CHAPTER 1

Introduction

Abstract

In this chapter, a brief introduction to the problems considered in the thesis and basic concepts studied has been presented. Related work, the main contributions of this thesis and the layout of the thesis has also been stated.

In section 1.1, graphs, types of graphs, applications of graphs and a few graph problems based on the concept of path length and ranking have been discussed. In section 1.2, various sources of uncertainty in real life have been described. Section 1.3 presents the definition of fuzzy sets and other concepts used for modelling uncertainty has been stated in section 1.4. In section 1.5, the literature survey related to the graph problems and models and algorithms considered in the thesis has been discussed. In section 1.6, the motivation has been stated. Finally, section 1.7 highlights the main contributions of this thesis and section 1.8 presents the layout of the thesis.

1.1 Graphs

In the area of Computer Science and Mathematics, graphs are structures that can represent the pair wise relations between objects from a collection. It can be depicted as an ordered pair $G = (V, E)$ where $V = \{v_1, v_2, \dots..\}$ denotes the set of vertices and $E = \{e_1, e_2, \dots..\}$ signifies the set of edges such that each edge e_t is associated with an ordered pair of vertices(v_s, v_d). An edge having the same vertex as both its end points i.e. (v_l, v_l) is termed as a *self-loop* and if there is more than one edge connecting the same pair of vertices i.e. if there are two edges e_1 and e_2 having the same end points (v_3, v_4) then they are called *parallel edges*. The number of edges incident on a vertex $v_i \in V$ denotes the *degree* $d(v_i)$ or *valency* of the vertex. A *walk* in a graph G is an alternating sequence of vertices and edges and a walk where a vertex can be visited at most once is termed as a *path*. A path that has the same initial and terminal vertex is called a *cycle* or a *circuit*. A walk in which all the vertices of a graph G are traversed and each and every vertex is visited exactly once is denoted as a *Hamiltonian path* and in case the source and target vertex of the walk are same then it represents a *Hamiltonian cycle* or *Hamiltonian circuit* (Deo, 1974).

1.1.1 Types of Graphs

(a) Directed and Undirected Graph: A directed graph, also called a diagraph is a graph in which each edge has a specific direction and the edge can be traversed only in that direction. However, in an undirected graph, edges do not have any orientation. For example, if e_1 is an edge connecting the nodes (v_1, v_2), then the edge can be traversed only from v_1 to v_2 in case of a directed graph and can be traversed both ways from v_1 to v_2 and v_2 to v_1 in case of an undirected graph (Deo, 1974).

(b) Complete Graph: A complete graph is one in which there exists an edge for each pair of vertices belonging to the vertex set V (Deo, 1974).

(c) Finite and Infinite Graphs: In a finite graph $G = (V, E)$, both the sets (edge and vertex sets) are finite and if either of the sets (edge or vertex sets) or both are infinite, then it is an infinite graph (Deo, 1974).

(d) Connected and Disconnected Graphs: In an undirected graph G, if there exists a path between any two vertices v_s and v_d then it is called a connected graph else it is a disconnected graph (Deo, 1974).

(e) Weighted Graphs: A graph G with some weight (w) i.e., a number associated with each edge of the graph is called a weighted graph. These weights signify the parameters cost, time, delay, distance, length etc. depending upon the requirement of the problem at hand (Deo, 1974).

(f) Regular Graphs: In a graph G, if the degree of all the vertices is same then it is termed as a regular graph (Deo, 1974).

(g) Simple Graph and Multigraph: In a simple graph, there exist no loops and parallel edges. However, a graph consisting of loops and parallel edges is termed as a multigraph (Deo, 1974).

1.1.2 Applications of Graphs

Graph theory is about links and associations between objects and entities and in our world almost everything is linked. Graph theory finds application in several fields and real life areas. The cities and countries are connected through streets, rail and flights. The components of an electric circuit and computer chips are linked. The web pages on the internet are connected through hyperlinks. In the medical field, DNA sequencing and the paths of disease outbreaks can be modelled as networks. In order to understand and analyse these networks and optimize the resource utilization, scientists, engineers etc., use the concept of graph theory (Deo, 1974).

Graphs find varied application in several practical areas as it can represent different types of relations between processes and few of such applications are discussed here. In designing of the integrated chips which consists of several transistors that needs to be joined in such a way that the connections are optimized and the best performance of the chip is assured. Also, the concept of graphs is used to determine the links between the web pages of the internet, used in the social networking sites to determine the connections between different communities and the hierarchies that exist between the substructures, in the GPS system for determining the shortest interconnect between the source and destination, in computer network security to avoid worm propagation, etc. (Deo, 1974).

