CHAPTER IV

MULTIOBJECTIVE OPTIMAL PMU PLACEMENT FOR MAXIMUM OBSERVABILITY

4.1. INTRODUCTION

As technology advanced, the time frame of synchronized information has been steadily reduced from minutes to microseconds. However, determination of locations of Phasor Measurement Units (PMUs) in the system in order to get desired measurements with least number has been a challenging job. Several algorithms and approaches have been published in literature reviewed in Chapter 1 for the OPP problem in power system. This chapter presents a Binary Gravitational Search Algorithm (BGSA) methodology for the optimal placement of PMUs to minimize the total number of PMUs installed at various buses, which in turn minimize installation cost of the PMUs and improves observability redundancy by including an additional objective. Thus the PMU placement problem has been expressed as a multi-objective problem. Besides, single PMU outage or single line outage cases in the presence of zero injection buses have been investigated. The proposed BGSA has been applied to the IEEE 14-bus, IEEE 30-bus, IEEE 118-bus, Northern Regional Power Grid 246-bus Indian system, and Polish 2383-bus system. Case studies reveal that the lower number of PMUs or equal number of PMUs have been produced by the proposed method compared to methods reported in the literature. In cases of equal number of PMUs, the observability produced by proposed method are higher or at least equal.

4.2. CONCEPT OF MAXIMUM OBSERVABILITY (MO)

If the results have more than one solutions, then the question of best solution arises. Because, by changing the location of optimal phasor measurement units, system observability may vary. For example, phasor measurement unit placement as given in Figure 4.1 (a), phasor measurement unit at bus 2 observe the buses 1, 2, 3, 6 and 7, therefore, the system observability by PMU at bus 2 is 5. Similarly, PMU at bus 5 observe the buses 4 and 5, therefore, observability by PMU at bus 5 is 2. Hence, the total system observability of the system in Figure 4.1 (a) is 7. Alternatively, for PMU placement in Figure 4.1 (b), system observability by PMU at bus 2 is 5 from previous data. And PMU at bus 4 observe the buses 3, 4, 5 and 7. Therefore, the observability by PMU at bus 4 is 4. Hence, the total observability of the system of Figure 4.1(b) is 9, which is more as compare to system observability of Figure 4.1 (a). From the above results, it is clear that the selection of appropriate locations of PMU buses is an important task in OPP problem for the maximum observability of the system.

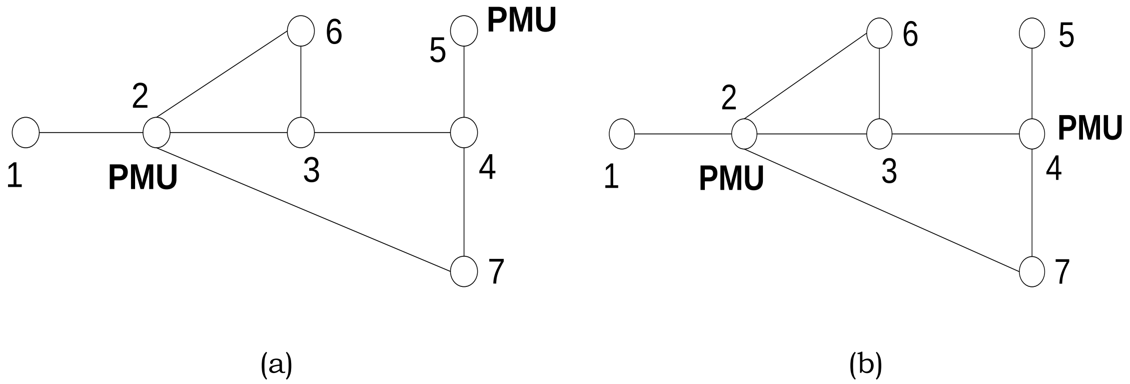

Figure 4.1. 7-bus system (a) PMUs at buses 2 & 5, (b) PMUs at buses 2 & 4

4.3. MATHEMATICAL FORMULATION OF OPP

Real-time observation of power system is achieved by synchronous phasor measurements. Due to economic consideration, optimal placement of PMUs is vital. PMU provides the voltage phasors at installed bus and some or all the current phasors of lines incident to that bus. With the help of Kirchhoff's current law and Ohm's law, voltage phasors of neighbor buses can be calculated. The procedure for building the constraint equations has been described for four possible cases in this chapter.

4.3.1. OPP without ZI buses

In reference [29], the author proposed a methodology to place the optimal PMU with maximum observability. For this, a proper value of λ should be chosen. Otherwise, the number of PMU can increase. Unlike the method, no such variable is involved in proposed method. The basic objective of this investigation is to identify the minimum number of locations of PMUs in the system such that all the desired states are observed and observability is maximum. In order to fulfil above mentioned requirement, the objective function consisting of two components has been suggested in this investigation. The first component is responsible for restricting the number of PMUs to the minimum. Whereas, the second component decides those locations which yield maximum observability. This can be mathematically expressed, for n bus system, as:

$$Minimize \;\; h = h_1 + h_2 \tag{4.1}$$

where,

$$h_1 = \sum_{i=1}^{n} wt_i c_i \tag{4.2}$$

$$h_2 = \frac{1}{\sum_{i=1}^{n} e_i f_i} \tag{4.3}$$

Subject to observability rule,

$$f_i = \sum_{j=1}^{n} a_{ij} c_j \geq b_i \qquad \forall i \in I \tag{4.4}$$

For objective function h, to be minimum, it is desired that its two components h_1 and h_2 are independently minimum. This means, none of the components should restrict other to be minimum. It can be understood that the second term of the objective function will assume lower value with increase in observability. Moreover, h_2 will always be less than unity. Thus it will not affect the first component from producing lower value. Therefore, the proposed objective function expressed by Equation (4.1) would produce lowest number of PMUs such that the observability is highest. In present case, wt_i is assumed to be unity. If any bus say i, is to be given importance or to be recognized as a distinguished bus, the term e_i helps in assigning such bus. In this chapter, the value of e_i is assumed to be unity. The value of c_j can be determined using Equation (4.5).

$$c_j = \begin{cases} 1 & if \; a \; PMU \; is \; needed \; at \; bus \; j \\ 0 & otherwise; \end{cases} \tag{4.5}$$

In Equation (4.4), a_{ij} is the element of connectivity matrix (A), between buses i and j which is defined as Equation (3.5). b_i is the column vector whose all entries are unity as given in Equation (4.6).

$$b = [11\ldots1]^T \tag{4.6}$$

Finally, the system observability can be computed as,

$$\text{Observability}(Obs.) = \sum_{i=1}^{n} f_i \tag{4.7}$$

4.3.2. OPP with ZI buses

The characteristics of ZI buses present in the system can be exploited to further reduce the number of PMUs. This can be reflected in the above model by modifying Equation (4.4) as given in [19]:

Observability rule (constraint),

$$f_i = \sum_{j \in I} a_{ij} c_j + \sum_{j \in I} a_{ij} z_j y_{ij} \qquad \forall \ i \in I \tag{4.8}$$

$$\sum_{i \in I} a_{ij} y_{ij} = z_j \qquad \forall \ j \in I \tag{4.9}$$

where, z_j is equal to 1 if bus j is ZI bus ; otherwise 0. In Equation (4.9), z_j is 1 for ZI bus, it means only one auxiliary binary variable is equal to one. If y_{ij} is equal to 1 it means, bus i is observable due to bus j that is ZI bus.

