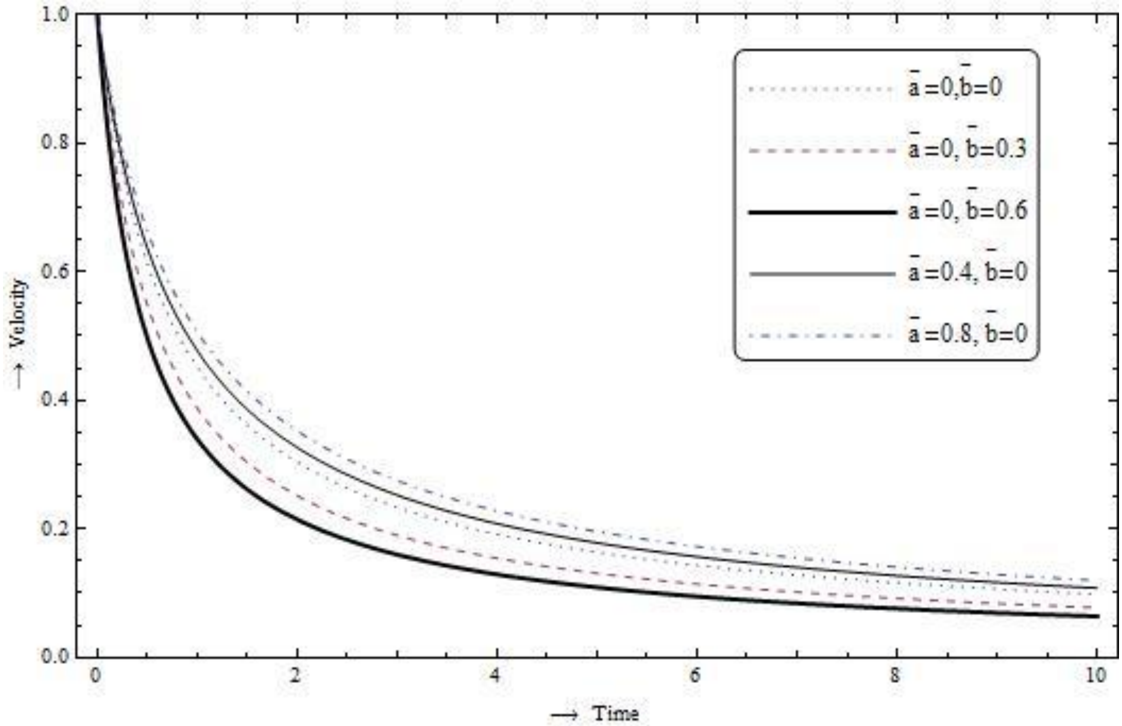

Fig.4.6 Variation of velocity of sawtooth profile in absence of magnetic field in cylindrical flows.