Graphs can be used in intelligent transportation systems and air traffic controllers to minimize the overall congestion and reduce the heavy traffic condition by determining efficient routes. It is used in the field of operations research to perform the task of scheduling, for the allocation of frequencies in GSM mobile phone networks, in communication networks, coding theory etc. In the field of linguistics, graph-theoretic methods can be used to explore the syntax and semantics of the natural languages. To study the molecular structures and topology of atoms in physics and chemistry, graph concepts can be used. Some other applications includes the utilization of graphs in sociology to explore the relations, in biology to determine the breeding patterns, growth and spread of diseases, in mathematics for studying the topology and geometry etc. (Deo, 1974).

1.1.3 Graph Problems

As discussed above, graphs find applications in various fields and several problems can be represented as graphs. Here, a few of them are defined, specifically those problems which implement the concept of distance.

- ***Shortest Path Problem (SPP):*** For a given weighted undirected graph, the shortest path is the path linking the source and the destination such that the sum of the weights of the edges connecting the two nodes is minimized. This problem finds wide range of applications like it can be used in the computer games to determine the shortest path for moving from one point to the other, it can be used in the GPS navigation system to determine the cheapest/shortest path in terms of time or distance connecting one city to the other or connecting two locations within the same city, in circuit wiring to determine the most optimized connections, in flight itineraries etc. (Cormen, Leiserson, Rivest, & Stein, 1990).

- ***Constrained Shortest Path Problem (CSPP):*** A variant of the shortest path is the constrained shortest path problem. In this problem there is more than one constraint which makes the problem intractable and it falls under the class of NP-Complete problems. In this problem, the requirement is to determine a minimum cost path connecting the source and the target such that the total delay of the path is less than or equal to threshold value. This problem finds application in the telecommunication networks, real time and multimedia applications like video teleconferencing, web broadcasting, remote diagnosis etc. where providing quality of service (QoS) assurance is essential (Chen, Song, & Sahni, 2008).

- ***Travelling Salesman Problem (TSP):*** Given a complete weighted undirected graph G, the task is to obtain a Hamiltonian cycle of minimal cost. It is an NP-Hard combinatorial optimization problem and is a problem of interest as well as importance as it finds several practical applications in the fields of logistics, transportation, scheduling, planning, routing, DNA sequencing, design of microchips etc. Some real applications are devising the school bus route for the pick-up and drop of children within a city, determining the route of trucks for the pick-up and delivery of parcels, movement of the farming equipment from one point to the other for the testing of soil, delivery of meals to employees working at different places etc. (Cormen, Leiserson, Rivest, & Stein, 1990).

- ***Orienteering Problem (OP):*** This is a variant of TSP and can be observed as a combination of two well-known problems viz. TSP and the Knapsack problem. It is an NP-Hard problem and given a weighted undirected graph with weights associated with both the vertices as well as edges, the task is to compute a Hamiltonian path connecting the initial and final nodes and visit as many intermediate nodes as possible such that the reward collected can be maximized within the stated time bound. Some important applications of this problem are found in the tourism industry, robot path planning in the disaster management etc. where the goal is maximum utilization of the resources with some restriction on the time available (Vansteenwegen, Souffriau, & Oudheusden, 2011).

- ***Minimum Spanning Tree Problem (MST):*** Given a connected weighted undirected graph G, the goal is to determine a subgraph of G which is a tree connecting all the vertices of the graph. Such a sub graph of G is called a spanning tree. For a graph, several spanning trees can be generated and the one among these multiple spanning trees that has the minimum total cost of the edges is called the minimum spanning tree. An application of this problem is found in the cable television, telecommunication (for telephone and internet) etc. industry for the designing of the layout of cable or wire networks in order to connect a point to the other either directly or indirectly such that the overall cost can be minimized (Cormen, Leiserson, Rivest, & Stein, 1990).

- ***Steiner Tree Problem (STP):*** For a given weighted undirected graph $G = (V, E, w)$ where w is a weight function that represents a mapping of the edge set E to the set of non-negative numbers and another set $(S \subseteq V)$. The problem is to generate a sub graph of G without cycles that covers the nodes in S in such a manner that the total cost of the edges is minimized. It finds application in the fields of computational biology, network designing, wireless communication, facility location problem, designing of circuit layouts etc. (Kou, Markowsky, & Berman, 1981).

- ***Chinese Postman Problem:*** This problem was first considered by a Chinese mathematician Kwan (1962) and therefore the problem was named the Chinese postman problem. The problem was related to a postman whose job is to collect the mails from the post office, deliver it to their desired locations and return back to the post office. His job is bound by the constraint that he should visit each street in his area at least once and to accomplish this task he wishes to select such a route that can minimize his walking. Therefore, for a given weighted connected graph, the aim of the problem is to determine a minimum-weight tour. Some of the applications of this problem includes analysing DNA, routing robots and determining the cheapest path for snow removal, cleaning of the streets, garbage collection etc. (Edmonds, & Johnson, 1973).

