Chapter 1

Introduction

1.1 Non-linear waves and hyperbolic equations

Probably the most physical concept of a wave includes the cases of a clearly identifiable disturbance, that may either be localized or non-localized, and which propagates in space with increasing time, a time dependent disturbance throughout space that may or may not be repetitive in nature and which frequently has no persistent geometrical feature that can be said to propagate, and even periodic behaviour in space that is independent of the time. The most important single feature that characterizes a wave when time is involved, and which separates wave-like behaviour from the mere dependence of a solution on time, is that some attribute of it can be shown to propagate in space at a finite speed. This form of a wave is thus inherently connected with motion of some kind involving the space R^n and the time t, so that it gives rise naturally to problem of an evolutionary behaviour with respect to time. For this reason the time variable will always need to be distinguished from the other independent variables.

Mainly there are two types of fluids one is compressible and other is incompressible. Here our main attention is to study the problems arising in the area of compressible fluid dynamics. We are familiar with the propagation characteristics of light and sound waves. Violent disturbances such as, resulting from detonation of explosives, flow through rocket nozzles, supersonic flight of projectiles, or from impact on solids differ from the linear phenomena of sound, light or electromagnetic signals. In contrast to the latter, their propagation is governed by non linear differential equations, and as a consequence the familiar laws of superposition, reflection, and rarefaction ceases to be valid but even more novel features appear, among which the occurance of shock fronts is the most significant. Across shock fronts the medium undergoes sudden change in velocity, pressure and temperature. Even when the start of the motion is perfectly continuous, shock discontinuities may latter arise automatically. Yet, under other conditions just the opposite may happen, initial discontinuities may be smoothed out immediately. Both these possibilities are essentially connected with the nonlinearity of the underlying equation.

The wave propagation may be described by partial differential equations which may be classified either as hyperbolic or parabolic type; here we shall consider only quasilinear hyperbolic partial differential equations. Further if the governing system of equations is non-linear, it is not possible to apply the principle of superposition of solutions as in case of linear partial differential equations. In most physical situations hyperbolic partial differential equations provide the basic mathematical tool to describe the wave propagation. The mathematical theory of hyperbolic equations is dominated by the concept of characteristic hypersurfaces and their geometry. Across these hypersurfaces a continuous solution may exhibit Lipschitz discontinuities in its first or higher order normal derivatives. These hypersurfaces act as transporters of these discontinuities when they exist; they also transport elements of a solution hypersurface when it is differentiable. Further in the case of one-dimension space and time these characteristic hypersurfaces reduce to the family of characteristic curves in the (x,t) plane, along each of which may be transported a Lipschitz discontinuity in a derivative of the solution normal to the characteristics. The solution hypersurface, then reduces to an ordinary smooth surface on which a Lipschitz discontinuity in the first derivative of the solution normal to the characteristics curve manifest itself in the form of a crease on the surface. This crease in the solution surface $R^n \times t$ may be interpreted as representing a clearly defined propagating wavefront. The solution on the side of the wavefront towards which, the propagation of wave takes place is called an undisturbed solution ahead of the wavefront while the solution on the other side regarded as a propagating disturbance flow which is entering in a region occupied by the undisturbed solution.

A large number of physical problems arising in gasdynamics, lead to the formulation of a quasilinear system of first order partial differential equations and these equations are linear in the first derivative of dependent variables, but the coefficients may be functions of dependent variables. For studying the mathematical description let us consider a general quasilinear system of first order equation in $R^n \times t$ written as

$$A_0\left(U,\bar{x},t\right)U_t + \sum_{i=1}^{n} A_i\left(U,\bar{x},t\right)U_{x_i} + B\left(U,\bar{x},t\right) = 0, \qquad (1.1.1)$$

where $U\left(\bar{x},t\right)$ is the column vector with the m elements $u_1\left(\bar{x},t\right)$, $u_2\left(\bar{x},t\right),\ldots\ldots.,u_m\left(\bar{x},t\right)$, $\bar{x} = \left(x_1, x_2, \ldots\ldots\ldots. x_n\right)$ is a vector in R^n, $A_i\left(U,\bar{x},t\right)$ are $m \times n$ matrices and $B\left(U,\bar{x},t\right)$ is the column vector with m elements $b_1\left(U,\bar{x},t\right)$, $b_2\left(U,\bar{x},t\right),\ldots\ldots,b_m\left(U,\bar{x},t\right)$. Here the subscripts t and x_i denote the partial differentiation unless stated otherwise.