4.3.3. OPP considering a single PMU outage

Equation (4.4), is written on the premise that all the buses must be observed at least once by a PMU for full system observability. Therefore, the entries of vector b are all ones. However, for a single PMU outage, all the buses must be observed at least twice by PMUs. Hence, Equation (4.4) can be modified to Equation (4.10), [19];

$$f_i + \sum_{j \in I} a_{ij} y_{ij} \geq 2 \qquad \forall i \in I \tag{4.10}$$

From Equation (4.10), it can be seen that a single PMU outage cannot affect the full system observability. If ZI buses are also present in the system, the value of $\sum_{j \in I} a_{ij} y_{ij} = 1$. If ZI buses are not present in the system, then $\sum_{j \in I} a_{ij} y_{ij} = 0$ in Equation (4.10). Accordingly, $f_i \geq 2$ which indicates that all the buses are observed at least twice by a PMU.

4.3.4. OPP considering a single line outage

PMU placement for single line outage is the subset of OPP for single PMU outage. The optimal number of PMUs increases as further line outages are considered. If the outage of a radial line occurs, observability of the radial bus on this line will be lost. However, this loss of observability will not affect the observability of the remaining system as the radial bus will be isolated from rest of the system [36]. This can be incorporated by eliminating the radial lines from the set of lines, L.

The case of line outage can mathematically be expressed by modifying the relation for connectivity matrix expressed by Equation (4.4). For example, if a line between bus *i-j* gets disconnected, then the elements of connectivity matrix $a_{ij} = a_{ji} = 0$. So, Equations (4.4, 4.8 and 4.9) need to be uploaded to incorporate this change for each line outage. This process should be repeated until the outages of all lines have been incorporated. The expressions for constraints in this case can be expressed as follows [19]:

$$f_i^k = \sum_{j=1}^{n} a_{ij}^k c_j \geq b_i \qquad \forall i \in I, \ \ \forall k \in L \tag{4.11}$$

where,

$$f_i^k = \sum_{j \in I} a_{ij}^k c_j + \sum_{j \in I} a_{ij}^k z_j y_{ij}^k \qquad \forall i \in I, \ \ \forall k \in L \tag{4.12}$$

$$\sum_{i \in I} a_{ij}^k y_{ij}^k = z_j \qquad \forall i \in I, \ \ \forall k \in L \tag{4.13}$$

4.3.5. OPP considering uncertainties

The uncertainty of PMU may arise due to line outages or loss of a PMU causing unobservability of buses. However, which of the contingencies has occurred may not be known a priory. This necessitates formulation of the PMU placement problem such that none of the buses is left unobservable in such uncertainty. Though the PMU and line outages have been dealt with, in sections 4.2.3 and 4.2.4 respectively, but in the case of uncertainty, suitable modification need to be done in above formulation such that both the eventualities are dealt in together. For this purpose, the constraints of the single PMU outage and single line outage cases need to be combined. In order to do this, the right-hand side inequality constraint of expression (4.11) has been multiplied by 2 to provide the observability of each bus at least twice by the PMUs so that loss of a PMU does not affect the system observability. Now, the resulting expression (4.11) and expressions (4.12 and 4.13) should be repeatedly solved until the outage of all lines has been incorporated. Thus the system would be fully observable irrespective of a line outage or a PMU outage.

4.4. IMPLEMANTATION OF OPP PROBLEM IN BGSA

The detail procedure to apply the BGSA based on Newton's Law of Gravity and Mass interactions for solving the OPP problem is as follow:

Step 1. Read bus data and line data of the test system.

Step 2. Identify the search space.

Step 3. Obtain the connectivity matrix (A).

Step 4. Initialize BGSA parameters T, Q, G_o and β.

Step 5. Initialize population within min and max values of the control variables.

Step 6. Calculate the fitness values of each agent in the population for the OPP problem.

Step 7. Update $G(t)$, $best(t)$, $worst(t)$ and $M_i(t)$ for i =1, 2, ….., Q based on fitness value.

Step 8. Calculate total force in different directions using Equation (3.11).

Step 9. Modify acceleration of each agent using Equation (3.12).

Step 10. Update velocity and position of each agent using Equations (3.25) and (3.26) respectively.

Step 11. Repeat steps 6-10 until the termination criterion is reached.

Step 12. Stop

4.5. CASE STUDIES

The proposed formulation has been implemented on MATLAB platform and tested on IEEE 14-bus, IEEE 30-bus, IEEE 118-bus, NRPG 246-bus and Polish 2383-bus systems. The data for IEEE 14-bus, IEEE 30-bus and IEEE 118-bus has been taken from [125], NRPG 246-bus from [127] and Polish 2383-bus from [128]. The data of IEEE 14-bus, 30-bus and 118-bus test systems are given in Appendix I. Table 4.1 shows the details of the above mentioned test systems. The results obtained for four cases for test systems under consideration have been compared with those available in literature. Following four case studies under two different conditions have been studied:

Case 1: Normal case with and without ZI buses

Condition 1: Without ZI bus

Condition 2: With ZI bus

Table 4.1. Details of all the test systems

Test system	No of Lines	No. of ZIB	ZIB Number	No. of radial buses	Radial bus Number
IEEE 14-bus	20	1	7	1	8
IEEE 30-bus	41	6	6, 9, 22, 25, 27, 28	3	11, 13, 26
IEEE 118-bus	186	10	5, 9, 30, 37, 38, 63, 64, 68, 71, 81	7	10, 73, 87, 111, 112, 116, 117
NRPG 246-bus	376	60	51, 53, 54, 56, 58, 61-63, 69, 70-75, 80, 81, 86, 102-104, 107, 122, 126, 129, 131, 147, 154, 155, 167, 175, 179, 180, 183, 209, 210-217, 221, 222, 226, 229-234, 236-241, 243, 244	31	2,4,5,12, 30, 31, 38, 41,47, 51-53, 58, 76, 77, 112, 120, 123, 124, 135, 149, 153, 156, 159, 172, 178, 189, 208, 224, 242, 246
Polish 2383-bus	2896	552	-	500	-