1.2 Uncertainty in Real Life

In each of the above described problems, we consider a weighted graph for the representation and the weights assigned to the edges and nodes represent parameters like cost, time, length, delay, distance, capacities, demand, rewards etc. In practice, it is difficult to determine these parameters precisely and accurately and the uncertainty that exists can be modelled through fuzzy sets. For example, let us consider the problem of determining the shortest path that connects two locations and represent the interconnections between the two points of concern through a weighted graph, where weights represent the time taken to move from one location to the other. Practically, while following the path, the time taken depends upon the traffic conditions and several other factors which cannot be predicted exactly beforehand. Therefore, the time taken to traverse an edge might exceed the real value assigned to the edge. To deal with such a situation, it would be beneficial to represent the parameter "time" with a fuzzy number instead of a real number that not only considers the existing uncertainty but also provides a more realistic solution. Similarly, in several other applications of each of the above stated problems and various other graph problems, the major concern is to deal with the imprecise nature of the parameters so that a more practical output can be generated.

1.3 Fuzzy Sets

The fuzzy set theory, introduced by Zadeh (1965) is a generalization of the classical set theory. The conventional sets also called crisp sets consider those objects which possess some precise properties i.e., it is a two-valued logic which means that if a question is asked like "Does this object belong to that set?" then the answer will always be either "yes" or "no", "is" or "isn't", "1" or "0". Therefore, in case of crisp sets there is no ambiguity regarding the belongingness of an element to that set and the boundary of the set is very well defined, clear and precise. This is valid for both stochastic as well as deterministic cases.

In case of probability theory that deals with the statistical uncertainty the question can be formulated like "What is the probability that this element belongs to that set?". To this the answer can be "The probability that this element is a member of that set is 90%". However, even this answer leads to the conclusion of the type "is" or "is not" a member of the set i.e., the chances of correctly predicting that the element belongs to the set is 90%. Hence, the degree of membership or non-membership cannot be indicated through probability theory.

There are several real world applications which cannot be represented and tackled through crisp sets because in our day to day life there exists a lot of vagueness and the data and information available to deal with a problem are imprecise. Let us consider an example to understand this concept. Suppose a person is learning driving. Now, the student has to be taught that when to apply the brakes if a red light is encountered.

What should be the instruction?

"Apply the brakes 74 feet before the crossing".

Or

"Apply the brakes well in time".

The first instruction is very precise but not very useful. However, the latter instruction carries a lot of vagueness still is a more appropriate instruction because the decision should be taken at the spot and the uncertainty due to traffic conditions etc. cannot be predicted beforehand. Thus, fuzzy sets were introduced to deal with the non-statistical uncertainties of the practical world. The fuzzy model helps in identifying, representing, interpreting and manipulating the uncertainties closely associated with the real world problems (Bezdek, 1993; Chen, & Pham, 2001).

In fuzzy sets, the concept of membership grade denoted as μ, is defined which considers the degree of belongingness of an element to a particular set. Let us consider an example, if we say that a person with 50yrs or more of age is "old" then what is the answer to the following question?
"Is a person with 49yrs of age old? "

The answer to the question is no for crisp sets because the entity with 49yrs does not belong to the set "old" but in case of fuzzy sets, the concept of degree of membership is introduced, according to which a person with 49yrs of age is old with a degree of 0.9 and that with 25yrs of age is old with a degree of 0.5 as depicted in figure 1.