The basic idea underlying the hyperbolicity of a system is that the Cauchy problem should be well posed for it. For the first order system (1.1.1) the Cauchy problem amounts to specifying U at points on some initial manifold S in $R^{n-1} \times t$, so that the system will be hyperbolic when this data is sufficient to determine a unique solution that depends continuously on the data specified at points on S. In view of this data and in keeping with the geometrical approach to wavefronts that has been adopted so far, let us now seek to determine when it is possible to so group terms of (1.1.1) that they express the derivative of U normal to S in terms of derivative of U in S.

Further a new coordinate system $(\bar{\xi}, t')$ is introduced, where $\bar{\xi} = (\xi_1, \xi_2, ..., \xi_n)$ with $\xi_i = \xi_i(\bar{x}, t)$ being differentiable functions of their arguments and $t = t'$. The manifold S is associated with the coordinates ξ_k and to have the equation $\xi_k(\bar{x}, t) = c$ (constant), and apart from this restriction, the other ξ_i' will be chosen arbitrarily. The transformation thus becomes

$$t = t', \quad \xi_i(\bar{x}, t) = \text{constant, for } i = 1, 2, ..., n. \tag{1.1.2}$$

Also it is assumed that initially the transformation is non-singular in the vicinity of S. Using the transformation (1.1.2), Eq. (1.1.1) may be written as

$$A_0(U, \bar{x}, t)\left(\frac{\partial U}{\partial t'} + \sum_{j=1}^{n} \frac{\partial \xi_j}{\partial t} \frac{\partial U}{\partial \xi_j}\right) + \sum_{i,j=1}^{n} A_i(U, \bar{x}, t) \frac{\partial \xi_j}{\partial x_i} \frac{\partial U}{\partial \xi_j} + B(U, \bar{x}, t) = 0. \tag{1.1.3}$$

As the derivative of U is to be expressed normal to S, and S has been embedded in the family of coordinate manifolds $\xi_k(\bar{x}, t) = \text{Const.}$, it follows that the required derivative is $\partial U / \partial \xi_k$, which may be separated from (1.1.3) and may be written in the following form:

$$\Lambda \partial U / \partial \xi_k + R = 0, \tag{1.1.4}$$

where

$$\Lambda = \left(A_0(U, \bar{x}, t) \frac{\partial \xi_k}{\partial t} + \sum_{i=1}^{n} A_i(U, \bar{x}, t) \frac{\partial \xi_k}{\partial x_i}\right), \tag{1.1.5}$$

and R is the column vector with its m elements dependent upon $U, \bar{x}, t$ and $\partial U/\partial \xi_k$ with $i \neq k$. Consequently, the derivative $\partial U/\partial \xi_k$ normal to S is to be derivative from (1.1.4) provided Λ^{-1} exists, which implies the condition

$$\det \Lambda \neq 0. \tag{1.1.6}$$

Dividing $\det \Lambda$ by $\left|\nabla_x \xi_k\right| = \left(\sum_{i=1}^{n} \left(\partial \xi_k / \partial x_i\right)^2\right)^{1/2}$ and setting

$$-\lambda = \frac{\partial \xi_k / \partial t}{\left|\nabla_x \xi_k\right|},\ \upsilon_i = \frac{\partial \xi_k / \partial x_i}{\left|\nabla_x \xi_k\right|} \text{ for } i = 1, 2, ..., n, \tag{1.1.7}$$

so that the unit vector $\bar{\upsilon} = \left(\upsilon_1, \upsilon_2, ...\upsilon_n\right)$ is then the normalized spatial gradient $\nabla_x \xi_k$ of ξ_k.