Case 2: Single PMU outage with and without ZI buses

Condition 1: Without ZI bus

Condition 2: With ZI bus

Case 3: Single Line outage with and without ZI buses

Condition 1: Without ZI bus

Condition 2: With ZI bus

Case 4: Uncertainty of PMUs with and without ZI buses

Condition 1: Without ZI bus

Condition 2: With ZI bus

4.5.1. Case 1: Normal case with and without ZI buses

It has been assumed that initially there is no PMU installed in the system. The locations of PMUs have been determined such that the entire system becomes observable. Table 4.2 shows the results in terms of optimal number of PMUs and their locations for all the above mentioned test systems for both the conditions of without and with ZIB under this case. From the Table 4.2, it is clear that the numbers of PMUs required are reduced when ZIB are present in the system. The proposed algorithm has been applied for allocation of PMU in order to achieve maximum observability under both the conditions. A comparison of present results with other results reported in the literature has been tabulated in Table 4.3 for three test systems under two conditions of with and without ZIB. It can be seen from Table 4.3 that the results obtained by proposed method produced same number of PMUs as obtained by other reported methods except for method reported in [26], [38] in which number of PMUs are more than that in proposed method for IEEE 118-bus system. However, the locations of PMUs obtained by proposed method are different from other methods. For IEEE 14-bus, all methods produced same number of PMUs and observability. For IEEE 30-bus system all methods produce same number of PMUs but observability in case of presence of ZI buses is highest (i.e. 36) in proposed method but equal to methods reported in [21]-[22], [25]-[26]and [133]. Whereas, observability in without ZI buses case by proposed method and methods reported in [22], [38], [133]-[134]is same i.e. 52. In IEEE 118-bus system, mostly all methods in both the conditions with and without ZI buses produced same number of PMUs but observability redundancy is maximum for both the conditions in proposed method and equal to [22]. In IEEE 30-bus test systems,

Table 4.2. Optimal number and location of PMUs for the test systems (Case 1)

Test System	**Without ZIB**		**With ZIB**	
	No. of PMUs	**Locations of PMUs**	**No. of PMUs**	**Locations of PMUs**
IEEE 14-bus	4	2, 6, 7, 9	3	2, 6, 9
IEEE 30-bus	10	2,4,6,9,10,12,15,19,25,27	7	2,4,10,12,15,19,27
IEEE 118-bus	32	3,5,9,12,15,17,21,25,28,34,37,40, 45,49,52,56,62,64,68,70,71,76,79, 85,86, 89,92, 96,100,105,110,114	28	3,8,11,12,17,21,27,31,32,34,37, 40,45,49,52,56,62,72,75,77,80, 85,86,91,94,101,105,110
NRPG 246-bus	70	6, 11, 21, 23, 24, 29, 34, 35, 40, 44, 48, 54-56, 61-63, 65, 69, 71, 74, 75, 80, 82, 83, 88, 91, 93, 94, 98, 100, 101, 106, 109, 122, 125, 126, 128, 129, 132, 134, 139-142,147, 157,158,160,167, 168,173,181,185,187, 190, 191,194,199,201-203,215,216, 219, 229, 234, 235, 243, 245	51	6, 11, 15, 21, 24, 27, 29, 34, 40, 44, 48, 49, 56, 65, 67, 82, 83, 88, 89, 91, 96, 106, 109, 113, 117, 121, 125, 128, 132, 134, 140-142, 157, 158, 160, 165, 166, 168, 181, 185, 187, 190, 191, 194, 199, 202, 203, 219, 235, 245
Polish 2383-bus	746	-	553	-

Table 4.3. Results comparison with available techniques for base case (Case 1)

Method	IEEE 14-bus				IEEE 30-bus				IEEE 118-bus			
	With ZIB		Without ZIB		With ZIB		Without ZIB		With ZIB		Without ZIB	
	No. Of PMUs	Obs.	No. Of PMUs	Obs.	No. Of PMUs	Obs.	No. Of PMUs	Obs	No. Of PMUs	Obs.	No. Of PMUs	Obs.
Proposed	3	15	4	19	7	36	10	52	28	147	32	164
Ref. [21]	-	-	-	-	7	36	-	-	28	142	-	-
Ref. [22]*	3	15	4	19	7	36	10	52	28	147	32	164
Ref. [25]*	3	15	-	-	7	36	-	-	28	147	-	-
Ref. [26]*	3	15	-	-	7	36	-	-	29	148	-	-
Ref. [27]*	-	-	4	19	-	-	10	43	-	-	-	-
Ref. [28]*	3	15	-	-	-	-	-	-	28	141	-	-
Ref. [36]	3	15	4	19	7	34	10	50	-	-	-	-
Ref. [38]*	3	15	4	19	7	31	10	52	29	148	32	145
Ref. [39]	3	15	-	-	7	33	-	-	28	137	-	-
Ref. [126]	3	15	4	19	7	31	10	50	28	138	32	155
Ref. [133]	3	15	4	19	7	36	10	52	28	147	32	164
Ref. [134]	3	15	4	19	7	33	10	52	28	139	32	163
Ref. [135]	-	-	4	19	-	-	-	-	-	-	32	157

* : Multi-objective function with maximum redundancy

observability of proposed method is higher than [36] for both the conditions. However, the method reported in [36] is not a multiobjective method rather it includes a distinct constraint to obtain maximum observability. Figure 4.2 shows the single line diagram of IEEE 30-bus test system with installed PMUs for condition 1 under case 1 from the proposed work. As compared to Figure 3.4, the observability of the system by installed PMUs in Figure 4.2 is maximum.

Reference [28] proposed the multi-objective formulation but produced observability value of 141 only, in case of IEEE 118-bus system which is less as compared to proposed method. Similarly, observability in [38] is 145 which is less as compared to proposed method in absence of ZI buses. Having suggested different locations, the proposed method produced better observability as seen from Table 4.3.

