Contents

Acknowledgement iv

List of Figures ix

List of Tables xiii

Preface xv

1. Introduction 1
- 1.1 Motivation . . . 1
- 1.2 Introduction . . . 2
- 1.3 Review of Wavelets . . . 2
- 1.4 Preliminaries . . . 5
  - 1.4.1 Definition of wavelets . . . 5
  - 1.4.2 Basis of wavelets . . . 6
  - 1.4.3 Orthonormality of wavelets . . . 7
  - 1.4.4 Legendre Polynomials . . . 7
  - 1.4.5 Two-dimensional shifted Legendre polynomials . . . 9
  - 1.4.6 One dimensional Legendre wavelets . . . 9
  - 1.4.7 Two dimensional Legendre wavelets . . . 10
  - 1.4.8 One-dimensional Chebyshev wavelet . . . 10
  - 1.4.9 Two-dimensional Chebyshev wavelet . . . 11
  - 1.4.10 One-dimensional Bernoulli wavelet . . . 12
  - 1.4.11 Two-dimensional Bernoulli wavelet . . . 13
  - 1.4.12 Kronecker multiplication . . . 13
- 1.5 Review of fractional partial differential equations . . . 14
- 1.6 Review of operational matrices . . . 16

2. Two-Dimensional Shifted Legendre Polynomial Collocation Method for Electromagnetic Waves in Dielectric Media via Almost Operational Matrices 19
- 2.1 Introduction . . . 19
  - 2.1.1 Function approximation . . . 22
- 2.2 Operational matrices . . . 23
  - 2.2.1 Operational matrix of differentiation . . . 23
  - 2.2.2 Almost operational matrix of integration with singularity for variable t . . . 25
- 2.3 Method of the solution . . . 26
- 2.4 Convergence Analysis . . . 29
- 2.5 Error Analysis . . . 34

2.5.1 Error bound . . . 34
2.5.2 Numerical error . . . 39
2.6 Numerical examples . . . 40
2.7 Conclusion . . . 49

3. Two-Dimensional Wavelets Collocation Method for Electromagnetic Waves in Dielectric Media 50
3.1 Introduction . . . 50
3.2 Function approximation . . . 52
3.3 Operational matrices . . . 54
3.3.1 Legendre wavelet operational matrix of differentiation with respect to variable t . . . 54
3.3.2 Legendre wavelet operational matrix of differentiation with respect to variable x . . . 54
3.3.3 Chebyshev wavelet operational matrix of differentiation with respect to variable t . . . 55
3.3.4 Chebyshev wavelet operational matrix of differentiation with respect to variable x . . . 56
3.3.5 Legendre wavelet almost operational matrix of integration with singularity for variable t . . . 56
3.3.6 Chebyshev wavelet almost operational matrix of integration with singularity for variable t . . . 58
3.4 Method of the solution . . . 58
3.5 Convergence analysis: . . . 62
3.5.1 For Legendre wavelet . . . 62
3.5.2 For Chebyshev wavelet . . . 68
3.6 Error analysis . . . 73
3.7 Numerical examples . . . 76
3.8 Conclusions . . . 88

4. Comparative Study of Two-Dimensional Wavelets Scheme for Linear and Nonlinear Volterra Weakly Singular Partial Integro-Differential Equation Based on Operational Matrices 90
4.1 Introduction . . . 90
4.2 Function approximation . . . 93
4.3 Operational matrices . . . 98
4.3.1 Operational matrix of integration for Legendre wavelet . . . 98
4.3.2 Operational Matrix for integration for Chebyshev wavelet . . . 99
4.3.3 Legendre wavelet operational matrix of differentiation . . . 99
4.3.4 Chebyshev wavelet operational matrix of differentiation . . . 100
4.3.5 Almost operational matrix of integration with singularity for variable x . . . 101
4.3.6 Product operational matrix . . . 102
4.4 Numerical method of the solution . . . 102
4.4.1 Type I-Numerical solution for linear PIDEs . . . 103
4.4.2 Type II-Numerical solution for nonlinear PIDEs . . . 105
4.4.3 Type III-Numerical solution for nonlinear PIDEs . . . 107
4.5 Convergence Analysis . . . 109

4.5.1 For Legendre Wavelet . . . 109
4.5.2 For Chebyshev Wavelet . . . 110
4.6 Error Bound . . . 112
4.7 Error Estimation . . . 115
4.8 Numerical examples . . . 116
4.9 Conclusion . . . 130

5. Numerical Wavelets Scheme for Complex Partial Differential Equation Arising from Morlet Continuous Wavelet Transform 132
5.1 Introduction . . . 132
5.1.1 Function approximation . . . 135
5.2 Operational matrices . . . 136
5.2.1 Bernoulli wavelet operational matrix of differentiation with respect to variable a . . . 136
5.2.2 Bernoulli wavelet operational matrix of differentiation with respect to variable b . . . 137
5.2.3 Legendre wavelet operational matrix of differentiation with respect to variable a . . . 137
5.2.4 Legendre wavelet operational matrix of differentiation with respect to variable b . . . 138
5.3 Numerical solution of the problem . . . 139
5.4 Convergence analysis . . . 146
5.4.1 For Legendre wavelet . . . 146
5.4.2 For Bernoulli wavelet . . . 147
5.5 Error analysis . . . 149
5.5.1 Error for real part . . . 149
5.5.2 Error for imaginary part . . . 151
5.5.3 Absolute Error for w(a,b) . . . 153
5.6 Numerical examples . . . 153
5.7 Conclusion . . . 166

6. Appendix 168
6.1 Almost operational matrices for integration . . . 168
6.1.1 Legendre wavelets operational matrix of integration for variable t, M=M'=3 . . . 168
6.1.2 Chebyshev wavelets operational matrix of integration for variable t, M=M'=3 . . . 169
6.2 Piecewise almost operational matrices for integration . . . 171
6.2.1 For Legendre wavelets . . . 171
6.2.2 For Chebyshev wavelets . . . 173
6.3 Almost operational matrices for CPDEs . . . 176
6.3.1 Almost operational matrix of integration I . . . 176
6.3.2 Almost operational matrix of integration II . . . 177
6.3.3 Almost operational matrix of integration III . . . 177
6.3.4 Almost operational matrix of integration IV . . . 177
6.3.5 Almost operational matrix of integration V . . . 178
6.3.6 Almost operational matrix of integration VI . . . 178

References 180