Using Eq. (1.1.7), the condition (1.1.6) is written as

$$F\left(P; \bar{\upsilon}, \lambda\right) \neq 0, \tag{1.1.8}$$

where

$$F\left(P; \bar{\upsilon}, \lambda\right) = \left|\sum_{i}^{n} \upsilon_i A_i\left(P\right) - \lambda A_0\left(P\right)\right|. \tag{1.1.9}$$

Here the notation $A_i\left(P\right)$ represents the value of $A_i\left(U, \bar{x}, t\right)$ at point P of the manifold S. The expression $F\left(P; \bar{\upsilon}, \lambda\right)$ is homogeneous polynomial of degree m in the quantities $\left(-\lambda, \upsilon_1, \upsilon_2, ...\upsilon_n\right)$, which is also called characteristics polynomial of the system (1.1.1) with respect to S.

Here it may be noted that the normal derivative $\partial U/\partial \xi_k$ will be indeterminate at any point P of a manifold S for which

$$F\left(P; \bar{\upsilon}, \lambda\right) = 0. \tag{1.1.10}$$

The manifold S on which the condition (1.1.10) is satisfied are called characteristic manifold; and the manifolds for which (1.1.8) is satisfied are called non-characteristic.

The system (1.1.1) is said to be strictly hyperbolic in t-direction at any point P if the roots $\lambda^{(1)}$, $\lambda^{(2)}, \ldots\ldots\ldots \lambda^{(m)}$ of the characteristic equation $F\left(P;\bar{\upsilon},\lambda\right)=0$ are real and distinct for all choices of the unit vectors $\bar{\upsilon}$ and if the right eigenvectors $r^{(1)}$, $r^{(2)}, \ldots\ldots r^{(m)}$ satisfying

$$\sum_{i=1}^{m}\left[\upsilon_i A_i\left(P\right)-\lambda^{(j)}A_0\left(P\right)\right]r^{(j)}=0, \tag{1.1.11}$$

span the space E^m occupied by the m element eigenvectors. The system (1.1.1) will merely be said to be hyperbolic in t-direction if the eigenvectors span the space E^m but the eigen values, although all real, are not all distinct (Jeffery (1976), Courant and Friedrichs (1948)).

1.2 Simple waves and progressive waves

Let us consider the following system of quasilinear hyperbolic partial differential equations for the one-dimensional flow

$$U_t + A(U)U_x = 0, \tag{1.2.1}$$

where U is a vector in n-dimensional Euclidian space and the coefficient matrix A is a function of U.

The solution vector U is said to define a simple waves if it can be expressed in terms of variables by means of a single function; the corresponding flows are called simple wave flow (Germain (1972), Witham (1974)).

The system of equation (1.2.1) is said to be hyperbolic if the coefficient matrix A has n real eigenvalues $\lambda^{(i)}$ $(i=1,2,3.....n)$ and $r^{(i)}$ the corresponding eigenvectors.

The simple wave solutions of the system (1.2.1) are solution of the form

$$U(x,t) = U(\bar{X}), \tag{1.2.2}$$

where $\bar{X} = \bar{X}(x,t)$.

Thus the equation (1.2.1) yields

$$A\left(\frac{dU}{d\bar{X}}\right) = -\left[\left(\partial\bar{X}/\partial t\right)/\left(\partial\bar{X}/\partial x\right)\right]\left(\frac{dU}{d\bar{X}}\right), \tag{1.2.3}$$

which implies that $dU/d\bar{X}$ is an eigenvector of the matrix $A(U)$ associated with the eigenvalues

$$\lambda^{(i)} = \left(\left(\partial\bar{X}/\partial t\right)/\left(\partial\bar{X}/\partial x\right)\right). \tag{1.2.4}$$

The function $U(\bar{X})$ is determined by the integration of the following equation

$$\frac{dU}{d\bar{X}} = r^{(i)},$$

i.e., of the following system

$$\frac{dU_1}{r_1^{(i)}} = \frac{dU}{r_2^{(i)}} = ... = \frac{dU}{r_n^{(i)}} = d\bar{X}. \tag{1.2.5}$$

After determining the value of $U(\bar{X})$, we can find $\bar{X}(x,t)$ by integration of Eq. (1.2.4). Thus we have

$$x = r^{(i)}t + f(\bar{X}), \tag{1.2.6}$$

where the function $f(\bar{X})$ is arbitrary. The curves along which the function $\bar{X}$ is constant are called simple waves, i.e. the simple waves are the straight lines given by the Eq. (1.2.6). The surface $\bar{X}$ =constant is called wavelets. Therefore, U remains constant if and only if one stays on the wavelet.

