Preface

The thesis consists of five chapters. Chapter 1 is the collection of the definitions, notations and some fundamental results which are used in subsequent chapters.

In chapter 2, we present an approximate method for solving the Generalized Abel's integral equations. The approximate method is based on the collocation method for solving Volterra integral equations. Generalized Abel's integral equations could be considered as a more general form of Volterra integral equations. Collocation method in sense of Atkinson's approach is applied to get the approximate solution of Generalized Abel's integral equations. The convergence analysis of the presented method is also established. The different polynomials such as 1) Jacobi polynomials 2) Legendre polynomials 3) Chebyshev polynomials and 4) Gagenbauer polynomials are considered to get the numerical solution of the Generalized Abel's integral equations. Illustrative examples with different solutions are considered to show the validity and applicability of the proposed method. Numerical results show that the proposed method works well and achieve good accuracy even for less number of polynomials. Further, the performance of the proposed method is compared under the effect of different polynomials. The entire chapter in form of a paper has been published in Journal of Computational and Applied Mathematics.

In chapter 3, we provide Bernstein approximation and hybrid Bernstein approximation to solve the generalized Abel's integral equations (GAIEs) via collocation approach. The discussion is started with the basic definitions of Bernstein and hybrid Bernstein polynomials and some basic results related to Bernstein approximation. Bernstein polynomial and hybrid Bernstein functions are used to approximate the solution of

GAIEs. Convergence analysis for the proposed methods are proved in detail. Several numerical examples are considered from the literature to show the accuracy of the proposed methods. In last section of this chapter, application of the proposed method is shown through Abel inversion in tomography.

In chapter 4, we describe the collocation and Galerkin's approaches for fractional integro-differential equations (FIDEs). We explain the application of Jacobi polynomials to solve the FIDEs which converts the problem into a system of algebraic equations. To approximate the solution of FIDEs by Jacobi polynomials, a suitable variable transformation is applied which assures that the solution of the transformed FIDEs is sufficiently smooth. This results in a rapid convergence of both the methods with Jacobi polynomials even when the solution is not smooth. The error estimate and convergence analysis for presented numerical methods are provided. To perform the numerical simulations, two test examples (linear and nonlinear) are considered with non-smooth solutions, and numerical results are presented. Further, the comparative study of the presented schemes with some existing numerical schemes is provided. The entire chapter in form of a paper has been accepted for publication in Iranian Journal of Science, Technology and Transactions A: Science.

In chapter 5, we study a numerical approach for some class of generalized fractional integro-differential equations (GFIDEs) defined in terms of the B-operators presented recently. We develop collocation method for linear and nonlinear form of GFIDEs. The numerical approach uses the idea of collocation methods for solving integral equations. Legendre polynomials are used to approximate the solution in finite dimensional space with convergence analysis. The obtained approximate solution recovers the solution of the fractional integro-differential equation (FIDE) defined using Caputo derivatives in a special case. FIDEs containing convolution type kernels appear in diverse area of science and engineering applications therefore some test examples varying the kernel in the B-operator are considered to perform the numerical investigations. The numerical results validate the presented scheme and provides good accuracy using few Legendre basis functions. The entire chapter in form of a paper has been published in Journal of Computational and Applied Mathematics.