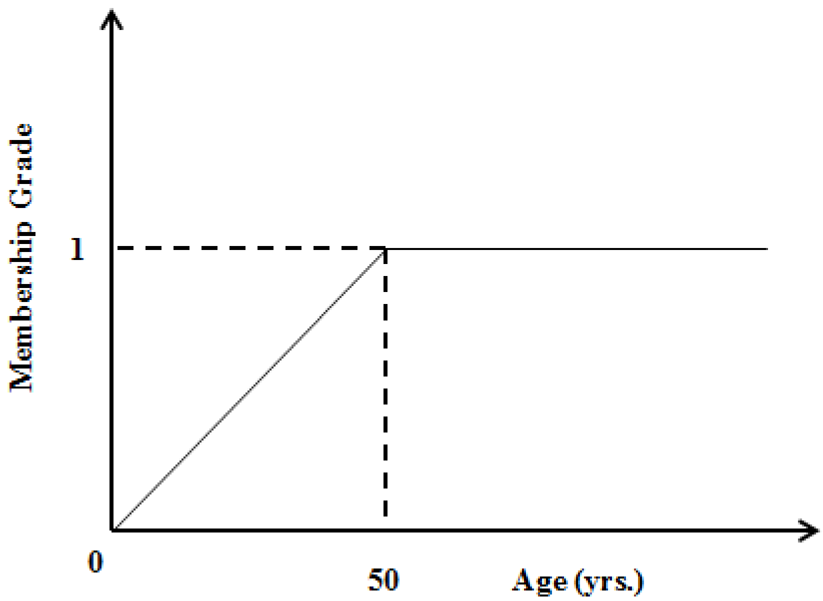

Fig. 1.1: Pictorial Representation of degree of membership

1.4 Other Concepts Modelling Uncertainty

1.4.1 Rough Sets

The idea of rough sets was proposed by Pawlak (1982) which is based on the concept of approximations. It is a mathematical approach to deal with imprecision or vagueness that can be represented as a boundary region of a set. Two kind of topological operations are involved in rough sets, namely interior and closure which are termed as *approximations.* A few basic concepts and notations used in rough set theory are described below (Walczak, & Massart, 1999):

- Let U be a finite set of objects called the *Universe.*
- Let a binary relation R be given such that $R \subseteq U \times U$ called the indiscernibility relation. It denotes our lack of knowledge about the universe U.
- R is assumed to be an equivalence relation.
- The *approximation space* can be represented by a pair (U, R) where U is the universe and R is an equivalence relation on U.
- Let us consider X as a subset of U i.e., $X \subseteq U$. The aim is to characterize X with respect to R.
- The equivalence class of R that is determined by element X can be represented as $R(x)$.
- The equivalence classes of R are called *granules* and they denote that portion of knowledge which can be perceived due to R.
- The set of all objects that can be classified with *certainty* as members of X with respect to R signify the $R - lower\ approximation\ of\ X$ and can be represented as

$$R_*(X) = \{x: R(x) \subseteq X\}$$

- The set of all objects that can be classified as being *possible* members of X with respect to R denote the $R - upper\ approximation\ of\ X$ and can be represented as

$$R^*(X) = \{x: R(x) \cap X \neq \phi\}$$

- The set of all objects that cannot be classified either as members or non-members of X signify the $boundary\, region\, of\, X$ and can be represented as

$$RN_R(X) = R^*(X) - R_*(X)$$

- Therefore, if the boundary region of a set X with respect to R is empty, then it is called a crisp set and if the boundary region is non-empty, it is called a ***rough set***.

1.4.2 Intuitionistic Fuzzy Sets or Vague Sets.

The concept of intuitionistic fuzzy sets (IFS) was introduced by Atanassov (1983). It is a generalization of the fuzzy sets. In IFS, the advantage is that it considers both the degree of membership as well as non-membership. Therefore, few real life situations that could not be dealt with fuzzy sets can be easily tackled through IFS as it helps in modelling the uncertainty in a much better way. Another idea was introduced by Gau and Buehrer (1993) as a generalization of fuzzy sets, which they termed as vague sets. Later, Bustince and Burillo (1996) specified in their work that both the concepts of vague sets and intuitionistic fuzzy sets are same. Mathematically, the definitions of IFS and vague sets are presented below:

Let U denote the universe of discourse.

If A is an IFS in U, then it can be represented as

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in U\}$$

Where, μ and ν whose values lie within the interval $[0,1]$, signifies the degree of membership and non-membership respectively, such that the constraint $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ is satisfied for each $x \in U$. Another term that can be associated with every element $x \in U$ is the hesitancy degree which can be represented as $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ such that $0 \leq \pi_A(x) \leq 1$ (Atanassov, 1983).

If B is a vague set in U, then B can be characterized by two functions, the truth-membership function (t_B) and the false-membership function (f_B) such that $t_B + f_B \leq 1$ and $t_B: U \rightarrow [0,1]$, $f_B: U \rightarrow [0,1]$. Here, t_B specifies the lower bound on the membership grade of x that has been derived from the evidence in favour of $x \in U$ and f_B indicates the lower bound on the negation of x that has been derived from the evidence not in favour of $x \in U$ (Gau, & Buehrer, 1993).

1.4.3 Soft Sets

The concept of soft sets was introduced by Molodtsov (1999). It is a general mathematical tool that was proposed to tackle the non-clearly defined, fuzzy and uncertain objects. Based on the idea of Molodtsov, Maji *et al*. (2002) presented some operations and applications of soft sets. According to Molodtsov, soft sets can be defined as follows:

- Let U denote the universal set.
- Let K denote the set of parameters.
- A pair $\langle T, K\rangle$ is called a *soft set* iff T represents a mapping of K into the set of all subsets of U.
- Let $P(U)$ be the power set of U, then the mapping T can be stated as:
- $T: K \rightarrow P(U)$ such that for $\forall x \in K, T(x)$ may be concluded as the set of $x - approximate\ elements$ of the soft set $\langle T, K\rangle$.