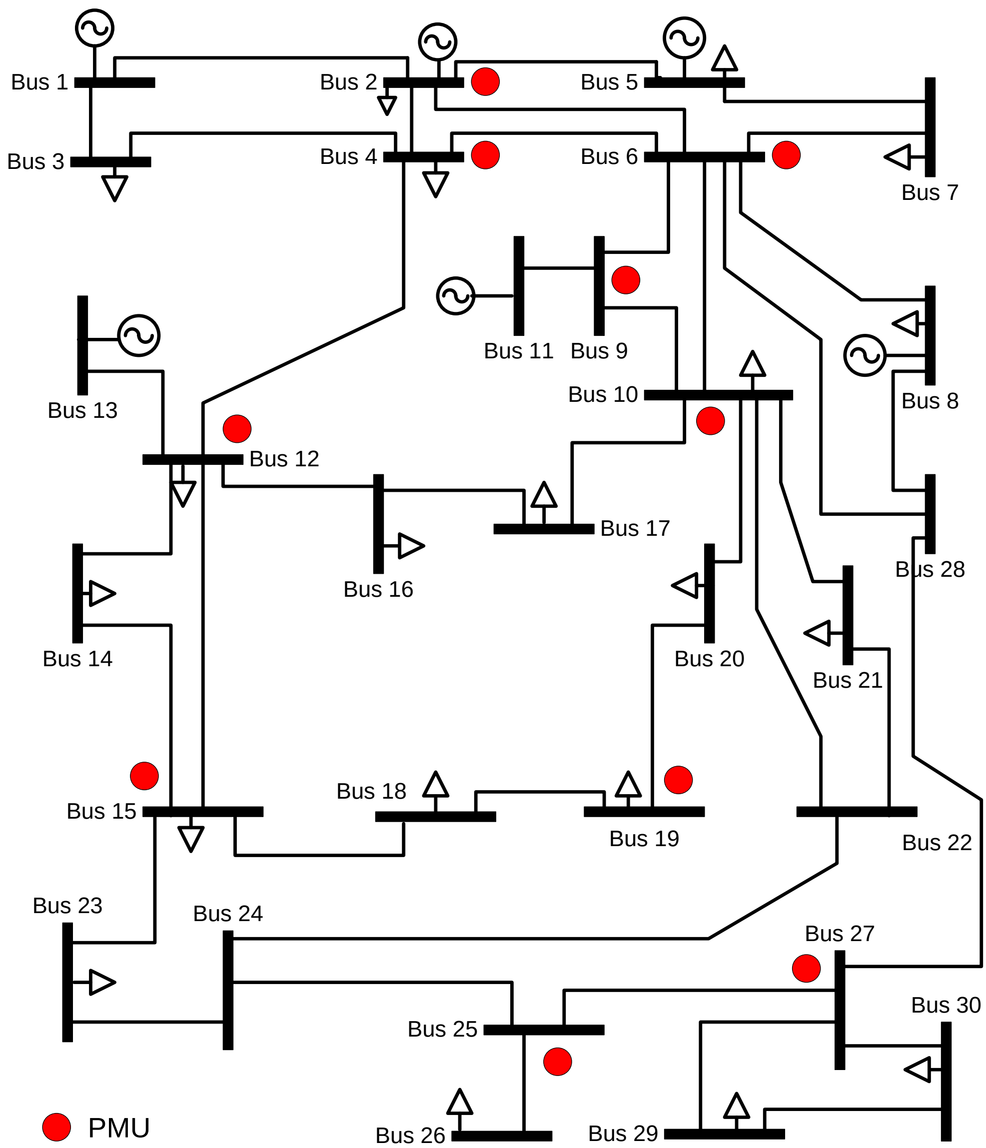

Figure 4.2. IEEE 30-bus test system with installed PMUs (Condition 1 of Case 1)

4.5.2. Case 2: Single PMU outage with and without ZI buses

In this case, locations of PMUs are determined such that all the buses are observable by at least two PMUs. In this way all the buses would be observable even if a single PMU is out. Based on this premise BGSA is applied for such cases. Optimum numbers of PMUs and their locations so obtained under both the conditions are shown in Table 4.4.

The number of PMUs and observability obtained by proposed method and other existing methods have been tabulated in Table 4.5 for three IEEE test systems under two conditions of with and without zero injection buses for comparison. The optimal number of PMUs for IEEE 14-bus is same but observability is as high as two other reported methods [19], [25] could produce in case of ZI buses as shown in Table 4.5. The optimal number of PMUs produced by proposed method is less than other methods for IEEE 30-bus and 118-bus system in presence of ZI buses as shown in Table 4.5. Under condition one (without ZI buses), the optimal number of PMUs for IEEE 118-bus system is same by proposed method as reported in [20] and [133]. However, the observability obtained by proposed method is higher than above mentioned method [20] i.e. 309 against 301 respectively.

4.5.3. Case 3: Single line outage with and without ZI buses

In case of single line outage, locations of PMUs have been determined such that any single line outage does not affect the observability of the system. In this way, all the buses would be observable even if a single line is out. Taking cognizance of this criterion, the proposed algorithm has been implemented. Optimum numbers of PMUs and their locations obtained by proposed method has been shown in Table 4.6 under both the conditions 1 and 2. The results have been tabulated in Table 4.7 for comparison with three IEEE test systems under both the conditions. It can be seen from this table that though optimal numbers of PMU are same in IEEE 14-bus test system but observability of proposed method is higher than [19], [25]. In [25], maximum redundancy function is included in the objective function even then the observability is less than proposed method. In IEEE 30-bus test system for condition-2, optimal number of PMU is 10 in proposed method which is less than [19], [25] but equal to [36]. In IEEE 118-bus system, number of PMU is less in proposed method as compared to [19], [20], [25] but equal to [28], [29].

Table 4.4. Optimal number and location of PMUs for test systems (Case 2)

Test System	Without ZIB		With ZIB	
	No. of PMUs	Locations of PMUs	No. of PMUs	Locations of PMUs
IEEE 14-bus	9	2, 4-10, 13	7	2, 4-6, 9, 11, 13
IEEE 30-bus	21	1,2,4-6,9-13,15,17,19,20,22,24-28,30	13	1,2,4,7,10,12,13,15,17,19,20,24,27
IEEE 118-bus	68	1,3,5,7,9-12,15,17,19,21,22,24,25,27,28,30-32, 34, 36, 37,40,42, 44-46,49,51, 52,54,56,57,59,62, 64-66,68,70,71,73, 75,77,79,80,83,85-87,89,91,92,94,96,100, 101,105,106,109-112,115-118	59	2,3,7,8,11,12,15,17,19-21,23,27,28,30-32,34,35,40,42,44-46,49,51,52, 54,56, 57,59,62,66,70,71,75-77,79,80,83,85-87,89,90,92,94,96,100,102,105,106, 109,110-112,114,117
NRPG 246-bus	153	2, 4, 5, 6, 7, 10, 11, 12, 15, 21, 23, 24, 27, 29-34, 38, 40, 41, 42, 44, 45, 47, 48, 50-58, 60-65, 67, 69, 71, 73, 74, 75, 76, 77, 80, 82, 83, 84, 88, 89, 91, 93-97, 100, 101, 103, 105, 106, 108, 109, 111-113, 117-120, 122-126, 128, 129, 132-135, 138-142, 144, 145, 147-149, 153, 154, 156-161, 164-169, 172, 174, 175, 178, 180-182, 185, 187-191, 193, 194, 195, 197, 199, 201-203, 205, 207, 208, 213, 216, 217, 219, 221, 223, 224, 226, 229, 234, 235, 237-239,242,243,245,246	109	2, 5, 6, 7, 10, 11, 15, 21, 22, 24, 27, 29, 30, 31, 33, 34, 38, 40-42, 44, 45, 47-49, 55-57, 60, 65, 66, 68, 75, 77, 82, 83, 84, 88, 89, 91, 93, 95, 96, 100, 101, 106, 109, 113, 116-119, 121, 123-125, 128, 132, 133-135, 138, 139, 140-142, 144, 147-149, 152, 156-161, 164-166, 168, 169, 172, 174, 178, 181, 182, 185-187, 189, 190, 191, 193-195, 197, 199, 201-203, 205, 207, 208, 219, 223, 224, 235, 245
Polish 2383-bus	1681	-	1087	-

Table 4.5. Results comparison with available techniques considering single PMU outage (Case 2)

