CHAPTER III

OPP FOR FULL OBSERVABILITY USING BINARY GRAVITATIONAL SEARCH ALGORITHM

3.1. INTRODUCTION

Many optimization techniques have been proposed in the literature which strive to optimize the number of PMUS in the power networks for providing the full observability of the system. These techniques can be broadly classified into two categories, i.e. conventional and non-conventional technique. Reliance on the selection of initial solution in conventional techniques makes it inadequate for application to large number of non-linear complex optimization problems. The absence of dependence on selection of initial solution in case of non-conventional technique makes them advantageous as random selection of initial solution can be done.

Owing to the above mentioned advantages of non-conventinal techniques and presence of discerete binary variables in the basic OPP problem, BGSA has been proposed in this work. It is modified form of GSA whereby discrete binary variables are incorporated in optimization process. BGSA parameters play a major role in achieving the optimal solution hence the procedure of parameter selection along with the basic overview of BGSA methodology has been given in this chapter.

3.2. BASIC PROBLEM FORMULATION OF OPP

The OPP formulation finds a minimal set of PMUs such that a bus must be reached at least once by the PMUs. The optimal placement of PMUs for n bus system is formulated as follows:

$$Min \ \sum_{i=1}^{n} wt_i c_i \tag{3.1}$$

$$\text{Subject to, } AC \geq b \tag{3.2}$$

$$C = \left[c_1, c_2, \ldots\ldots\ldots, c_n\right]^T \tag{3.3}$$

$$c_i \in \{0,1\} \tag{3.4}$$

where, b is the unit vector matrix, wt_i is the weight factor accounting for the cost of installed PMU at bus i, for convenience purpose wt_i is taken to be equal to 1. C is the binary variable vector whose entries are defined as equation (3.4). If c_i=0 then PMU is not available at bus i, if c_i=1 then PMU is available at bus i. The entries of connectivity matrix (A) are defined as follows:

$$a_{ij} = \begin{cases} 1 & if \;\; i = j \\ 1 & if \;\; i \;\; and \; j \;\; are \;\; connected \\ 0 & otherwise \end{cases} \tag{3.5}$$

After getting the optimal number of PMUs, we can easily check the observability of each bus of the system and expression for total observability (O_{total}) is given as:

$$O_{total} = \sum_{k=1}^{P} A_{Lp(k)} \tag{3.6}$$

where, P is the total optimal number of PMUs and Lp is the location of PMUs at the power system buses.

3.2.1. Solution methodology

Let us assume that there are no zero injection (ZI) buses in the test system under consideration. Now, in order to form the constraint set, the binary connectivity matrix A, will be formed first. Matrix A can be directly obtained from the bus admittance matrix by transforming its entries into binary form.

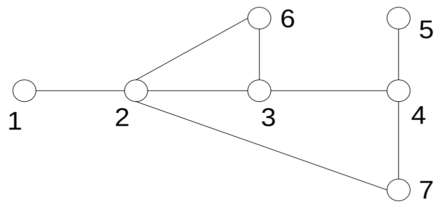

Figure 3.1 7-bus system

Consider the 7-bus system and its measurement configuration shown above. Building the A matrix for the 7-bus system of Figure 3.1 is as follows:

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \tag{3.7}$$

The constraints (3.2) for this problem can be formed as:

$$f(C) = \begin{cases} f_1 = c_1 + c_2 & \geq 1 \\ f_2 = c_1 + c_2 + c_3 + c_6 + c_7 & \geq 1 \\ f_3 = c_2 + c_3 + c_4 + c_6 & \geq 1 \\ f_4 = c_3 + c_4 + c_5 + c_7 & \geq 1 \\ f_5 = c_4 + c_5 & \geq 1 \\ f_6 = c_2 + c_3 + c_6 & \geq 1 \\ f_7 = c_2 + c_4 + c_7 & \geq 1 \end{cases} \tag{3.8}$$

The operator "+" serves as the logical "OR" and the use of 1 in the right hand side of the inequality ensures that at least one of the variables appearing in the sum will be non-zero. The constraint $f_1 \geq 1$ implies that at least one PMU must be placed at either one of buses 1 or 2 (or both) in order to make bus 1 observable. Similarly, the second constraint $f_2 \geq 1$ indicates that at least one PMU should be installed at any one of the buses 1, 2, 3, 6, or 7 in order to make bus 2 observable.