The set of all soft sets over U may be represented by $S(U)$.

Example 1: Let U be a set of buildings and $U = \{b_1, b_2, b_3, b_4, b_5\}$ and K be the set of parameters i.e., $K = \{k_1, k_2, k_3, k_4\} = \{single\ storey, multi\ storey, office, residential\}$. Now suppose, $T(k_1) = \{b_2, b_3, b_4\}, T(k_2) = \{b_1, b_5\}, T(k_3) = \{b_1, b_2\}, T(k_4) = \{b_3, b_4, b_5\}$. Then the soft set $\langle T, K\rangle$ can be represented as a collection of approximation as shown below:

$$\langle T, K\rangle = \{single\ storey\ buildings = \{b_2, b_3, b_4\}; multi\ storey\ buildings = \{b_1, b_5\};\ office\ buildings = \{b_1, b_2\}; residential\ buildings = \{b_3, b_4, b_5\}\}.$$

The tabular representation of the above stated example is presented below:

K \ U	k_1	k_2	k_3	k_4
b_1	0	1	1	0
b_2	1	0	1	0
b_3	1	0	0	1
b_4	1	0	0	1
b_5	0	1	0	1

1.5 Related Work

Mainly two graph problems have been considered, namely the shortest path/constrained shortest path problem and the orienteering problem. To tackle the two problems, fuzzy models, intuitionistic fuzzy models and heuristic algorithms have been proposed.

The shortest path problem is an important problem in graph theory and finds several practical applications like in routing, transport etc. (Elizabeth, & Sujatha, 2011; Hernandes, Lamata, Verdegay, & Yamakami, 2007). The weights associated with the edges of the graph in such applications represent parameters like time, distance, cost etc. which are represented as real numbers. However, since these parameters cannot be determined precisely it is beneficial to represent those using fuzzy numbers instead of real numbers in order to tackle the prevailing uncertainty. This leads to the concept of fuzzy shortest path problem (FSPP). In any given graph, fuzziness can be introduced in five ways according to Blue *et al*. (2002), which are as follows:

- Type I: Fuzzy set of crisp graphs.
- Type II: Crisp vertex set and fuzzy edge set.
- Type III: Crisp vertices and edges with fuzzy connectivity.
- Type IV: Fuzzy vertex set and crisp edge set.

- Type V: Crisp graph with fuzzy weights.

To tackle the non-statistical uncertainty in problems, fuzzy set theory was proposed by Zadeh (1965) and using this theory a lot of work has been done in the area of shortest path problem. Dubois and Prade (1980) first introduced the "Fuzzy Shortest Path Problem" and found the Fuzzy Shortest Path Length (FSPL) in a network using the Floyd's algorithm and the Ford's algorithm. However, the corresponding shortest path may not be present in the network. A dynamical programming recursion-based fuzzy algorithm was proposed by Klein (1991) and in 2006 another dynamic programming approach was used to solve FSPP using triangular fuzzy numbers (Kung, Chuang, & Lin, 2006). Later, an attempt was made to determine the Fuzzy Shortest Path present in the network corresponding to the calculated FSPL using different fuzzy numbers (Triangular, Trapezoidal and Discrete fuzzy numbers) as arc lengths and was successfully accomplished by using the concept of "Similarity Measure" where a comparison was made between each path length and the shortest path length, and the one with the highest similarity degree was concluded as the Fuzzy Shortest Path (FSP) (Chuang, & Kung, 2005; Chuang, & Kung, 2006; Sujatha, & Elizabeth, 2011). A linear multi objective programming method (Yu, & Wei, 2007), multi-criteria decision method based on vague similarity measure (Dou, Zhu, & Wang, 2012), graded mean integration method (Deng, Chen, Zhang, & Mahadevan, 2012) etc. were proposed to deal with FSPP. A signed distance ranking method was suggested by Yao and Lin (2003). Other than similarity measures, ranking index has also been used for determining the Fuzzy Shortest Path in the network by comparing individual path lengths with the shortest path length and assigning ranks in order to determine the FSP (Elizabeth & Sujatha, 2011).

A lot of research has been done regarding the MST problem and many algorithms have been suggested. The initial algorithms were given by Boruvka (1926) and Jarnik (1930). However, the most commonly used algorithms to solve the MST problem are two greedy algorithms that run in polynomial time called Prim's (1957) and Kruskal (1956) algorithm. Some parallel algorithms have also been suggested to solve the MST problem faster than the sequential algorithms. Some researchers have also studied the fuzzy MST problem and

Itoh and Ishii (1996) presented a necessity measure to deal with the problem. An overall existing ranking index was suggested in (Chang, & Lee, 1994) to rank the fuzzy numbers in a spanning tree and this was used by Chang and Lee (1999) to deal with the MST problem. Later Liu (2002; 2004) suggested a credibility measure as a fuzzy ranking method that was used by Gao and Lu (2005) to formulate and solve a fuzzy MST problem. Janiak *et al*. (2008) determined the minimum spanning tree with fuzzy cost using the concept of possibility theory. A signed distance ranking method was proposed by Mohanty *et al*. (2012) to deal with MST problem with triangular fuzzy edge weights.

