MATHEMATICAL MODELING ON ADVANCED THERMOELASTICITY AND MAGNETO THERMOELASTICITY

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By

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Chapter-6

Summary of the thesis and scope for future work

6.1 Summary of the work

The present thesis is concerned with the investigation of wave propagation in homogeneous isotropic thermoelastic and magneto-thermoelastic material in the contexts of some recent models of heat conduction. We consider different unsolved problems and investigate them in order to analyze the mutual interactions of elastic field with other fields like, thermal and magnetic field. We pay special attention to the effects of employing different heat conduction models for such problems. We highlight special features in each case. Green-Naghdi models, dual-phase-lag model, fractional order heat conduction model and recently developed model related to the concept of 'memory dependent derivative' have been selected to study the propagation of waves in elastic solid subjected to thermoelastic or magneto-thermoelastic loading effects. Furthermore, we have attempted to obtain the solution of a mixed boundary initial value problem by boundary integral equation formulation in the context of fractional order thermoelasticity. The effect of magnetic field has been investigated in some problems and compared with the results in the absence of magnetic field. Our investigation brings to highlight the following features:

6.1.1 Some aspects of magneto thermoelasticity under Green-Naghdi models

Green and Naghdi proposed the thermoelasticity theory in a completely alternative way to address a broad class of problems. Their theory is divided into three parts and they are called as Green-Naghdi theories of type-I, II and III. Chapter 4.1 deals with a detailed investigation of electro-magneto-thermoelastic plane waves in the presence of magnetic field in the context of Green and Naghdi type-II theory of thermoelasticity with the consideration of finite conductivity of the medium. We have subdivided our problem into two subcases to study the longitudinal and transverse waves propagating through the medium. Small and high frequency asymptotic approximation techniques have been employed for obtaining analytical solutions for various important components of waves of different modes. Numerical computation has been carried out to verify our analytical results as well as for better illustration. From analytical and numerical results in two cases, we have concluded the following features:

(i) Longitudinal and transverse mode waves corresponding to two separate cases are obtained in which the transverse mode wave is completely independent of the thermal field, but mechanical and electrical fields are coupled together. Hence, we identify quasi-magneto-elastic shear wave and quasi-electro-magnetic shear wave.

(ii) The longitudinal wave is found to be coupled with the thermal field. We identify quasi-magneto elastic mode longitudinal wave and quasi-magneto-thermal mode longitudinal wave. Both waves are almost similar in nature. They are dispersive in nature and show constant limiting speed as frequency increases to high value. However, both waves propagate with negligible specific loss and very high penetration depth.

(iii) Presence of magnetic field plays very important role in the results of longitudinal waves. In absence of magnetic field, we obtain the dispersive relation coupled with purely elastic and thermal fields. However, we observe that phase velocity of elastic and thermal mode waves are constant and are frequency independent. Furthermore, specific loss is completely zero and penetration depth is infinite implying that there is no decay of waves and waves will penetrate the materials till an infinite depth. However, in our present work in Chapter 4.1, we consider the presence of magnetic field and due to that we observe that phase velocity of elastic mode wave and thermal mode wave is dependent on frequency, ω. Phase velocities of both the waves start from a constant values and then increase as the frequency increases but finally they attain constant limiting values. Furthermore, we note a small value of specific loss and a very high value of penetration depth in presence of magnetic field.

(iv) We obtain quasi-electro-magnetic shear wave and quasi-magneto-elastic shear wave in which phase velocity of quasi-magneto elastic mode shear wave is always constant, i.e. frequency independent and its value is 0.5. However, quasi-electro-magnetic shear wave is slowly dependent on frequency. Like the case of longitudinal waves, the specific loss for the transverse mode waves show very less value and penetration depth has a very high limiting value, although they are dispersive in nature. It is believed that the results of the present investigation reveal several specific features of magneto-thermoelastic interactions under thermoelasticity without energy dissipation which has not been investigated earlier by any researcher.

In Chapter 4.2, dispersion relation solutions for the plane wave propagating in a magneto-thermoelastic media with finite electrical conductivity have been determined by employing Green and Naghdi theory of thermoelasticity of type-III. We have made a comparative study of GN-I, GN-II and GN-III theory of thermoelasticity in presence of an external magnetic field. Significant resemblance and non-resemblance among the results under GN-I, GN-II and GN-III theory of thermoelasticity have been identified. The most important features of the present investigation can be summarized as follows:

(i) The phase velocity of thermal mode wave is found to be an increasing function of frequency under GN-I and GN-III models.