$U(x,t)$ is said to describe a progressive wave if there exists a family of propagating wavelets $\bar{X}=$constant, with

$$F(x,t)=\bar{X}\ , \tag{1.2.7}$$

such that the magnitude of the rate of change of U or eventually its derivatives, when x is moving with such a wavelet is small compared with the magnitude of rate of change of U when x is kept fixed (Germain (1972)).

In order to determine the progressive wave solution one has to introduce $\bar{X}$ either by replacing one of the independent variables such as t or to add an extra one which describes the wavelet. Following this definition one may write

$$\bar{X}=\psi X=F(x,t), \tag{1.2.8}$$

where ψ is small parameter. Now U can be written as

$$U(x,t)=U(x,t,X). \tag{1.2.9}$$

In the equation (1.2.9) the right had side U is a function of three independent scalar variables. In order to study the physical description, one may replace X by its value given by the Eq. (1.2.8). Also we can see that the rates of change are given by

$$\frac{\partial U}{\partial t}=\frac{1}{\psi}\left(\frac{\partial U}{\partial X}\right)\frac{\partial F}{\partial t}+\frac{\partial U}{\partial t}\ , \tag{1.2.10}$$

$$\frac{\partial U}{\partial x}=\frac{1}{\psi}\left(\frac{\partial U}{\partial X}\right)\frac{\partial F}{\partial X}+\frac{\partial U}{\partial x}\ , \tag{1.2.11}$$

Since ψ has been taken to be small parameter, so we may assume that the first partial derivative of U and F are bounded, say $O(1)$. Along any given path $x(\eta)$, $t(\eta)$ one may write

$$\frac{\partial U}{\partial \eta}=\left(\frac{1}{\psi}\right)\left(\frac{\partial U}{\partial x}\right)\left(\frac{dF}{d\eta}\right)+\frac{\partial U}{\partial t}\left(\frac{dt}{d\eta}\right)+\left(\frac{\partial U}{\partial x}\right)\left(\frac{dx}{d\eta}\right).$$

The left had side of above Equation is of the order of $O\left(\psi^{-1}\right)$; but if $F(\eta)$ is taken to be constant it is only of $O(1)$. Also the results which have been used to obtain general theorems for linear and non-linear systems were given by Ludwig (1960), Lewis (1965) and Courant and Hilbert (1962).

1.3 Ideal and non-ideal gas

The ideal gas law is an extension of experimentally discovered gas laws i.e. Charles's law and Boyle's law. The equation of state of an ideal gas is written as $PV=nRT$, where n is the number of molecules of the gas, R is the gas constant, T is the absolute temperature, P is the pressure and V is the volume of the gas.

The equation of real gases is $\lim\limits_{P\to\infty} PV/RT=1$ also the compressibility factor Z defined as

$$Z(P,T)=PV/RT\ . \tag{1.3.1}$$

It may be noted that the deviation from unity in the value of Z indicates the degree of departure from ideal behaviour.

In theoretical derivations of the ideal gas law it is necessary to make two assumptions i.e. the gas molecules are too small to neglect their volume and the molecules are non-interacting. If the temperature of the gas is very high and density is too low, the assumption that the gas is ideal is no longer valid; therefore the alternative to the ideal gas is simplified van der Waals model. Dutch Physicist van der Waals derived the equation of state without the assumptions of ideal gas, which is known as the van der Waals equation of state and is written as

$$\left(P+\frac{a}{V^2}\right)(V-b)=RT\,, \tag{1.3.2}$$

where a denotes the amount of intermolecular force of attraction between the gas particles and b denotes the neglected volume which is associated with the volume of the gas.

Wu and Roberts (1996) presented the similarity solutions and determined conditions for the stability of strong spherical implosions for both ideal and van der Waals gases. They found that when the van der Waals excluded volume is sufficiently large, a new type of solution is found and the shock may be linearly stable. Wu and Roberts (1996) investigated the problem of structure and stability of strong spherical shock waves whereas Somogyi and Roberts (2007) analyzed numerical stability of an imploding spherical shock wave in van der Waals gas.