3.2.2. Concept of observability

In recent years, phasor measurement have been extensively placed in the power system. They provide the synchronized voltage and current measurements in microsecond. The presence of a PMU at a bus, calculate the voltage phasor of that bus and current phasors of each branch which are directly connected to that bus [36]. Therefore, a given system is said to be fully observable when all the buses of the system are observable through direct or indirect measurement.

The voltage phasors of the buses next to the PMU installed bus can be determined by calculating the current phasors of the branches, bus voltage phasor and line parameters [124]. Therefore, the placement of PMUs at a bus not only to monitor that bus but also all the neighboring buses attached to it. As displayed in Figure 3.2(a), assume PMUs are placed at buses 2 & 3. PMU at bus 2 will observe buses 1, 2, 3, 6 and 7 and PMU at bus 3 will observe buses 2, 3, 4 and 6. All the buses of the 7-bus system are observable except bus 5, therefore the system is not fully observable in Figure 3.2(a). Similarly, in Figure 3.2(b), PMUs at buses 2 and 7 will observe the buses (1, 2, 3, 6 and 7) and buses (2, 4 and 7) respectively, and finally bus 5 will be unobservable. Therefore, the system is not fully observable in Figure 3.2(b). Figures 3.2 (c) and (d) show the fully observable system because all the buses in both the figures are observable through directly or indirectly measurement.

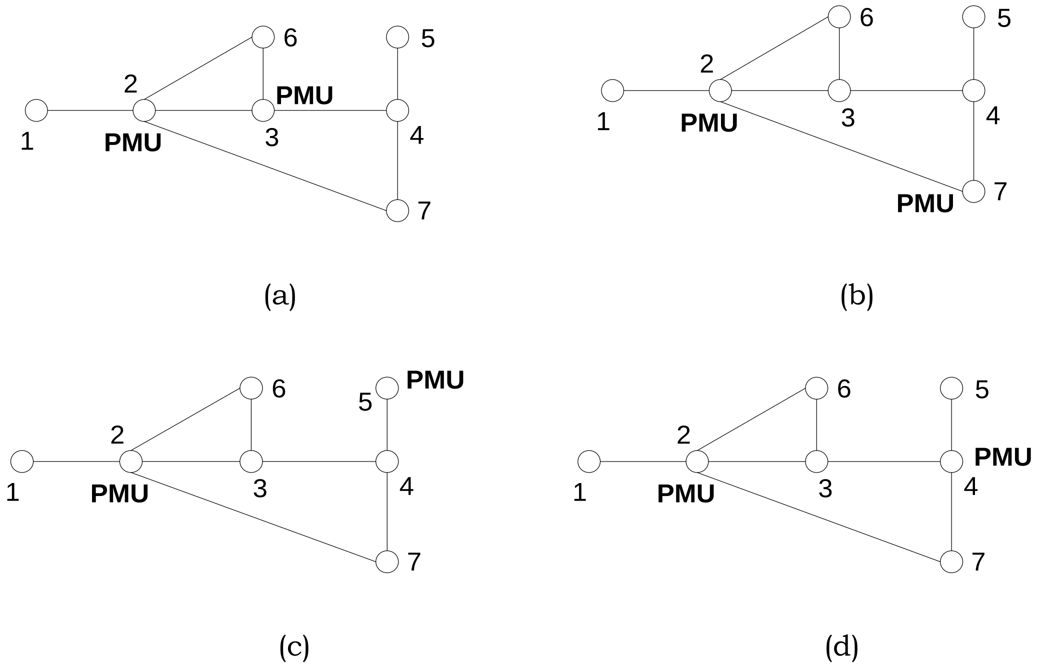

Figure 3.2 7-bus system (a) Unobservable system (b) Unobservable system (c) Observable system (d) Observable system

3.3. GRAVITATIONAL SEARCH ALGORITHM (GSA)

The GSA is a stochastic algorithm that uses the concept of gravity and laws of motion to provide a solution for an optimization problem. It is a well-known fact that any two particles in the universe attract each other by a gravitational force directly proportional to the product of their masses and inversely proportional to the distance between them [52]. In GSA, this concept is utilized along with the laws of motion, where agents are considered as objects and their performance is measured by their masses with a gravitational force acting as a mode of communication between them.