Method	IEEE 14-bus				IEEE 30-bus				IEEE 118-bus			
	With ZIB		Without ZIB		With ZIB		Without ZIB		With ZIB		Without ZIB	
	No. Of PMUs	Obs.	No. Of PMUs	Obs.	No. Of PMUs	Obs.	No. Of PMUs	Obs.	No. Of PMUs	Obs.	No. Of PMUs	Obs.
Proposed	7	33	9	39	13	54	21	85	59	274	68	309
Ref.[19]	7	33	-	-	15	57	-	-	63	288	-	-
Ref. [20]	-	-	-	-	-	-	-	-	61	274	68	301
Ref. [21]	-	-	-	-	15	58	-	-	61	283	-	-
Ref. [25]*	7	33	-	-	14	53	-	-	61	281	-	-
Ref. [28]*	7	LNG	-	-	-	-	-	-	63	LNG	-	-
Ref. [29]*	7	LNG	-	-	-	-	-	-	63	LNG	-	-
Ref. [39]	7	31	-	-	15	61	-	-	62	274	-	-
Ref. [126]	7	31	-	-	15	63	-	-	64	291	-	-
Ref. [133]	-	-	9	39	-	-	21	85	-	-	68	309

* : Multi-objective function with maximum redundancy

Due to unavailability of optimal locations of PMUs in [28], [29], their observability could not be given in Table 4.7 for comparison with proposed method. Having suggested different locations with minimum PMUs, the proposed method produced higher observability as seen from Table 4.7.

4.5.4. Case 4: Uncertainty of PMUs with and without ZI buses

In this case, locations of PMUs are determined such that any one of the single line outage or single PMU outage does not affect the observability of the system. Based on this premise BGSA has been applied for such cases. Results obtained by proposed method have been tabulated in Table 4.8 along with other reported methods. These results correspond to ZIB as reported in the literature with whom these results have been compared for fair comparison. The test results reveal that the minimum number of PMUs has been obtained with maximum observability as compared to other methods and every bus is observable irrespective of a line or a PMU outage.

4.5.5. Performance on large size system

The performance of the proposed method has further been studied on two large-scale systems namely NRPG 246-bus and Polish 2383-bus systems. Optimum numbers of PMUs and their locations for NRPG 246-bus system are shown in Tables 4.2, 4.4 and 4.6 under both the conditions 1 and 2. However, comparison of results with other methods has been shown in Table 4.9 for NRPG 246-bus system under all the cases and conditions. Optimal number of PMUs under condition-1 has been found to be 70 in case 1, whereas, it has been reported to be 77 in [48]. The author of [49] also reported the same number of PMUs as obtained by proposed method. However, the greater observability has been found by proposed method compared to above mentioned method. In condition-2 of case 1 and case 2, optimal number of PMUs in proposed method is less as compared to [48]. In case 3, line outage, under both the conditions, the number of PMUs and observability are also given in Table 4.9. The number of PMUs under condition-1 and condition-2 are 87 and 104 respectively. The observability in these two conditions are 428 and 505 respectively. These results for other methods [48], [49] are not given in respective literature; therefore they could not be compared.

Table 4.6. Optimal number and location of PMUs for test systems (Case 3)

Test System	Without ZIB		With ZIB	
	No. of PMUs	Locations of PMUs	No. of PMUs	Locations of PMUs
IEEE 14-bus	7	1,3,6,7,9,10,13	7	2,4-6,9,10,13
IEEE 30-bus	15	2,3,7-10,12,15,17,19,22,24,25,27,30	10	1,4,5,10,12,15,16,19,24,27
IEEE 118-bus	55	1,5,7,9,11,12,15,17,19,21,23,24,25,27, 29,30,32,34,35,37,40,42,44,46,49,50,51, 53,56,59,62,64,66,68,70,71,75,76, 78,80,83,85,86,89,91,92,94,96,100, 101,105,106,108,110,114	50	1,6,8,11,12,15,17,19,21,23,25,27, 29,32,34,35,40,42,44,46,49,50,51, 53,56,59,62,66,69,70, 72,75,76,78, 80,83, 85,86,89,90,92,94,96,100, 101,105,106,108,110,114
NRPG 246-bus	104	1, 3, 6, 7, 9, 10, 11, 15, 18, 21, 23, 27, 32, 33, 34, 40, 43, 48, 50, 54, 56, 57, 61, 62, 63, 64, 65, 68, 70, 74, 75, 80, 83, 84, 85, 87, 88, 91, 92, 93, 97, 101, 105, 106, 109, 117, 118, 121, 122, 125, 126, 127, 128, 130, 132, 138, 139, 140, 141, 142, 144, 146, 147, 154, 157, 158, 160, 161, 163, 165, 168, 169, 173, 175, 181, 182, 185, 187, 188, 190, 191, 192, 194, 195, 197, 199, 201,202,203,204,205,206,213, 218, 219,221,223,229,235,237,238,239,243,244	87	1, 6, 7, 9, 11, 15, 18, 21, 23, 27, 32, 33, 34, 39, 40, 42, 43, 48, 50, 54, 56, 57, 65, 68, 70, 71, 74, 83, 84, 88, 91, 92, 93, 94, 96, 98, 100, 101, 106, 109, 113, 117, 118, 121, 125, 128, 132, 140, 141, 142, 143, 146, 147, 151, 157, 158, 160, 161, 163, 165, 166, 168, 169, 173, 181, 182, 185, 187, 188, 191, 192, 194, 195,197,199,201, 202,203,204, 205,206,211,219,227,229,235,245

Table 4.7. Results comparison with available techniques considering single line outage (Case 3)

Method	IEEE 14-bus				IEEE 30-bus				IEEE 118-bus			
	With ZIB		Without ZIB		With ZIB		Without ZIB		With ZIB		Without ZIB	
	No. Of PMUs	Obs.	No. Of PMUs	Obs.	No. Of PMUs	Obs.	No. Of PMUs	Obs.	No. Of PMUs	Obs.	No. Of PMUs	Obs.
Proposed	7	33	7	27	10	44	15	62	50	243	55	267
Ref.[19]	7	25	-	-	13	42	-	-	53	225	-	-
Ref. [20]	-	-	-	-	-	-	-	-	53	225	59	247
Ref. [25]*	7	25	-	-	13	42	-	-	53	239	-	-
Ref. [28]*	7	LNG	-	-	-	-	-	-	50	LNG	-	-
Ref. [29]*	7	LNG	-	-	-	-	-	-	50	LNG	-	-
Ref. [36]	7	33	7	27	10	44	15	62	-	-	-	-

* : Multi-objective function with maximum redundancy

Table 4.8. Comparison of results in case of uncertainty of PMUs (Case 4, with ZIB)

Method		IEEE 14-bus	IEEE 30-bus	IEEE 118-bus	NRPG 246-bus
Proposed	**No. of PMUs**	7	15	61	114
	Location of PMUs	2,4-6, 9,10,13	2-6, 10,12, 13, 15,17, 18, 20,24, 27,29	2,3,6,8,9,11,12,15,17,19,21,22,24,25,27,29, 31,32,34,36,40,42,44-46,49,51, 53,54,56,57, 59,62,66,68,70,72,75,77, 78,80,83,85-87,89, 91,92,94,96,100,101,105,106,109-112,115, 117,118	-
	Obs.	33	65	279	500
Ref.[19]	**No. of PMUs**	8	17	65	-
	Obs.	33	60	289	-
Ref. [20]	**No. of PMUs**	-	-	61	-
	Obs.	-	-	266	-
Ref. [25]*	**No. of PMUs**	8	16	62	-
	Obs.	35	61	278	-
Ref. [29]*	**No. of PMUs**	7	-	63	-
	Obs.	LNG	-	LNG	-