1.4 Dusty gas

Dusty gas is a homogenous mixture of solid particles and gas. The solid particles may occupy less than or equal to 5% of the total volume and mix well with the fluid in the flow field. This is known as the dilute phase of the two-phase flow of a mixture of solid particles and fluid. Or we simply call it the two-phase flow of a mixture of solid particles and a fluid in a narrow sense. The paper of Marble (1963) is the first attempt to apply the modern technique of fluid dynamics to the research of the two phase flow of gas and solid particles. He introduced many important concepts of the problem in his analysis such as temperature and diameter of the solid particle in the distribution function of solid particles. These are very important in the development of the fundamental equations of the mixture of gas and solid particles. There are some assumptions for the solid particles in the mixture as; the size of the solid particles should be uniform, the shape of the solid particles should be spherical because body of different shapes have different drag coefficient and heat transfer rate. Collision between particles of different size is not considered in this thesis. It is also assumed that the solid particles are uniformly distributed in the gas. The distance between particles are small as compared with the cross sectional of the duct. The boundary layer effect and heat transfer with the duct walls are not considered here.

Here we will derive some simple relations for a mixture of gas and small solid particles. Let us consider the thermodynamic equilibrium condition such that $T_p = T_g = T$, where T_p and T_g are the temperature of the solid particles and gas respectively. The density of the mixture is given as

$$\rho = Z\rho_{sp} + \left(1 - Z\right)\rho_g , \tag{1.4.1}$$

where Z is the volume fraction of the solid particles in the mixture and is defined as $Z = V_p / V$, where V_p is the volume occupied by the solid particles in the mixture and V is the total volume of the mixture. ρ_{sp} is the species density of the solid particles and is defined as $\rho_{sp} = M_p / V_p$, where M_p is the mass of the solid particles in the volume V. ρ_g is the species density of the gas.

The mass concentration of the solid particles is defined as

$$k_p = \frac{Z\rho_{sp}}{\rho} . \tag{1.4.2}$$

The pressure of the mixture is given as

$$p = p_p + p_g , \tag{1.4.3}$$

where p_p and p_g are the partial pressure of solid particles and gas respectively.

Using the perfect gas law, the total pressure of the mixture is given as

$$p = \rho_g R T_g . \tag{1.4.4}$$

From equations (1.4.1), (1.4.3) and (1.4.4), we get the following relation between the pressure and the density of the mixture as

$$p = \left(\frac{1-k_p}{1-Z} \right) \rho R T . \tag{1.4.5}$$

The internal energy of the mixture per unit mass E is related to the internal energies of the two species by the following relation

$$\rho E = Z \rho_{sp} c_{sp} T_p + \left(1-Z\right) \rho_g c_v T_g , \tag{1.4.6}$$

where c_{sp} is the specific heat of the species.

From equation (1.4.6) we have

$$E = k_p c_{sp} T_p + \left(1-k_p\right) c_v T_g \ . \tag{1.4.7}$$

For thermodynamic equilibrium condition, we have the specific heat of the mixture at constant volume c_{vM} as

$$c_{vM} = k_p c_{sp} + \left(1-k_p\right) c_v , \tag{1.4.8}$$

where c_v is the specific heat of the gas at constant volume.

Also for thermodynamic equilibrium condition, we have the specific heat of the mixture at constant pressure c_{pM} as

$$c_{pM} = k_p c_{sp} + \left(1-k_p\right) c_p , \tag{1.4.9}$$

where c_p is the specific heat of the gas at constant pressure.

The specific heats of the mixture are independent of the volume fraction Z but depend on the mass fraction k_p of the solid particles. The ratio of the specific heats of the mixture is

$$\Gamma = \frac{c_{pM}}{c_{vM}}. \tag{1.4.10}$$

Using equation (1.4.8) and (1.4.9) in equation (1.4.10) and after simplification, we have

$$\Gamma = \frac{\gamma\left(1+\lambda\beta\right)}{1+\lambda\beta\gamma}, \tag{1.4.11}$$

where $\gamma = c_p/c_v$, $\beta = c_{sp}/c_p$ and $\lambda = k_p/\left(1-k_p\right)$. Note that the value of Γ is always smaller than γ. If we put $k_p = 0$ in equation (1.4.11) then Γ turns to γ.