A survey of the problem shows that a lot of research has been done for STP and many algorithms have been suggested for different types of STP. An algorithm for rectilinear STP was presented in (Agarwal, & Shing, 1990) and because STP is an NP-Hard problem some approximation algorithms have been suggested for the Euclidean and rectilinear STP in (Arora, 1998; Berman, P., Fobmeier, Karpinski, Kaufmann, & Zelikovsky, 1994; Berman, & Ramaiyer, 1992; Berman, & Ramaiyer, 1994). Some heuristic algorithms were suggested in (Du, Smith, & Rubinstein, 2000; Kahng, & Robins, 1990). A Cheng's centroid point fuzzy ranking method was proposed to deal with the fuzzy STP (Seda, 2008). Here, the heuristic algorithm stated in (Kou, Markowsky, & Berman, 1981) has been considered and an algorithm for the fuzzy STP has been developed using Quasi Gaussian Fuzzy Numbers (QGFN) and the proposed ranking index, Link Preference Index (LPI) is used for ranking QGFN and finding the fuzzy STP.

One of the first algorithms, to solve CSPP was given by Joksh (1966). CSPP is an NP-Complete problem. Thus, most of the algorithms suggested are either Heuristic or Approximation algorithms and the strategies used to provide the solution are mostly either Dynamic Programming (DP) or Lagrangian Relaxation (LR) (Chen, Song, & Sahni, 2008; Xiao, Thulasiraman, Xue, & Juttner). Some initial techniques were suggested by Sahni (1977) and Ibarra and Kim (1975) that were later used by researchers to solve the CSPP using ε-approximation algorithms. Let m be the number of links and n be the number of nodes in the given network and $U\ and\ L$ be the upper bound and lower bound on the optimal cost respectively. Hassin (1992) suggested a Fully Polynomial

Time Approximation Scheme (FPTAS) for CSPP based on the technique of DP and scaling / rounding with a time complexity of $O(\log\log(U/L)\,[mn\,\varepsilon^{-1} + \log\log(U/L)])$ and Juttner *et al*. (2001) applied the technique of LR on the delay constrained least cost routing problem and solved it in $O(m^2 log^4 m)$ time. A heuristic algorithm was presented by Korkmaz and Krunz (2001) with the same time complexity as that of Dijkstra's algorithm. Handler and Zang (1980) used the strategy of LR and applied it to the k-th shortest path problem algorithm, to solve CSPP. Several authors have considered the fuzzy version of SPP however, only a few papers are available in the literature that deals with the fuzzy version of CSPP (Ji et al, 2007; Su & Li, 2009a; Su & Li, 2009b). A strongly polynomial time approximation scheme was proposed by Lorenz and Raz (2001) for general networks. Some heuristics for CSPP were proposed by Luo *et al*. (1999) and Ravindran et al (2002). The LR strategy was applied by Xue (2000) to the delay constrained least cost routing problem. A label setting algorithm in combination with several pre-processing methods was described by Dumitrescu and Boland (2003) to solve CSPP. Further, an enhanced version of this algorithm called the Lagrangian relaxation with near-shortest-paths enumeration (LRE) was proposed by Carlyle *et al*. (2008). Some latest algorithms for CSPP were proposed by Royset *et al*. (2009) and Lefebvre *et al*. (2011).

The orienteering problem is NP-Hard. Several heuristic algorithms and a few approximation algorithms are available in the literature for solving OP. One of the first heuristics for OP was the stochastic (S-Algorithm) and deterministic (D-Algorithm) suggested by Tsiligirides (1984). In 1987, a center-of-gravity heuristic was proposed by Golden *et al.* (1987). A four phase heuristic was introduced by Ramesh and Brown (1991). An artificial neural network based approach was proposed by Wang *et al*.(1995) and a five-step heuristic was suggested by Chao *et al*. (1996). A branch and cut heuristic for OP was stated by Fischetti *et al*. (1998). The first genetic algorithm for OP was developed by Tasgetiren (2001). Gendreau *et al*. suggested a tabu search heuristic in (1998). Liang *et al*. (2002) proposed and compared a tabu search and an ant colony optimization approach for OP. Two techniques called the pareto ant colony optimization algorithm and the variable neighbourhood search algorithm were proposed by Schilde *et al*. (2009) for the multi-objective variant of OP. A heuristic method based on the Greedy Randomized Adaptive Search Procedure and the Path Relinking methodologies was proposed by Campos *et al*. (2013). The approximations known for Prize Collecting Travelling Salesman Problem can be easily extended for the un-rooted version of OP (Awerbuch, Azar, Blum, & Vempala, 1999; Johnson, Minkoff, & Phillips, 2000) and for the rooted version, a constant factor approximation was suggested by Blum *et al*. (2003). Another approximation algorithm for the time dependent variant of OP was presented by Fomin *et al*. (2002).