(ii) Quasi- magneto dilatational and thermal mode waves propagate faster in the theory of type GN- I in comparison to GN-III theory of thermoelasticity. However, phase velocity of quasi magneto-electric wave is unaffected whether we employ GN-I theory or GN-III theory. Quasi-magneto dilatational and thermal mode wave are found to be non-dispersive, i.e., propagate with constant speed and there is no significant variation on the phase velocity of both the modified elastic wave and modified thermal wave with respect to frequency in the context of GN-II theory of thermoelasiticity.

(iii) Penetration depth has a less finite value in the case of GN-I and GN-III theory of thermoelasticity. However, in case of GN-II theory, we see that penetration depth for both waves namely, quasi-magneto dilatational wave and quasi-magneto thermal wave is infinite since waves are propagating with constant speed and there is no specific energy loss of waves. This is a very distinct feature of GN-II model.

(iv) In view of above points, we can conclude that for coupled magneto-thermoelastic problem, GN-II model exhibits realistic behavior in comparison to GN-I and

GN-III models with respect to the phase velocity of thermal wave. However, in the case when we analyze the behavior of penetration depth, we find that predictions by GN-I and GN-III theory is more realistic as compared to GN-II model as we obtain in this case an infinite penetration depth which is also physically unrealistic prediction by GN-II model.

(v) It is observed that when thermal conductivity rate, $k_1 < 1$, the plots of all wave fields in the context of GN-III theory show much resemblance with the plots of GN-I. This implies that the results of GN-III model of thermoelasticity are more prominently different as compared to GN-I model when the thermal conductivity rate is greater than 1.

6.1.2 Investigations under generalized magneto-thermoelasticity in context of theory of dual phase-lags

In Chapter 5 of the present thesis, we studied a problem of elastic half space with finite conductivity permeated by a primary uniform magnetic field employing dual phase-lag magneto-thermoelasticity theory. The boundary of the half space is subjected to a normal load and a thermal shock that originate magneto-thermoelastic waves inside the medium. We have employed magneto-thermoelastic dual phase-lag model-I (MTDPL-I) as well as magneto-thermoelastic dual phase-lag model-II (MTDPL-II). We have presented a thorough analysis of the effects of magnetic field on wave propagation and investigate the nature of distributions of temperature, displacement, stress and perturbed magnetic field in the media in the contexts of two models of magneto-thermoelasticity with dual phase-lags. Significant differences among the analytical results and numerical results predicted by MTDPL-I model and MTDPL-II model can be summarized in the following manner:

(i) In the case of MTDPL-I, we found that solution of each field consists of two parts: The first one is a wave part that is identified as modified elastic wave and the second part is not wave type, but of diffusive type. The non-dimensional speed of elastic wave is found to be finite and equal to 1, i.e. the dimensionless speed of elastic wave is not affected by any of the phase-lag parameters and it is also independent to magnetic field. But the attenuation coefficient of modified elastic wave is dependent on two phase-lag parameters. The solution of each field variable in case of MTDPL-II model consists of two coupled waves-modified elastic and modified thermal wave. The non-dimensional speed of both the waves are finite and dependent on the phase-lag parameters as well as on the magnetic field.

(ii) Furthermore, in the case of MTDPL-II, we observe that temperature, stress and perturbed magnetic fields have discontinuities with finite jumps at both the elastic and thermal wave fronts and displacement is observed to be continuous in nature at both the wavefronts in the context of MTDPL-II model. However, we obtain different results under MTDPL-I model. In this case,we observe that only stress and perturbed magnetic field show discontinuities having finite jumps at the elastic wave front but the temperature and displacement fields are free from any discontinuities.

(iii) We also observe significant differences in the numerical results predicted by two different models. It is noted that the non dimensional temperature achieves negative value for a region before approaching to zero value in MTDPL-II but it is always positive under MTDPL-I case. While observing the nature of stress, it is found that the minimum value of stress in MTDPL-I is greater than the minimum value of stress in MTDPL-II at higher time, although during initial time of interaction, the minimum value of stress in MTDPL-I is less than the minimum value of stress in MTDPL-II. Furthermore, the maximum numerical value of displacement in case of MTDPL-II model is greater than that in case of MTDPL-I model.

