Dedicated

To

My Grand Father

And

My Parents

CERTIFICATE

It is certified that the work contained in the thesis titled *"**SOME INVESTIGATIONS ON NON-LINEAR WAVE PROPAGATION IN GASEOUS MEDIA**"* by ***Mr. TRILOKI NATH*** has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

It is further certified that the student has fulfilled all the requirements of Comprehensive examination, Candidacy and SOTA for the award of Ph.D. Degree.

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DECLARATION BY THE CANDIDATE

I, ***TRILOKI NATH***, certify that the work embodied in this thesis is my own bonafide work and carried out by me under the supervision of ***Prof. L.P. SINGH*** from a period ***September 2012*** **to** ***December 2017*** at ***DEPARTMENT OF MATHEMATICAL SCIENCES***, Indian Institute of Technology, Varanasi. The matter embodied in this thesis has not been submitted for the award of any other degree/diploma. I declare that I have faithfully acknowledged and given credits to the research workers wherever their works have been cited in my work in this thesis. I further declare that I have not willfully copied any other's work, paragraphs, text, data, results, *etc*., reported in journals, books, magazines, reports dissertations, theses, *etc*., or available at websites and have not included them in this thesis and have not cited as my own work.

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It is certified that the above statement made by the student is correct to the best of my knowledge.

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Preface

Fluid dynamics is the branch of applied science which is concerned with the movement of fluids. Mainly there are two types of fluid, incompressible and compressible fluids. Here our main attention is to study the problems of compressible fluid. The first International congress on gasdynamics was convened in Rome in 1935. The intensive development of gasdynamics began during and after the second world war in connection with the wide use of gasdynamics in technology: jet aviation, rocket weaponry, rocket and jet engines; supersonic aircraft and missiles etc. We are familiar with the propagation characteristics of light and sound waves. Violent disturbances such as, resulting from detonation of explosives, flow through rocket nozzles, supersonic flight of projectiles or from impact on solids differ from the linear phenomena of sound, light or electromagnetic signals. In contrast to the latter, their propagation is governed by non-linear partial differential equations, and as a consequence the familiar laws of superposition, reflection, and rarefaction ceases to be valid but even more novel features appear, among which the occurrence of shock fronts is the most significant. Across shock fronts the medium undergoes sudden change in velocity, pressure and temperature.

The theoretical and mathematical foundation of gasdynamics is formed by the application of basic laws of mechanics and thermodynamics to a moving volume of compressible fluid. The German scientist B. Riemann (1860), the English scientist W. Rankine (1870), and the French scientist H. Hugoniot (1887) investigated the propagation of shock waves in a gas. These waves appear only in compressible media, and their speed is greater than the speed of sound in the media. B. Riemann also developed the foundations of the theory of unsteady motion of a gas i.e. motion for which the parameters of gas flow at every point change with time.

The wave propagation may be described by partial differential equations which may be classified either as hyperbolic or parabolic type; here we shall consider only quasilinear hyperbolic partial differential equations. Further if the governing system of equations is non-linear, it is not possible to apply the principle of superposition of solutions as in case of linear partial differential equations. In most physical situations hyperbolic partial differential equations provide the basic mathematical tool to describe the wave propagation. The understanding and behaviour of finite amplitude high frequency pulses associated with the governing system, their subsequent culmination into shocks, and the propagation distance over which dissipation, dispersion, is very important. It is necessary to look for approximate analytical and numerical techniques whose objective is precisely to do this.

The main objective of the present thesis is to review some of the important problems in the area of compressible fluid alongwith the powerful mathematical methods as they take place currently. Also our aim is to study the problems of gasdynamics motion by suitably modifying the equation of state in a non-ideal gas medium, dusty gas medium, and non-ideal magnetogasdynamics medium. The present thesis is divided into six chapters. References to the equations are of the form (m.n.p), where m, n and p denote the chapter, section and equation number respectively.

Chapter-1 is introductory and gives a general idea about the mathematical theory of quasilinear hyperbolic partial differential equations and when and how the discontinuities occur. Certain terminologies which are commonly used in non-linear wave propagation have been defined and discussed. The necessary idea about simple waves and progressive waves and their mathematical theory have been discussed in brief. The general idea of magnetogasdynamics with their field equations is given. The physical properties of ideal gas, non-ideal gas, van der Waals gas, dusty gas and non-ideal gas in presence of magnetic field with their equation of state have been reviewed. Further an overview of Riemann problem of Euler equations is also described in this chapter.