The concept of fuzzy metric has been widely used in the fields of pattern recognition, routing, scheduling, transportation, fuzzy optimization etc. (Deng, 1982; Kaleva, & Seikkala, 1984; Menger, 1942). A metric space can be defined as (X, d) where X is a set of points and d is a metric on X i.e., a function that defines the distance between every pair of elements belonging to X. Two methods of defining fuzzy metric space include: (1) use of fuzzy numbers for defining a metric in an ordinary space and (2) use of real numbers for measuring the distance between fuzzy sets. Using these two approaches, fixed point theorems were given (Grabiec, 1988), Hausdorff topology (George, & Veeramani, 1994; George, & Veeramani, 1997), and fuzzy normed spaces (Katsaras, 1984; Santhosh, & Ramakrishnan, 2011) were defined by several authors. The idea of intuitionistic fuzzy sets (IFS) was proposed by Atanassov (1983). An IFS is more powerful as it allows geometrical interpretation and helps in managing the existing uncertainty in a much better way (Atanassov, 2003). Shannon and Atanassov (1994) introduced the idea of intuitionistic fuzzy graphs (IFGs). Some properties of IFGs were presented in (Atanassov, & Shannon, 1995). A modified definition of IFG and some operations on IFG were proposed by Parvathi *et al*. (2014). Later the concept of intuitionistic fuzzy metric space was introduced using continuous t-norms and continuous t-conorms (Park, 2004). Some theorems and properties were also stated for intuitionistic fuzzy metric space (Chauhan, Shatanawi, Kumar, & Radenovic, 2014). Xia and Guo (2004) suggested a method of defining fuzzy metric spaces. They used the concept of fuzzy points and fuzzy scalars to measure the distance between the fuzzy points. The concept of intuitionistic fuzzy point (IFP) was introduced by Coker and Demirci (1995). The idea of IFP was then used by researchers to prove some relations and theorems (Akram, 2012; Sardar, Mandal, & Majumder, 2011). The concept of IFP can be used to study some general structures like those introduced in (Bustince, Barrenechea, & Pagola, 2006; Bustince, Barrenechea, & Pagola, 2008). Here, the concept of IFP has been used to define an intuitionistic fuzzy metric space.

In the fuzzy and intuitionistic fuzzy versions of the above stated problems, the parameters involved like cost, score etc. are represented as fuzzy and intuitionistic fuzzy numbers respectively. Fuzzy and intuitionistic fuzzy numbers cannot be ordered naturally. Therefore, to determine their ordering, some ranking method is required. The idea of ranking fuzzy numbers was first proposed by Jain (1976). Since then a lot of ranking methods have been suggested. The concept of using the centroid for ranking a fuzzy number was first introduced by Yager (1980). The method suggested by Yager could not rank the fuzzy numbers in some cases and to overcome the drawbacks of Yager's method, a ranking method was proposed by Cheng (1998), which used the distance between the origin and the centroid to perform the ordering. Some other ranking methods based on the idea of area of the fuzzy number, left and right deviation degree etc. were proposed (Abbasbandy, & Asady, 2006; Asady, & Zendehnam, 2007; Chen, & Tang, 2008; Chu, & Tsao, 2002; Kumar, Singh, Kaur, & Kaur, 2011; Nejad, & Mashinchi, 2011; Wang, & Lee, 2008; Wang, Liu, Fan, & Feng, 2009; Yao, & Lin, 2000; Yao, & Wu, 2000). Later, few other ranking methods were introduced based on the idea of maximizing set / minimizing set, Circumcenter of Centroids, weighting function etc. (Chou, Dat, & Yu, 2011; Rao, & Shankar, 2011; Saeidifar, 2011). A new ranking method for ranking normal and non-normal trapezoidal and triangular fuzzy numbers based on the left and right area of the fuzzy number has been suggested by Parandin *et al*. (2014). Similarly, some ranking methods were proposed for the intuitionistic fuzzy numbers. Some of the initial ranking methods were proposed by Grzegorzewski (2003) and Mitchell (2004). A ranking method utilizing the lexicographic technique was suggested by Nan and Li (2010) for ranking the trapezoidal intuitionistic fuzzy number. Another ratio method for ordering was proposed by Li (2010) for triangular intuitionistic fuzzy numbers that uses two parameters, namely the value index and ambiguity index for ranking the intuitionistic fuzzy number. Few more ideas for the ranking of intuitionistic fuzzy numbers were proposed (Nayagam, Vankateshwari, & Sivaraman, 2008; Nehi, 2010) and a latest method for ranking the trapezoidal intuitionistic fuzzy number based on the ambiguity index has been suggested by De and Das (2014).