A review of Newton's gravitational laws can give a better understanding of GSA. Newton's first law of gravity can be stated mathematically as:

$$F = G\frac{M_1 M_2}{R^2} \tag{3.9}$$

Newton's second law says that a force applied to a particle is equal to the product of its mass and the particle acceleration (*Acc*). Mathematically

$$Acc = \frac{F}{M} \tag{3.10}$$

Both of the above mentioned laws can be rewritten as

$$F_{ij}^d = G\frac{M_{pi}(t) \times M_{aj}(t)}{R_{ij}(t) + \varepsilon}(x_j^d(t) + x_i^d(t)) \tag{3.11}$$

$$Acc = \frac{F_{ij}}{M_{ii}} \tag{3.12}$$

where M_{aj} represents the active gravitational mass of particle *j* and M_{pi} represents the passive gravitational mass of particle *i*. The acceleration, Acc_i is proportional to F_{ij} and inversely proportional to M_{ii}, inertial mass of particle *i*.

Now, consider a system with *Q* agents (masses). We define the position of the i_{th} agent by:

$$X_i = (x_i^1, \ldots, x_i^d, \ldots, x_i^n) \qquad \text{for } i = 1, 2, 3.., Q \tag{3.13}$$

where x_i^d represents the position of i_{th} agent in the d^{th} dimension. At a specific time 't', the force acting on mass '*i*' from mass '*j*' can be defined as following:

$$F_{ij}^d = G(t)\frac{M_{pi}(t) \times M_{aj}(t)}{R_{ij}(t) + \varepsilon}\left(x_j^d(t) + x_i^d(t)\right) \tag{3.14}$$

The total force acting on the i_{th} agent $\left(F_i^d(\mathrm{t})\right)$ is calculated as follows:

$$F_i^d(\mathrm{t}) = \sum_{\substack{j=k_{best},\\ j \neq i}}^{Q} rand_j F_{ij}^d(\mathrm{t}) \tag{3.15}$$

where k_{best} is a function of time, with the initial value of K_0 at the beginning and decreasing with time. This way, all agents apply the force at the beginning, and as time passes, k_{best} is decreased linearly and at the end, there will be just one agent applying force to the others. By the law of motion, the acceleration of the agent *i* at time *t*, in direction d_{th} is given by:

$$Acc_i^d(\mathrm{t}) = \frac{F_i^d(\mathrm{t})}{M_{ii}(\mathrm{t})} \tag{3.16}$$

Furthermore, the next velocity of an agent is a function of its current velocity added to its current acceleration. Therefore, the next velocity and the next position of an agent can be calculated as follows:

$$v_i^d(t+1) = rand_i \times v_i^d(t) + Acc_i^d(t) \tag{3.17}$$

$$x_i^d(t+1) = x_i^d(t) + v_i^d(t+1) \tag{3.18}$$

where $rand_i$ is a uniform random variable in the interval [0, 1]. The gravitational constant G, is initialized at the beginning of the problem and will be decreased with time to control the search accuracy [52]. In other words, G is a function of the initial value (G_0) and time (t)

$$G(t) = G(G_o, t) \tag{3.19}$$

Gravitational and inertial masses are simply calculated by the fitness evaluation. Both of the masses namely gravitational and inertial are assumed to be equal and their values are calculated using the map of fitness. The inertial and gravitational masses are updated by using following equations:

$$M_{ai} = M_{pi} = M_{ii} = M_i \tag{3.20}$$

where, $i = 1, 2, 3....,Q$

$$m_i(t) = \frac{fit_i(t) - worst(t)}{best(t) - worst(t)} \tag{3.21}$$

$$M_i(t) = \frac{m_i(t)}{\sum_{j=1}^{Q} m_j(t)} \tag{3.22}$$

where $fit_i(t)$ represents the fitness value of the agent i at time t. The *best* (t) and *worst* (t) in the population of agents respectively indicate the strongest and the weakest agent according to their fitness and can be defined as follows:

For a minimization problem:

$$best(t) = \min_{j\in\{1,2,..Q\}} fit_j(t) \tag{3.23}$$

$$worst(t) = \max_{j\in\{1,2,..Q\}} fit_j(t) \tag{3.24}$$

3.4. BGSA AND IT'S IMPLEMENTATION FOR OPP PROBLEM

This chapter proposes an algorithm for optimal PMU placement. The concept of GSA expressed by (3.9) through (3.24) can be used for OPP problem provided it can handle binary variables. This objective can be met by converting the agent velocity and position expressed by (3.17) and (3.18) respectively in binary form. The basic property of sigmoid function has been explored in this work to convert the agent position and velocity into binary form. This would allow to squeeze position and velocity components within the interval [0, 1]. Velocity component expressed by expression (3.17) in GSA can be expressed as

$$sigmoid(v_i^d(t)) = \frac{1}{\left(1+e^{-v_i^d(t)}\right)} \tag{3.25}$$

The mass position vector x_i^d expressed by expression (3.18) in GSA can be expressed as

$$x_i^d(t) = \begin{cases} 1, & if \;\; rand_j < sigmoid\left(v_i^d(t)\right) \\ 0, & otherwise \end{cases} \tag{3.26}$$

The gravitational constant G in expression (3.27) is updated as follows:

$$G(t) = G_o e^{-\beta \frac{t}{T}} \tag{3.27}$$

where, t is the current iteration and T is the total iterations. Figure 3.3 shows the flow chart of proposed BGSA.

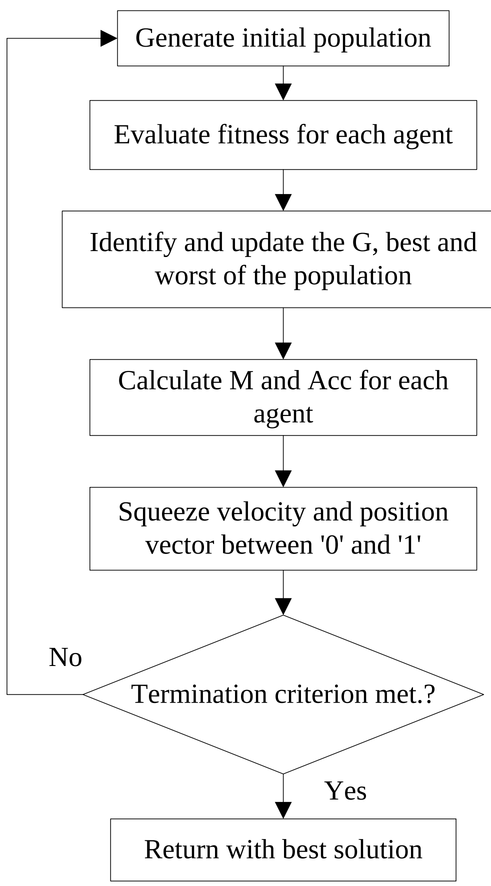

Figure 3.3 Flowchart of the BGSA

The detail procedure to apply the BGSA based on Newton's Law of Gravity and Mass interactions for solving the basic OPP problem is as follow:

Step 1. Read bus data and line data of the test system.

Step 2. Identify the search space.

Step 3. Initialize BGSA parameters: total iterations (T), population size (Q), initial gravitational constant (G_o) and user specified constant (β).

Step 4. Initialize population within min and max values of the control variables.

Step 5. Calculate the fitness values of each agent in the population.

Step 6. Update $G(t)$, $best(t)$, $worst(t)$ and $M_i(t)$ for i=1, 2, ….., Q based on fitness value.

Step 7. Calculate total force in different directions using Equation (3.15).

Step 8. Modify acceleration of each agent using Equation (3.16).

Step 9. Update velocity and position of each agent using Equation (3.25) and Equation (3.26) respectively.

Step 10. Repeat steps 6 to 9 until the termination criterion is reached.

Step 11. Stop

3.5. SELECTION OF BGSA PARAMETERS

Solution of a problem by BGSA is governed by four parameters namely, initial gravitational constant (Go), user specified constant (β), population size (Q) and total iterations (T). Therefore, it is imperative to select appropriate values of these parameters. However, the values of these parameters can only be determined sequentially one at a time keeping other three parameters fixed. The parameter Q affects the accuracy of the solution whereas T determines the accuracy and termination point. Therefore, selection process can begin with selecting some values of Q and T and Go and β can be determined by trial and error. Once the value of Go and β are determined, the values of Q and T can be determined a fresh following the similar procedure as used for Go and β.