If we consider the mixture as a homogenous medium, the first law of thermodynamics for the mixture gives

$$dQ = dE - \frac{1}{\rho^2} p d\rho, \tag{1.4.12}$$

where dQ is the heat addition to the mixture.

For isentropic change of state of the mixture we have $dQ = 0$, thus in view of equation (1.4.7) equation (1.4.12) gives

$$\frac{1}{\Gamma-1}\frac{dT}{T} = \frac{1}{1-Z}\frac{d\rho}{\rho}. \tag{1.4.13}$$

This on simplification gives

$$T\left(\frac{\rho}{1-Z}\right)^{-\Gamma} = \text{constant}. \tag{1.4.14}$$

If $Z \ll 1$, the isentropic change of state of the mixture has a similar relation as that for a pure gas with an effective ratio of specific heats Γ.

Similarly from equation (1.4.5) for a given k_p and $T_p = T$, we have

$$\frac{dp}{p} = \frac{dT}{T} + \frac{1}{1-Z}\frac{d\rho}{\rho}, \tag{1.4.15}$$

From equation (1.4.13) and (1.4.15), we have

$$\frac{dp}{p} = \frac{\Gamma}{1-Z}\frac{d\rho}{\rho}. \tag{1.4.16}$$

On simplification (1.4.16) we get

$$p\left(\frac{\rho}{1-Z}\right)^{-\Gamma} = \text{constant.} \tag{1.4.17}$$

Again if $Z << 1$, equation (1.4.17) is identical in form for the corresponding relation of a pure gas but an effective ratio of specific heats.

Now we can find the equilibrium speed of sound of the mixture a from equation (1.4.17) as follows

$$a^2 = \left(\frac{dp}{d\rho}\right) = \frac{\Gamma\left(1-k_p\right)RT}{\left(1-Z\right)^2}. \tag{1.4.18}$$

If $Z << 1$, equation (1.4.18) is identical in form as that for a pure gas, but the effective ratio of specific heats and effective gas constant R are used.

1.5 Magnetogasdynamics

Magnetogasdynamics is the branch of science in which we study the motion of electrically conducting fluids through magnetic field. The interest in magneto hydrodynamics particularly in problems associated with the behaviour of high temperature plasma, in the presence of magnetic field, was enhanced by the advent of studies of thermonuclear fusion reaction (Branover (1978)).

The interaction between gasdynamics phenomena and the magnetic field is examined by combining the field equation with those of gasdynamics. In most of the electromagnetic problems involving conductors the maxwell's displacement currents are ignored (Pai (1962), Kantrowitz and Petschek (1966), Anile and Greco (1978) and Spitzer (1967)). The magnetic permeability of the media considered in magnetogasdynamics differ only slightly from unity and therefore, is taken here as unity in the application. The field equations are then

$$\nabla \times \bar{E} = -\frac{1}{c}\bar{H}_t\,, \tag{1.5.1}$$

$$\nabla \times \bar{H} = \frac{4\pi}{c}\bar{J} = \frac{4\pi}{c}\sigma\left(\bar{E} + \frac{\bar{u}\times\bar{H}}{c}\right), \tag{1.5.2}$$

$$\nabla \cdot \bar{H} = 0\,, \tag{1.5.3}$$

where $\bar{E}$ is the electric intensity, $\bar{H}$ is the magnetic field, σ is the electrical conductivity, $\bar{J}$ is the current density, $\bar{u}$ is the velocity of fluid and c is the velocity of light.