In **Chapter-2**, the theory of progressive waves is used to analyze the main features of the finite amplitude disturbances, moderately small amplitude disturbances in a dusty gas for generalized geometry. The conditions, under which a complete history of the evolutionary behaviour of shock waves including weak shock can be traced out, are determined. It is also assessed as to how the presence of dust particles in the gas affects the existence of shock or no shock. Here it is observed that the amplitude dispersion depends on the amplitude of the wavelets which is dependent on the values of the mass fraction of the dust particles. Also the shock formation distance varies according to the variation of mass fraction of the dust particles. Further the effect of variation of mass fraction of the dust particles on the growth and decay behaviour of shock in cylindrically symmetric and spherically symmetric flows are discussed.

In **Chapter-3**, we use the method of multiple time scales to derive the asymptotic solution of system of one-dimensional quasilinear hyperbolic equations for the generalized geometry in van der Waals gas. The transport equation for the amplitude of resonantly interacting high frequency waves propagating into non-ideal gas is derived. The theory of weakly non-linear geometrical acoustics has been used to derive those conditions in which wave interactions occur resonantly. Also the interaction coefficients are determined which measures the strength of the coupling between different wave modes. Further, we discuss the cases when the initial data for the wave amplitude is of 2π periodicity. The effect of van der Waals parameter on the solution of the problem is discussed. Also the growth and decay behaviour of wave amplitudes for the nonplanar waves are studied. The evolutionary behaviour of non-resonant wave modes culminating into shock wave and its location are examined in van der Waals fluid.

In **Chapter-4**, our aim is to analyze the main features of weakly non-linear waves propagating in a compressible, inviscid, non-ideal gas with infinite electrical conductivity modelled by van der Waals equation of state permeated by transverse magnetic field. An asymptotic approach is used to derive the evolution equation, which characterizes the wave phenomena in a high frequency domain. The growth equation of an acceleration wave is derived as a special case. Further, we discuss the propagation of disturbances in the form of sawtooth profile. The effect of magnetic field and van der Waals parameter on the decay of sawtooth profile is presented. A remarkable difference between planar and nonplanar flows in magnetic case and nonmagnetic case has been drawn. Also the variation in velocity profile between planar and nonplanar flows has been discussed.

In **Chapter-5**, our aim is to investigate the growth and decay behaviour of acceleration waves in one-dimensional unsteady, inviscid, compressible, planar and non-planar flows in dusty gas. The solution of the problem of propagation of acceleration waves is obtained along the characteristic path by using the characteristics of the governing system of quasilinear hyperbolic partial differential equations as the reference coordinate system. It is shown that a linear solution in the characteristic plane exhibits non-linear behaviour in the physical plane. The transport equation describing the growth and decay behaviour of acceleration waves in dusty gas medium is derived. The effect of dust particles on the formation of shock in planar and non-planar flows has been discussed. Also those cases are discussed where the compressive wave fronts may terminate into shock waves. The effect of variation of mass fraction of the dust particles and the ratio of specific heat of solid particles and the specific heat of the gas at constant pressure with fixed volume fraction on the behaviour of amplitude of the acceleration waves in the case of planar, cylindrically symmetric and spherically symmetric flows is discussed.

In **Chapter-6**, The Riemann problem for a quasilinear hyperbolic system of equations governing the one dimensional unsteady flow of an ideal polytropic gas with dust particles is solved analytically without any restriction on magnitude of the initial states. The elementary wave solutions of the Riemann problem, that is shock waves, rarefaction waves and contact discontinuities are derived explicitly and their properties are discussed, for a dusty gas. The existence and uniqueness of the solution for Riemann problem in dusty gas is discussed. Also the conditions leading to the existence of shock waves or simple waves for a 1-family and 3-family curves in the solution of the Riemann problem are discussed. It is observed that the presence of dust particles in an ideal polytropic gas leads to more complex expression as compared to the corresponding ideal case; however all the parallel results remain same. Also, the effect of variation of mass fraction of dust particles with fixed volume fraction (Z) and the ratio of specific heat of the solid particles and the specific heat of the gas at constant pressure on the variation of velocity and density across the shock wave, rarefaction wave and contact discontinuities are discussed.