In present work, selection of parameters begun with determination of parameter Go, keeping parameters Q, T and β fixed. Parameter Go was varied from 1 to 40 in steps of 1 keeping Q, T and β at 100, 300 and 3 respectively. The value of Go at which the solution converged was assumed to be the best value of Go. Once, Go was determined, the value of β was varied from 0.2 to 10 in steps of

0.2 till convergence was achieved. It is to be noted that the *Go* and *β* were determined for fixed values of *Q* and *T*. Afterward, *Q* was varied from 10 to 200 in steps of 10 for previously determined values of *Go* and *β* and fixed value of *T* till convergence was achieved. Choosing this value of *Q* and predetermined values of *Go* and *β*, *T* was varied from 80 to 500 in steps of 10 till convergence. The value of *T* was chosen to be the final value at which solution converged. Above exercise was carried out for base case of all the systems on which test was to be carried.

The values of the four parameters were varied according to the above procedure. Various values of the parameters, starting from beginning to end, are shown in Table 3.1. It can be observed from this table that the set of *Go*, *β*, *Q* and *T* of values 29, 4, (40-200) and (140-500) produced converged solution for all the tested systems. However, convergence was achieved for a range of (40-200) and (140-500) of *Q* and *T* respectively, indicating any value in this range is suitable. Vowing this fact, a value of 50 for *Q* and 150 for *T* lying in the above range was finally selected in this work. Finally a set of *Q*, *T* and *Go* and *β* was chosen as shown in Table 3.2 which yield converged solution for all the test systems. It can further be observed from Table 3.2 that the parameters selected for three systems worked well for other system also. However, it is also observe that the number of iteration (*T*) increases with increase in system size. Moreover, increase in search space may also be required for larger systems.

3.6. TEST RESULTS

This chapter proposed the BGSA for optimizing the number and location of PMUs in the power network. For this, a basic OPP formulation has been used to check the effectiveness of proposed optimization algorithm. The proposed algorithm has been tested on IEEE 14-bus, 30-bus, and 118-bus test systems and data of test systems has been taken from [125]. The locations of PMUs have been determined such that the entire system becomes observable. Table 3.3 shows the results in terms of optimal number of PMUs and their locations for all the above mentioned test systems. For IEEE 14-bus test system, optimal number of PMUs for full system observability is 4 and their locations are 2, 8, 10 and 13. In IEEE 30-bus test systems, 10 PMUs are obtained for full observability of the system. Figure 3.4 shows the IEEE 30-bus test system with installations of obtained PMUs of Table 3.3. Column four in Table 3.3 represents the total observability of the test system for obtained PMUs. The given objective function contained only the optimal PMUs function, besides, maximum observability (MO) function has been considered in objective functions of Chapter 4 and Chapter 5.

Table 3.1. Selection of BGSA parameters

Parameters	Range	Step Size	Fixed parameters	Selection of parameters	
				Value of parameter	**Converged system size**
Q	10-200	10	T, Go, β	10-20	NOS
				30	14, 30
				40-200	14, 30, 118, 246
T	80-500	10	Q, Go, β	80-110	NOS
				120	14
				130	14, 30, 118
				140-500	14, 30, 118, 246
Go	1-40	1	Q, T, β	1-24, 26, 28, 30-31, 33-40	NOS
				25	14, 118
				27	30, 246
				29	14, 30, 118, 246
				32	118
β	0.2-10	0.2	Q, T, Go	0.2-3.4, 4.6-10	NOS
				3.6	118
				3.8	30
				4	14, 30, 118, 246
				4.2	14, 118
				4.4	246

