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List of symbols

ϕ	The Scaling Function
ψ	The Wavelet Function or *Mother wavelet*
A	Constraint Matrix
W	Spectrum of the desired signal in passband
Y	Spectrum of the matched wavelet in passband
1	vector of length L whose entries are 1's only
$\mathbb{Z}$	Set of integers
$\mathbb{R}$	Set of real numbers
$L^2(\mathbb{R})$	Set of all square integrable functions $\mathbb{R}$
$L^2(\mathbb{R}^n)$	Set of all square integrable functions $\mathbb{R}^n$
$(W_\psi)f(a,b)$	Continuous wavelet Transform of function f with respect to wavelet ψ
Subscripts	
Λ_Ψ	Group delay of the matched wavelet
Λ_F	Group delay of the desired signal
C_ψ	Admissibility constant
Acronyms	
STFT	Short-time Fourier transform
CWT	Continuous Wavelet Transform
DWT	Discrete Wavelet Transform
MRA	Multi-Resolution Analysis
CR	Compression Ratio
PRD	Percent root mean square difference

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