* : Multi-objective function with maximum redundancy

Table 4.9. Results comparison of NRPG 246-bus

Method	Normal Case (Case 1)				Single PMU Outage (Case 2)				Single Line Outage (Case 3)			
	With ZIB		Without ZIB		With ZIB		Without ZIB		With ZIB		Without ZIB	
	No. Of PMUs	Obs.	No. Of PMUs	Obs.	No. Of PMUs	Obs.	No. Of PMUs	Obs.	No. Of PMUs	Obs.	No. Of PMUs	Obs.
Proposed	51	280	70	357	109	470	153	649	87	428	104	505
Ref. [48]	57	301	77	LNG	117	565	-	-	-	-	-	-
Ref. [49]	-	-	70	330	-	-	-	-	-	-	-	-

Table 4.10. Results comparison of Polish 2383-bus

Method	Normal Case (Case 1)				Single PMU Outage (Case 2)			
	With ZIB		Without ZIB		With ZIB		Without ZIB	
	No. of PMUs	Obs.	No. of PMUs	Obs.	No. of PMUs	Obs.	No. of PMUs	Obs.
Proposed	553	2411	746	3309	1087	4001	1681	5870
Ref. [22]	553	LNG	746	LNG	-	-	-	-
Ref. [25]*	553	LNG	-	-	1224	LNG	-	-
Ref. [34]*	553	2344	-	-	-	-	-	-
Ref. [134]	-	-	776	3447	-	-	-	-

* : Multi-objective function with maximum redundancy

Comparison of results with other methods for both the conditions, without and with ZIB has been shown in Table 4.10 for Polish 2383-bus system. The optimal number of PMUs in case-1 is 746 and 553 for conditions 1 and 2 respectively. Under condition-1 of case 1, optimal number of PMUs is less than 776 of Ref. [134]. For condition-2, observability of proposed method is 2411 which is higher than observability given in [34]. In single PMU outage case, optimal number of PMUs is 1681 and 1087 respectively for conditions 1 & 2 respectively. Maximum observability values for both the conditions are 5870 and 4001 respectively. Under condition-2, optimal number of PMUs is less than 1224 of Ref. [25].

4.5.6. Convergence characteristics and time

The simulations have been performed on a computer having the following configuration: processor @ 3.40 GHz, Intel core i5 and 4 GB RAM. The convergence characteristics of proposed BGSA method for three IEEE test systems, IEEE -14, -30 and -118 bus systems considering three cases (case 1, 2 and 3) for minimization of the number of PMUs have been shown in Figures 4.3. , 4.4, and 4.5 respectively. In Figure 4.3, proposed method in all the cases is converged between iterations 9 and 16. In IEEE 30-bus test system, proposed method is converged in 45 and 34 iterations for condition-1 of case 1, shown in Figure 4.4. In case 2, it converged in 52 iteration for condition-1 and 37 iteration for condition-2. In case 3, method converged in 39 and 33 iterations for condition-1 and condition-2 respectively. In Figure 4.5, the method reported in present chapter converged in 53 and 30 iterations under condition-1 and condition-2 respectively for case 1. In case 2, it converged in 65 and 61 iterations for condition-1 and condition-2 respectively. In case 3, problem converged in 61 and 58 iterations for condition-1 and condition-2 respectively. Thus the proposed method converged in less number of iterations in case 1 as compared to case 2 and case 3 for both the conditions. Convergence speed of proposed method could not be compared with other methods as their details were not available.

It was observed that the time taken by larger systems to arrive at the convergence point is larger compared to smaller size of systems. This phenomenon has been exhibited because of larger number of variables involved in larger systems. Moreover, the increase in time with system size exhibit exponential characteristics as depicted in system size versus solution time graph shown in Figure 4.6. Though the detailed results have been reported for IEEE

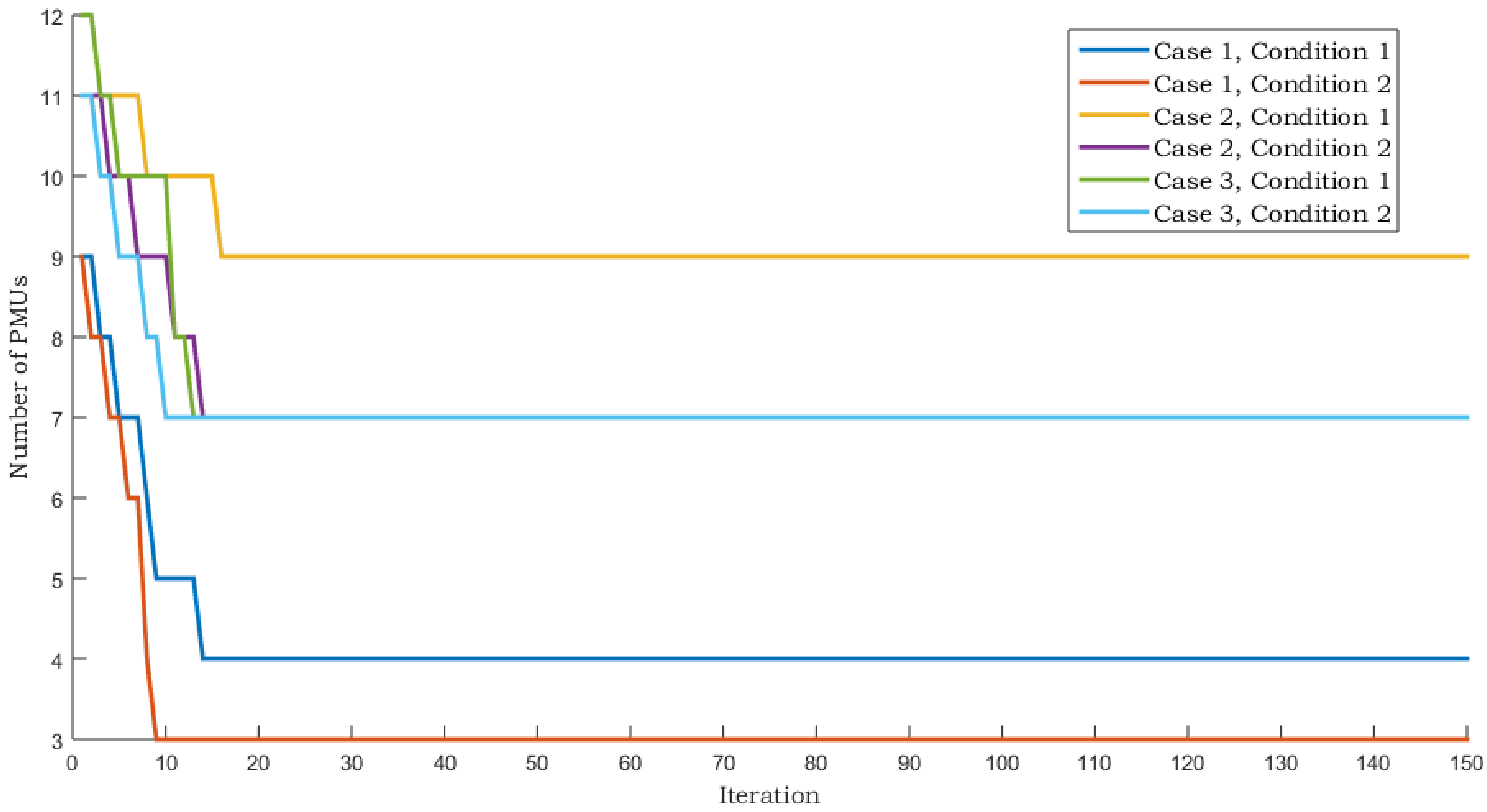

Figure 4.3. Convergence of BGSA for IEEE 14-bus system considering Case 1, Case 2, and Case 3