NOS: No Optimal Solution

Table 3.2. BGSA parameters

G_0	β	T	Q
29	4	150	50

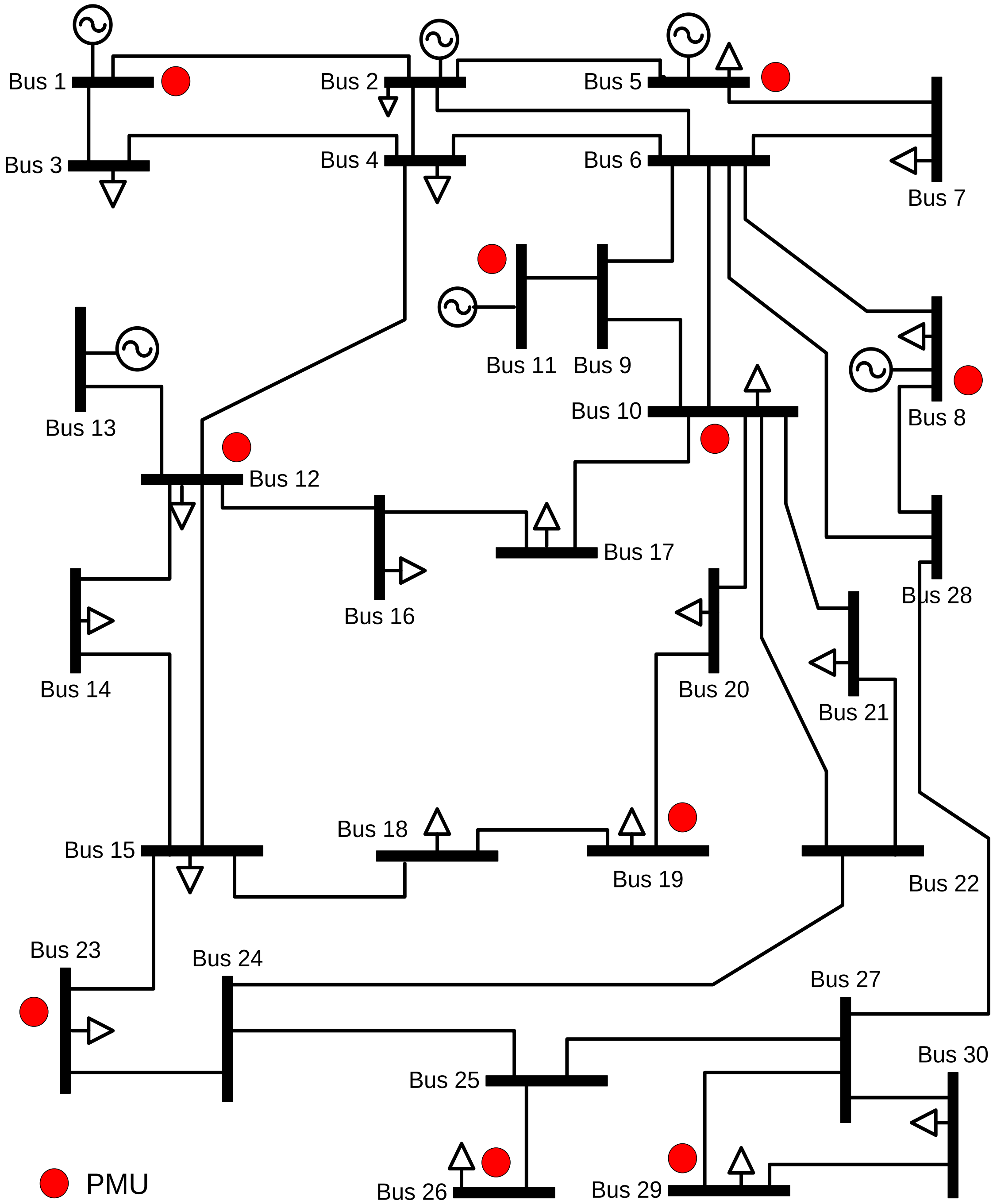

Figure 3.4 IEEE 30-bus with installed PMUs

A comparison of present result with other results reported in the literature has been tabulated in Table 3.4 for three test systems. It can be seen from Table 3.4 that the results obtained by proposed method produced same number of PMUs as obtained by other reported methods. This chapter contains only the basic case of PMU placement, therefore, there are no more differences in the proposed results with results reported in other literature.

In order to minimize the number of PMUs, Figure 3.5 shows the convergence characteristics of proposed BGSA method for all the test systems. A steep decline in objective function value is observed in Figure 3.5. It is observed from Figure 3.5 that the BGSA converged in ten iterations for IEEE 14-bus test system and suggested 4 PMUs for full observability of the system. Similarly, IEEE 30-bus and IEEE 118-bus test systems converged in 43 iterations and 53 iterations and suggest 10 PMUs and 32 PMUs respectively. The computational times of proposed BGSA method for IEEE 14-bus, IEEE 30-bus and IEEE 118-bus systems are 0.63 sec, 0.79 sec, and 2.71 sec respectively. Figure 3.5 also reveals that the number of iterations increases as the size of the system increases, whereas, it is not proportional to size of the system.