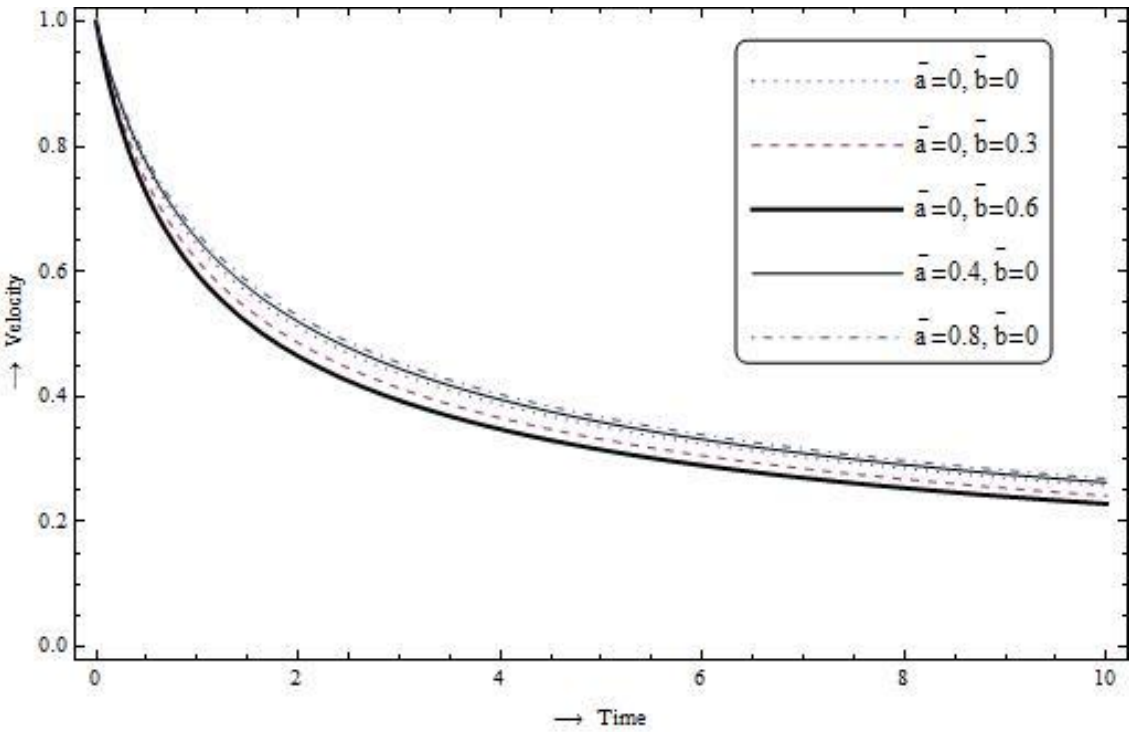

Fig.4.7 Variation of velocity of sawtooth profile in presence of magnetic field in planar flows.

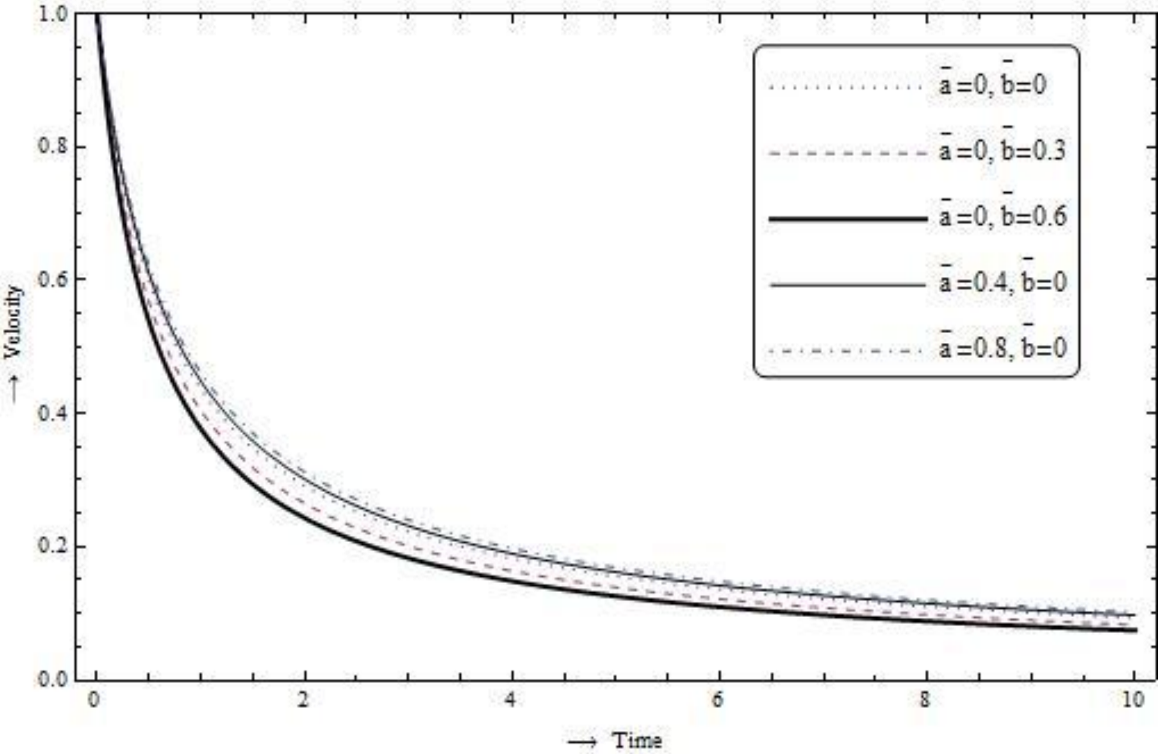

Fig.4.8 Variation of velocity of sawtooth profile in presence of magnetic field in cylindrical flows.

4.8 Conclusions

In the present chapter an asymptotic approach is used to analyze the main features of weakly non-linear waves in a compressible, inviscid, non-ideal gas with infinite electrical conductivity modelled by van der Waals equation of state permeated by transverse magnetic field. The analysis leads to an evolution equation, which characterizes the wave process in a high frequency domain. The growth equation of an acceleration wave is recovered as a special case. Further, we consider a sufficiently weak shock at the front and study the propagation of disturbances in the form of a sawtooth profile. It is observed that, an increase in the value of van der Waals parameter $\bar{b}$ at constant $\bar{a}$ causes an early decay of sawtooth wave, while an opposite trend is observed for an increasing value of van der Waals parameter $\bar{a}$ at constant $\bar{b}$. However the presence of magnetic field causes to slow down the decay process as compared to nonmagnetic case.