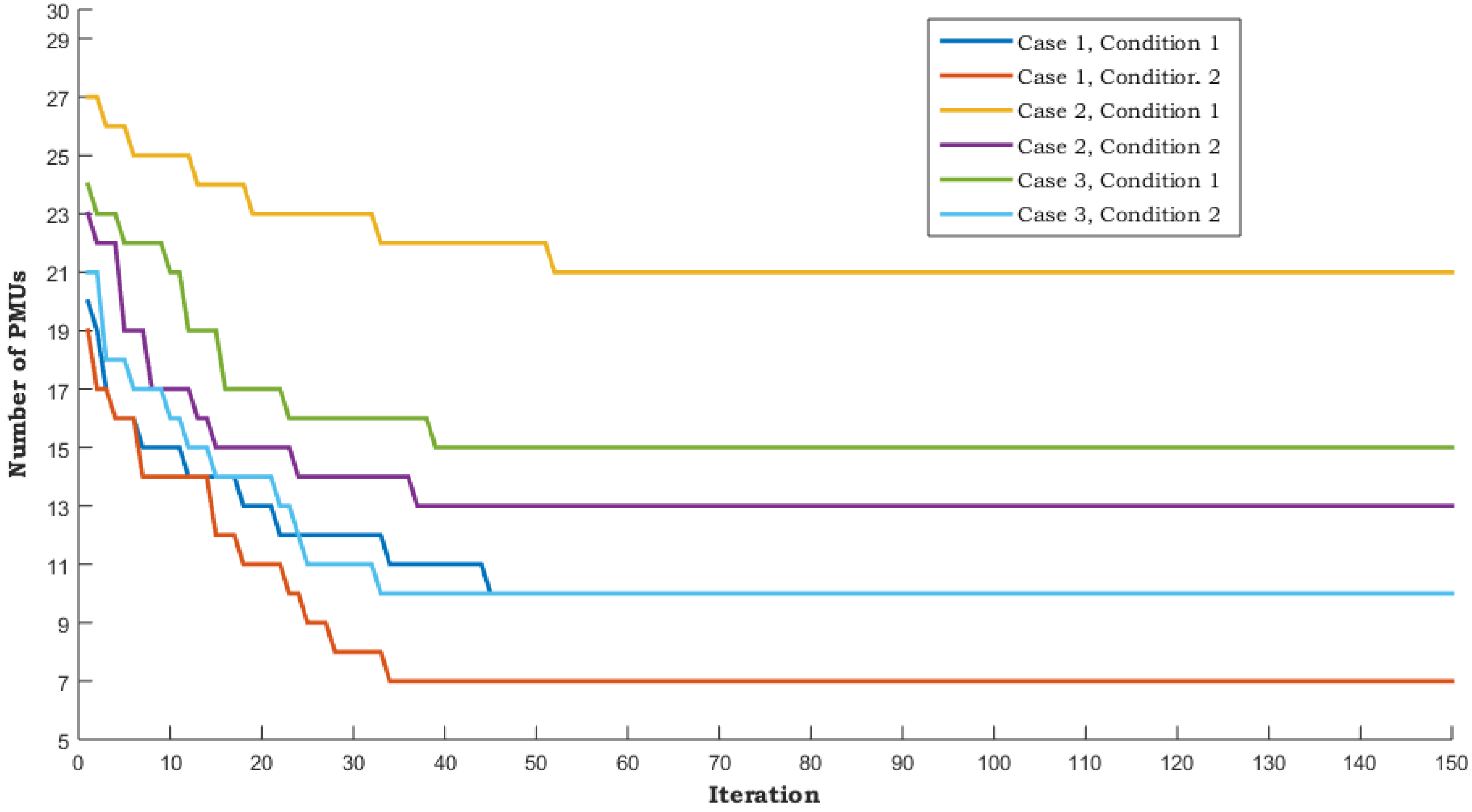

Figure 4.4. Convergence of BGSA for IEEE 30-bus system considering Case 1, Case 2, and Case 3