Table 3.3. Optimal number and location of PMUs for IEEE test systems

Test system	Optimal no. of PMUs	Optimal location of PMUs	O_{total}
IEEE 14-bus	4	2, 8, 10, 13	14
IEEE 30-bus	10	1,5,8,10,11,12,19,23,26,29	35
IEEE 118-bus	32	2,5,10,12,15,17,21,25,29,34,37,41,45,49,53,56,62,64,72,73,75,77,80,85,87,91,94,101,105,110,114,116	151

Table 3.4. Comparison of proposed results with other methods

Test system	Proposed	Ref. [22]	Ref. [27]	Ref. [36]	Ref. [38]	Ref. [126]
IEEE 14-bus	4	4	4	4	4	4
IEEE 30-bus	10	10	10	10	10	10
IEEE 118-bus	32	32	NA	NA	32	32

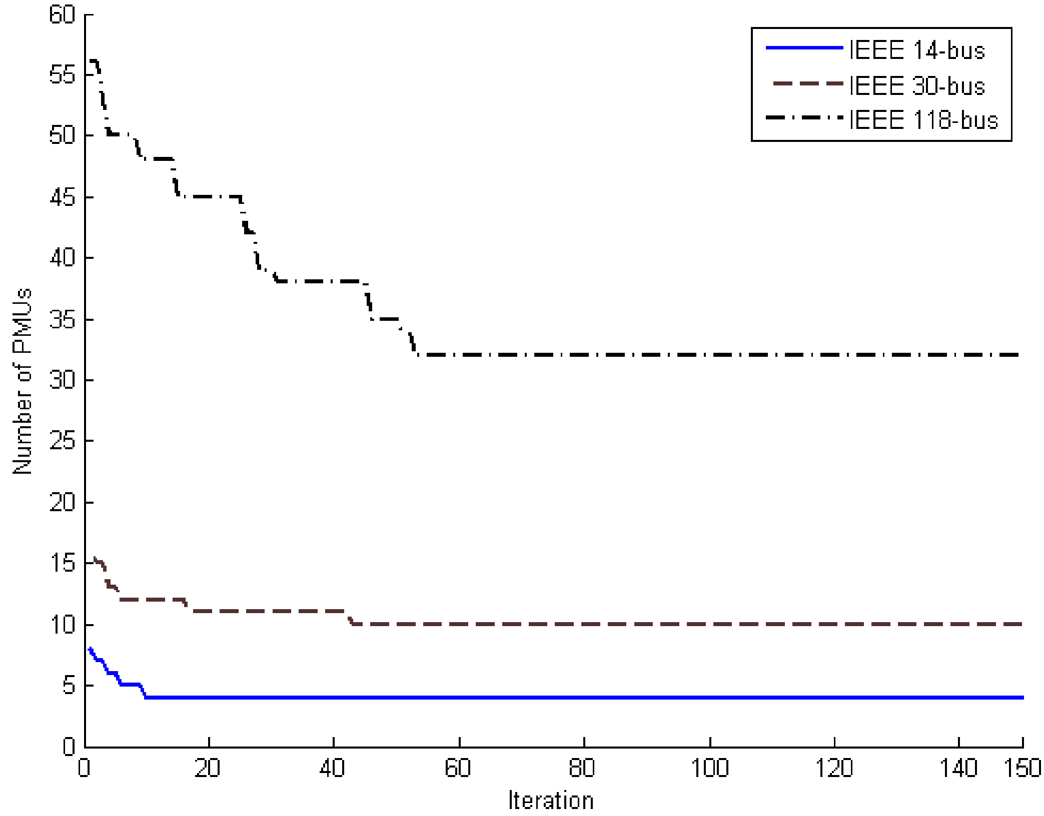

Figure 3.5 Convergence of BGSA for all the test systems

3.7. CONCLUSION

This chapter presents the basic problem formulation of PMU placement in the power system. To solve this problem, a technique has proposed, called as BGSA. This technique is basically based on the newton's law of gravity and mass interaction. The OPP problem deals with discrete binary variables. Therefore, the basic concepts of GSA has modified into the discrete binary variables. The procedure of appropriate parameters selection to achieve the global optimal solution has been explained. The test results reveal that the BGSA based basic OPP problem provides same number of PMUs as compared to other methods reported in the literature. The performance of novel PMU placement formulation using BGSA method on various test systems also on real system are evaluated and discussed in the subsequent chapters.