# CHAPTER 5

## Conclusion and Future Directions


### *Abstract*

*In this chapter, the contributions of this thesis are stated and the scope for future research has been discussed.*

*In section 5.1, the concluding remarks are presented chapter wise and in section 5.2, the possible scope for future work has been stated.*

## 5.1 Concluding Remarks

The research contribution of this thesis and the results achieved are as follows:

***Chapter 1*** presents the definition of graphs and various types of graphs. Also, the application of graphs has been discussed. There are several types of graph problems but those graph problems which are based on the idea of path length and ranking are considered in this thesis and a few such problems have been discussed in this chapter. Then the source of uncertainty in real life and the need to introduce fuzziness in the graph problems has been stated. Taxonomy of fuzzy sets has been discussed but the thesis focusses on the use of fuzzy sets and intuitionistic fuzzy sets only. The literature survey of the two problems considered viz. constrained shortest path problem and orienteering problem and a few ranking methods for fuzzy numbers and intuitionistic fuzzy numbers has been presented. Finally, the chapter discusses the research contributions and the layout of the thesis briefly.

***Chapter 2*** deals with the fuzzy numbers, their ranking methods and application to a few graph problems. This chapter considers the fuzzy version of the two problems, namely the constrained shortest path problem and the orienteering problem. A new ranking method has been proposed called the Link Preference Index that can rank the Quasi-Gaussian fuzzy numbers and has been applied on the shortest path problem, minimum spanning tree problem and Steiner tree problem. An extended Dijkstra's algorithm and Prim's algorithms has also been suggested that uses the link preference index and can work in the fuzzy environment.

Also, the trapezoidal fuzzy numbers have been used for modelling the parameter cost in the constrained shortest path problem and the other parameter involved i.e. delay, remains crisp. A fuzzy version of the Chen's path discretization algorithm has been presented and a few recently proposed ranking methods that include the Circumcenter of Centroid, maximizing set / minimizing set and weighing function methods have been compared to determine which one is more suitable for real life applications. An application of the trapezoidal fuzzy numbers and ranking method has also been suggested for the wireless sensor networks.

A max-min formulation has been proposed for the orienteering problem that works in the fuzzy environment and helps in providing latitude to the solution by considering the satisfaction and aspiration levels of the decision maker. A parallel formulation has been presented for the fuzzy orienteering problem in order to make the proposed method applicable for large instances and through theoretical analysis it has been shown that the parallel formulation is work-optimal.

***Chapter 3*** considers the intuitionistic fuzzy numbers, their ranking methods, intuitionistic fuzzy points and application to the two graph problems viz. the constrained shortest path problem and the orienteering problem. The trapezoidal intuitionistic fuzzy numbers have been used for representing the parameter cost of the constrained shortest path problem while the other parameter i.e. delay remains crisp. A centroid based ranking method has been proposed for the trapezoidal intuitionistic fuzzy number that uses an eight parameter representation instead of the already existing ranking methods in the literature that uses a six parameter representation and thus provides a more generalized representation than the already existing methods. Also, a centroid of centroids (CoC) method of ranking has been proposed for the trapezoidal intuitionistic fuzzy numbers. The idea of Quasi-Gaussian intuitionistic fuzzy number and a centroid based ranking method for the Quasi-Gaussian intuitionistic fuzzy number has been proposed and applied on the constrained shortest path problem.

The intuitionistic fuzzy version of the orienteering problem has also been considered. The two parameters involved i.e. score and time are represented as trapezoidal intuitionistic fuzzy numbers and a max-min formulation has been suggested that considers the hesitancy, degree of acceptability etc. of the decision maker and provides a latitude to the solution. In order to tackle the larger instances efficiently, a work-depth analysis has been presented for the intuitionistic fuzzy orienteering problem and theoretically, the algorithm has been shown to be work-preserving. Thus, it can be efficiently translated into a multiprocessor model like the PRAM model.

A new definition for the intuitionistic fuzzy metric space using the idea of intuitionistic fuzzy points and intuitionistic fuzzy scalars has been proposed which is different from the existing intuitionistic fuzzy metric spaces as it considers the uncertainty involved in the position or the location of the point of interest. The distance measure that has been proposed was applied on the intuitionistic fuzzy orienteering problem for the first time to model the uncertainty in the position of the point of interest by representing the nodes as intuitionistic fuzzy points and the uncertainty in the parameter score has been modelled using trapezoidal intuitionistic fuzzy numbers.

***Chapter 4*** presents heuristics and meta-heuristic for dealing with the two graph problems considered, namely the orienteering problem and the constrained shortest path problem. Through initial literature survey, it was observed that the methods proposed by different authors for tackling the orienteering problem can only be applied on complete graphs. In real life, it is not necessary that there be a direct link between the two points of interest and the triangle inequality property is satisfied. Therefore, complete graphs cannot model the practical situations appropriately. The need was to have an algorithm that can be implemented for incomplete graphs as well. In 2011, Ostrowski *et al*. (2011) suggested a genetic algorithm for the orienteering problem that can be applied on both complete as well as incomplete graphs. In this chapter, an algorithm $SEL\_OP$ has been proposed for the orienteering problem that can be applied on both complete as well as incomplete graphs. The algorithm initially considers four selection methods and through experimental analysis it has been shown that out of the four selection methods, the roulette wheel selection method has a better performance. Then the algorithm has been executed using the roulette wheel selection method ($RWS\_OP$) and its performance has been compared against the Ostrowski's algorithm for standard benchmark instances. It has been observed that the proposed algorithm shows a better performance than the Ostrowski's algorithm both in terms of the total collected score and the execution time.

A meta-heuristic called the flower pollination algorithm introduced by Yang (2012), has been applied for the orienteering problem and it has been shown experimentally that $FPA\_OP$ algorithm helps in achieving a better total collected score than the best known heuristic for the orienteering problem i.e. GRASP (Campos, Marti, Sanchez-Oro, & Duarte, 2013) and the proposed heuristic $RWS\_OP$ for larger values of $T_{max}$. Thus, when the decision maker is willing to achieve a higher total collected score at the cost of time delay, the $FPA\_OP$ algorithm can be preferred.

A bidirectional search heuristic has been implemented for the constrained shortest path problem and through experiments it has been shown that the proposed heuristic performs better (in the average case) than the Chen's path delay discretization algorithm for the constrained shortest path problem.

## 5.2 Future Scope

In future, the proposed fuzzy and intuitionistic fuzzy models can be applied on other graph problems which are based on the concept of path length and ranking like Minimum Spanning Tree Problem, Steiner Tree Problem, Travelling Salesman Problem, Hamiltonian Path Problem, Vehicle Routing Problem, Chinese Postman Problem, Longest Path Problem, Facility Location Problem, etc.

The heuristics and meta-heuristics proposed can be extended for other graph problems that are NP-Hard or NP-Complete and based on the idea pf path length and ranking like the Hamiltonian Path Problem, Vehicle Routing Problem etc.

Also, it would be interesting to study that how the known approximation ratios for the two considered problems, namely the orienteering problem and the constrained shortest path problem are effected by the fuzzy and intuitionistic fuzzy models, whether better approximation ratios can be obtained for the orienteering problem and the constrained shortest path problem and how the recently reported heuristics and meta-heuristics like the firefly algorithm, bat algorithm etc. performs when applied on the orienteering problem and the constrained shortest path problem.