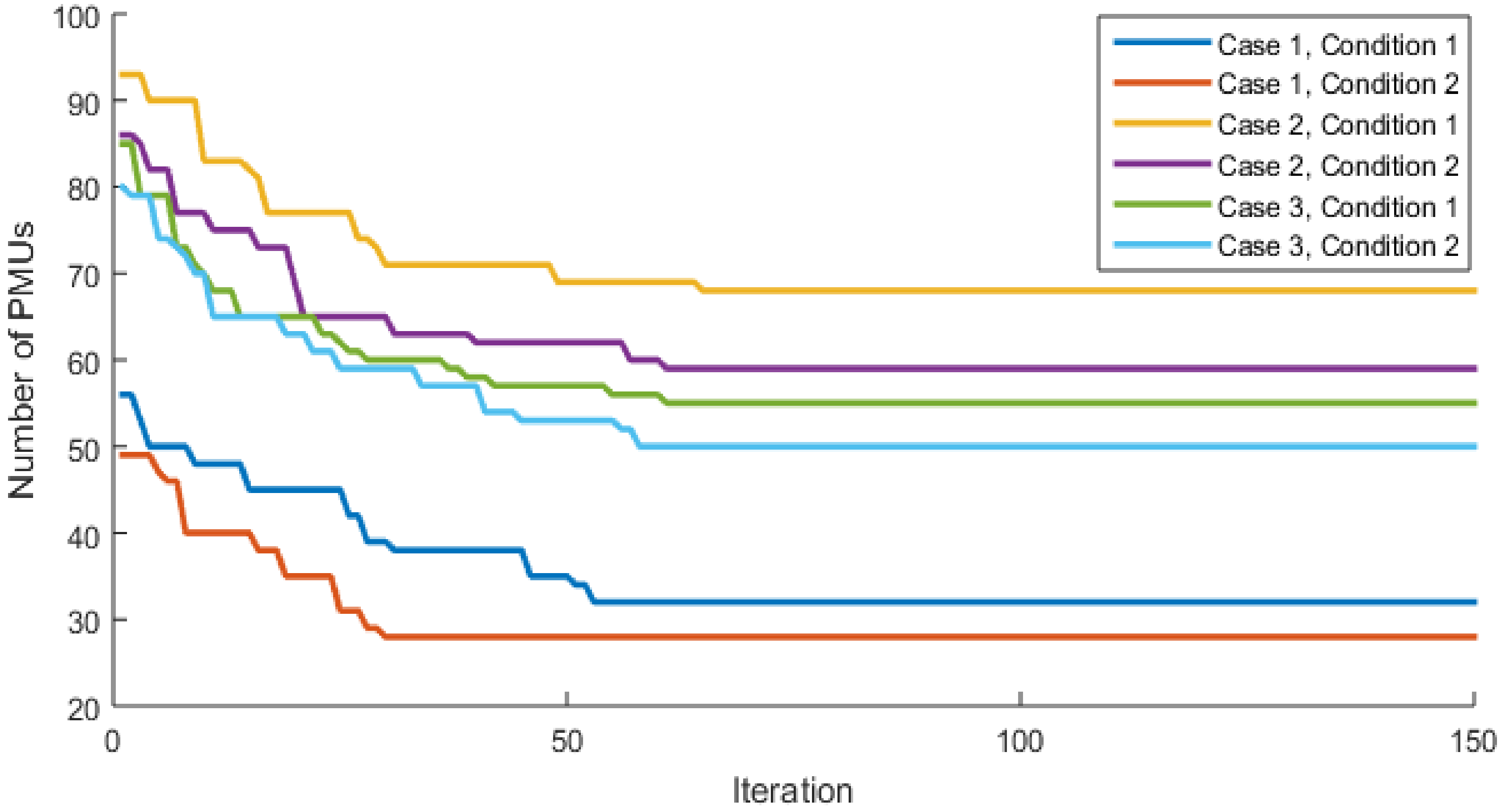

Figure 4.5. Convergence of BGSA for IEEE 118-bus system considering Case 1, Case 2, and Case 3

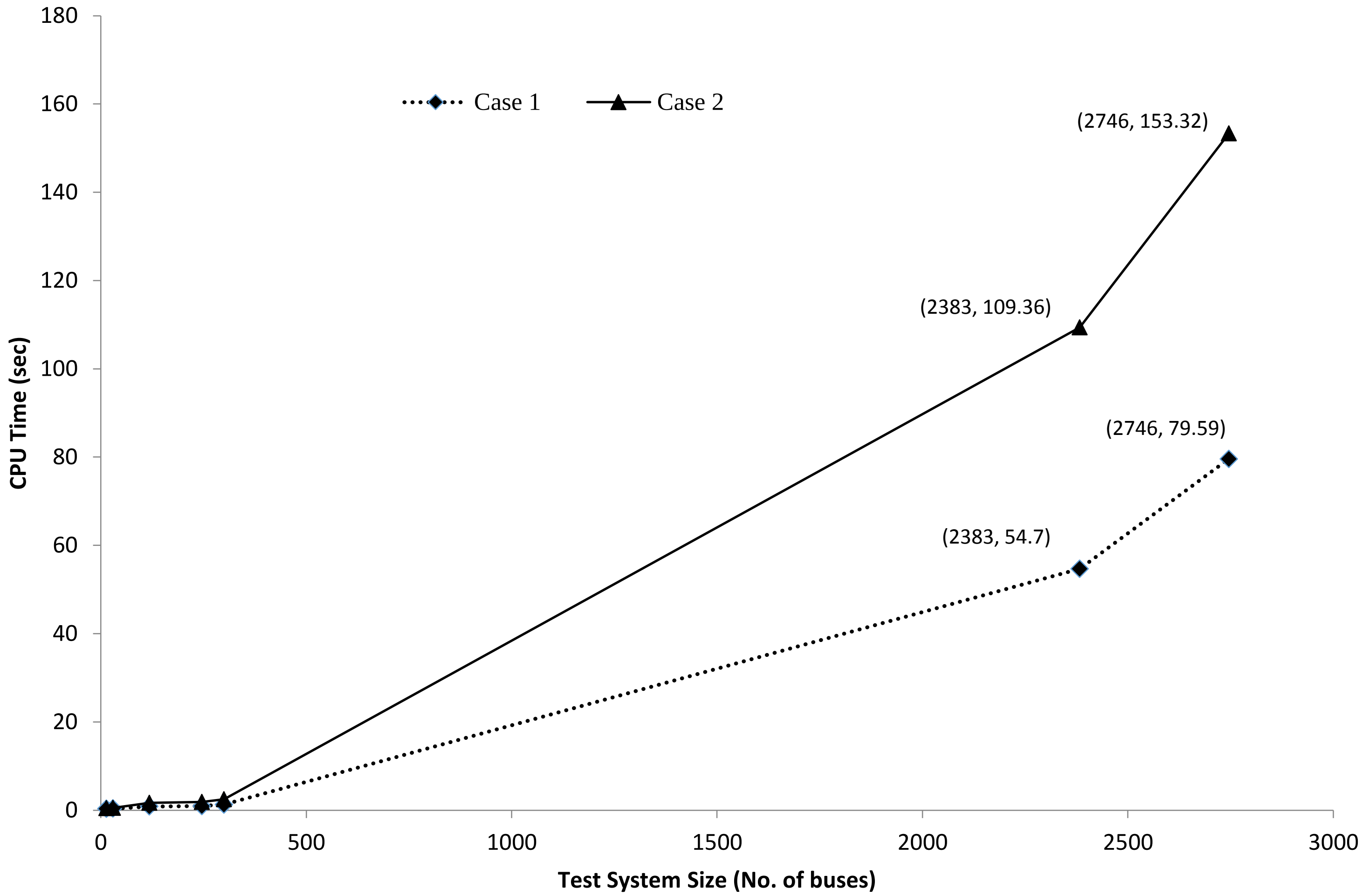

Figure 4.6. Variation in computational time with system size

14-bus, IEEE 30-bus, IEEE 118-bus, NRPG 246-bus and Polish 2383-bus systems but for more effective demonstration of time taken for larger systems, solution time for IEEE 300-bus and Polish 2746-bus systems have also been included as shown in Figure 4.6.

4.6. CONCLUSION

This chapter has proposed BGSA method for solving OPP problem that minimizes the number of PMUs and improves the observability redundancy of the power system. Observability redundancy is the major issue which depends on the best location of PMUs. In this chapter, the redundancy has also been incorporated in the objective function resulting in a multi-objective problem. Besides the placement of mere PMUs, this study considers the placement of PMUs when zero injection buses are also present in the system. Further application of proposed method has been extended for the outage of a PMU or single line outage for both the conditions of without and with zero injection buses. The method has been applied on IEEE 14-bus, IEEE 30-bus, IEEE 118-bus, NRPG 246-bus and Polish 2383-bus test systems, and its results have been compared with other methods, reported earlier in the literature. The simulation results indicate that the proposed PMU placement method provides system measurements with lower or at the most equal number of PMUs but higher observability compared to other methods. The latter has been achieved due to the inclusion of observability also in the objective function.