Let us consider σ being uniform in the medium. Substituting the Eq. (1.5.3) in (1.5.2) we get

$$\bar{H}_t - \nabla \times \left(\bar{u}\times\bar{H}\right) = \frac{c^2\nabla^2\bar{H}}{4\pi\sigma}. \tag{1.5.4}$$

In the case of infinite electrical conductivity, Eq. (1.5.4) becomes

$$\bar{H}_t + \left(\bar{u}\cdot\nabla\right)\bar{H} + \bar{H}\left(\nabla\cdot\bar{u}\right) - \left(\bar{H}\cdot\nabla\right)\bar{u} = 0\,. \tag{1.5.5}$$

Using equation (1.5.3) in (1.5.5) we get

$$\bar{H}_t + \left(\bar{u}\cdot\nabla\right)\bar{H} + \bar{H}\left(\nabla\cdot\bar{u}\right) = 0\,. \tag{1.5.6}$$

The equation (1.5.6) is used in conjunction with gas dynamic flow equations to incorporate the effect of magnetic field interaction.

In order to investigate the hydrodynamic shocks, the electrical conductivity of the medium is assumed to have infinite value. This assumption implies that the self-induction will prevent changes to magnetic field of the medium at rest (Hoffman and Teller (1950) and Kulikovski and Liubimov (1961)). Also the governing equations degenerate into the non-convex hyperbolic system, for which the characteristics surface may have unexpected singularities, making the wave structure much more complex than aerodynamic shocks (Courant and Hilbert (1962) and Jeffrey and Taniuti (1964)). Ideal magneto-hydrodynamics offers impressive potential applications but also generates many unanswered questions and uncertainties (Kantrowitz and Petschek (1966)).

1.6 Riemann problem

Riemann problem is a shcok tube problem. Consider a long, thin, cylindrical tube containing a gas separated by a thin membrane. We assume that the gas is at rest on both sides of the membrane, but that is of different constant pressure and densities on each side. At $t=0$, the membrane is broken, and the problem is to determine the ensuing motion of the gas. This problem was first studied by Riemann and is now known by his name.

Let (ρ_l, u_l, p_l) and (ρ_r, u_r, p_r) denote the density, velocity and pressure on both sides of the membrane. We consider the case where $u_l = u_r = 0$, $\rho_l > \rho_r$, $p_l > p_r$, and all these quantities are constant.

Furher Riemann problem is an initial value problem for the one dimensional Euler equations supplemented by discontinuous initial data and its solution constitutes the basic building block for the construction of a solution to the general initial value problem. The solution to the Riemann problem depends on the variables at the left and right states and is always similar, namely the values of the quantities are all constant on any ray issuing from the initial position of the jump. Moreover, the solution of the Riemann problem is composed of three waves, with always a contact discontinuity as the middle one while the other two are indifferently rarefaction or shock wave. If both external waves are rarefaction then it might occur to the formation of a vacuum region between two parts of the gas receeding from each other.

In mathematical form consider the one dimensional Euler's equation in conserved form as

$$\frac{\partial U}{\partial t}+\frac{\partial F(U)}{\partial x}=0, \tag{1.6.1}$$

where $U=\left(\rho,\rho u,\rho\left(u^2/2+E\right)\right)^{tr}$, $F(U)=\left(\rho u, p+\rho u^2,\rho u\left(u^2/2+E\right)+pu\right)^{tr}$, with E as the internal energy of the gas.

The Riemann problem for the system of Euler's equation (1.6.1) is an initial value problem with data of the form

$$U(x,0)=U_0(x)=\begin{cases}U_l & ,x<0\\ U_r & ,x>0\end{cases}, \tag{1.6.2}$$

where U_l and U_r are the left and right constant states, while $x=0$ is a point of discontinuity.