1.6 Motivation

The initial literature survey indicated that some ranking methods have been suggested for fuzzy and intuitionistic fuzzy numbers but have not been applied on real life problems. Their performances have been tested on some handpicked examples and compared against other existing methods. Also, most of the ranking methods were proposed for triangular and trapezoidal fuzzy and intuitionistic fuzzy numbers. Therefore, the need was to apply few of the latest ranking methods existing in the literature for the fuzzy and intuitionistic fuzzy numbers to some real life problems like the constrained shortest path problem (CSPP) and the orienteering problem (OP). Secondly, it was also necessary to represent the parameters involved like cost, time, delay, score etc. in the real life problems using some other fuzzy number models which are more appropriate and suitable considering the practical situation. Since the two problems considered fall under the class of NP-Complete (CSPP) and NP-Hard (OP) problems, the need was to propose some heuristic / meta-heuristic that improves the performance of some recent algorithms reported in the literature.

Intuitionistic fuzzy metric spaces have been defined using the concept of continuous t-norms and continuous t-conorms but a definition that can model the uncertainty in the position / location of the point of interest was missing which can be modelled by defining an intuitionistic fuzzy metric space using the concept of intuitionistic fuzzy point.

1.7 Research Contribution

The main contributions of this thesis are as follows:

- A ranking method has been proposed for Quasi-Gaussian Fuzzy Number.

- A new definition has been presented for Quasi-Gaussian Intuitionistic Fuzzy number.
- A centroid based ranking method has been suggested for Quasi-Gaussian Intuitionistic Fuzzy number.
- A centroid based ranking method has been proposed for trapezoidal intuitionistic fuzzy number with eight parameter representation.
- Another ranking method called the Centroid of Centroids has been suggested for trapezoidal intuitionistic fuzzy number.
- Fuzzy and Intuitionistic Fuzzy formulation of constrained shortest path problem and orienteering problem and their solutions has been presented.
- A max-min formulation has been suggested to deal with the fuzzy and intuitionistic fuzzy version of the orienteering problem.
- A parallel formulation and its theoretical analysis have been presented for the fuzzy orienteering problem.
- A Work-Depth analysis has been performed for the intuitionistic fuzzy orienteering problem.
- An intuitionistic fuzzy metric space using intuitionistic fuzzy points has been proposed.
- A roulette wheel selection based heuristic (RWS_OP) has been suggested for the orienteering problem that can be applied on complete as well as incomplete graphs.
- Flower pollination meta-heuristic (FPA_OP) has been presented for the orienteering problem that can be applied on complete graphs.
- Bidirectional search heuristic has been proposed for the constrained shortest path problem.

1.8 Layout of Thesis

Chapter 1 presents the introduction of the thesis. Graphs, various types of graphs and graph problems are defined in this chapter. Various sources of uncertainty present in real life and the concepts used to model them have been discussed. In chapter 2, fuzzy numbers, their ranking methods and applications have been considered. Two types of fuzzy numbers have been used viz. Quasi-Gaussian fuzzy number and trapezoidal fuzzy number. Ranking methods have been discussed for both types of fuzzy numbers and their applications have been suggested for some graph problems like the constrained shortest path problem, orienteering problem etc. A max-min formulation and theoretical analysis of the parallel formulation of fuzzy orienteering problem has been presented. Chapter 3 basically considers the intuitionistic fuzzy numbers (Quasi-Gaussian intuitionistic fuzzy number and trapezoidal intuitionistic fuzzy number), a few ranking methods proposed for the intuitionistic fuzzy numbers and their application to the constrained shortest path problem and the orienteering problem. Also, a max-min formulation has been presented for the intuitionistic fuzzy orienteering problem and a work-depth analysis has been performed for the intuitionistic fuzzy orienteering problem. A new intuitionistic fuzzy metric space that uses the idea of intuitionistic fuzzy point has been proposed. In chapter 4, a few heuristics and meta-heuristics have been suggested to deal with the two graph problems, namely the constrained shortest path problem and the orienteering problem. A roulette wheel selection based heuristic has been proposed for the orienteering problem that can be applied on incomplete graphs, a flower pollination meta-heuristic has been presented for the orienteering problem that can be applied on complete graphs and a bidirectional search heuristic has been suggested for the constrained shortest path problem. Finally, the conclusion of the thesis and the future scope of this research work have been stated in chapter 5.