1.7 Review of literature

When there is a relative motion between a body and a fluid, the disturbance (if infinitely small) caused by the body is propagated through the fluid with the speed of sound. The speed of sound in such cases is the speed with which rarefaction waves or compression waves of very small amplitude propagate. When the compressions in the flow are of finite amplitude, there usually occurs a discontinuous rise of pressure leading to a shock wave. In addition to discontinuities in pressure, there occur discontinuous increase in temperature, density, entropy and other fluid properties. The instant of formation of shock wave was largely studied by various authors. Macpherson (1971) studied the formation of a shock wave in dense Argon by applying a molecular-dynamic approach. Shifrin (1970) studied the formation of a shock wave in a plane flow of a perfect gas. Saldatov (1970) studied a symmetric two-way traffic flow and determined the instant of formation of a shock wave by using the Riemann method. Sharma et al. (1987) provided a non-linear analysis of traffic flow in which a shock wave appears. During the past decades many authors have studied the problem of growth and decay of shock waves propagating in a variety of media. Ardavan-Rhad (1970) studied the propagation of plane shock wave into a non-isentropic, non-viscous and non-heat conducting media. Boillatt (1965) presented the general theory of propagation of shock waves. Flack and Wittig (1971) presented the general solution for the case of normal shock wave moving through a medium in which all flow properties vary arbitrarily. Boillatt and Ruggeri (1979) studied the problem of reflection and transmission of discontinuity waves through a shock wave. By applying numerical methods Sod (1977) studied the propagation of one-dimensional shock wave with cylindrical and spherical symmetry. Chen and Gurtin (1970) and Colemann and Gurtin (1967) studied the growth and decay of shock waves with internal state variables. Chen (1971) studied the propagation of shock waves in elastic non-conductors. Thermodynamic influences on the propagation of shock waves have been studied by Chen (1973). Bowen and Chen (1974) studied the same problem in the ideal mixture with several temperature layers.

One of the interesting properties of the shock waves is the problem of determining the differential effects of shock fronts on the rear flow field. To this problem Thomas (1947) developed a tensorial approach which was further extended by Kanwal (1958) for three dimensional shocks in stationary, pseudo-stationary and unsteady flows of non-conducting gases. The problem of vorticity generation by a shock has also been solved by several authors like Trusdell (1952), Hayes (1957), Kanwal (1960) and Ram (1978). The evolution of weak discontinuities for quasilinear hyperbolic system was analyzed by Boillatt and Ruggeri (1979).

A considerable amount of work has also been done on the shock structure. A lot of work on the shock structure was carried out by Kuznetsov (1979), Goldman and Sirovich (1969), Boillatt and Ruggeri (1998). Wave fronts which are concave in the direction of propagation exhibit different kinds of behaviour depending on the strength of the wave-front. Generally, wave front propagates normal to itself and therefore has a tendency to converge. The shocks of weak strength are called weak shocks. Focusing of weak shock is an important problem. This problem of focusing of weak shocks was studied by Wanner et al. (1972). Observers of atomic explosions are also known to have seen shock waves of strong strength, called blast wave. Ram (1981) provided a closed form self similar solution to a MHD flow disturbed by propagating blast waves. Further, in the final stages of the collapse, the shock becomes very strong and the pressure ahead is neglected in comparison to the pressure behind the shock wave. This leads to similarity formulation of the problem. In the problem, the ratio of distance to a particular power of time is known as similarity exponent, which is not known a priori. Several numerical and analytical methods have been developed for the determination of similarity exponent of the problem e.g. Guderley (1942), Taylor (1950), Butler (1954), Sedov

(1959), Stanyukovich (1960), Welsh (1967), Zel'dovich & Raiser (1967) and Lazarus (1981), Chisnell (1998). Zen'kevich and Stepanov (2007) provided analytical solution of self similar equations in Lagrangian mass coordinate, describing the dynamics of the explosion and the propagation of a strong shock wave.

The exact solution of the Riemann problem is very useful as it represents the solution to a system of hyperbolic conservation laws subject to the simplest, non–trivial, initial conditions; in spite of the simplicity of the initial data the solution of the Riemann problem contains the fundamental physical and mathematical character of the relevant set of conservation laws. The solution of the general initial value problem may be seen as resulting from non–linear superposition of solutions of Riemann problems (Glimm (1965)). There is no exact closed–form solution to the Riemann problem for the Euler equations, not even for ideal gases; in fact not even for much simpler models such as the isentropic and isothermal equations. However, it is possible to derive iterative schemes whereby the solution can be computed numerically to any desired, practical, degree of accuracy. Godunov (1959) is credited with the first exact Riemann solver for the Euler equations. By today's standards Godunov's first Riemann solver is computationally inefficient. Later, Godunov (1976) proposed a second exact Riemann solver. Distinct features of this solver are: the equations used are simpler, the variables selected are more convenient from the computational point of view and the iterative procedure is rather sophisticated. Much of the work that followed contains the fundamental features of Godunov's second Riemann solver. Chorin (1976) independently, produced improvements to Godunov's first Riemann solver.