(iv) Magnetic field is not prominently effective in the distribution of temperature in the contexts of both the MTDPL-I and MTDPL-II models. However, the stress field and displacement field are affected by the presence of magnetic field under both the models.

6.1.3 Investigations under recently developed thermoelasticity theory: fractional order thermoelasticity

In chapters 2.1 and 2.2, we have applied heat conduction model in context of recently developed fractional order thermoelasticity theory. Firstly, in chapter 2.1, the fundamental solutions have been established in the Laplace transform domain for fractional order thermoelasticity. Then by employing a suitable reciprocal relation, we formulate boundary integral equations for a mixed boundary initial value problem. Further, we have illustrated our formulation in a better manner by considering a suitable example. This formulation will help to find out the numerical solution of a concrete problem under fractional order thermoelasticity by the BEM / BIEM method as it is capable to provide accurate results for several one and two-dimensional problems. In Chapter 2.1, the influence of fractional order parameter α

has been presented as we obtain the solution of the physical fields like temperature, displacement in terms of a function 'Mittag Leffler function' which interpolates between a purely exponential law and power-like behavior of phenomena governed by ordinary kinetic equations and their fractional counterparts.

In a similar way, Chapter 2.2 represents the influence of fractional order parameter α (which lies between 0 and 1) on the propagation of harmonic plane wave. We have attempted to compare the results predicted by fractional order thermoelasticity theory with respect to the classical coupled thermoelasticity theory as well as the generalized thermoelasticity theory. This chapter brings out some specific features of the proposed heat conduction law that involves fractional order derivative with fractional parameter, α. We summarize the following conclusions from Chapter 2.2:

(i) Two different modes of longitudinal waves have been obtained- one is elastic and other one is thermal in nature.

(ii) It is observed that the significant change occurs on the behavior of two modes of longitudinal waves (elastic and thermal mode wave) when α goes beyond $1/2$.

(iii) We note that when α goes beyond $1/2$, we achieve better results in comparison to the classical theory of thermoelasticity.

(iv) The effect of fractional order parameter, α is more prominent on thermal mode wave.

(v) While comparing our results under fractional order heat conduction model with the classical heat conduction model, we achieve that the fractional order heat conduction model is the better one. It gives better result in comparison to

Fourier heat conduction model (classical theory of thermoelasticity), specially when fractional order parameter α goes beyond 0.5. This fact is believed to be significant for further research in this direction.

6.1.4 Investigation on thermoelasticity under heat conduction with memory dependent derivative

Chapter 3 represents investigations on thermoelasticity theory coupled with memory dependent derivative heat conduction law. One kernel function and one time delay parameter are present in the derivative of memory dependent type. We make an attempt to investigate a problem of wave propagation in a homogeneous, isotropic and unbounded solid due to a continuous line heat source and understand the influence of memory dependent heat conduction model on the thermoelastic interactions. Potential function approach together with the Laplace and Hankel transform technique has been employed to derive the solutions in the transformed domain. Hankel inversion is performed analytically, and we achieve analytical solutions of displacement, temperature and stresses in Laplace transform domain. These results are affected by kernel function and time delay parameter.The analytical results are illustrated with numerical computation and graphical plots of distribution of the field variables for copper material. The trends of variation of the field variables are compared for different kernel functions and different delay time parameter. The significant facts concerning the effects of kernel function as well as time delay parameter can be summarized in the following way:

(i) Each solution consists of the combination of two coupled waves, namely elastic mode wave and thermal mode wave.

(ii) Distributions of temperature, radial stress and circumferential stress contains local maximum and local minimum values. However, displacement is free from any local minimum.

(iii) Effect of kernel function can be observed everywhere among the profiles of physical fields. This effect is more prominent in the profiles of displacement. Effect of kernel is more prominent at the vicinity of extreme points in each case.

(iv) Radial stress starts from a high negative value in comparison to circumferential stress in each case.

(v) Time delay parameter, ω also plays a significant role in the behavior of all physical fields and it is more prominent near the vicinity of extreme points. While observing the influence of time delay, ω in the variation of physical fields, we always obtain local maximum attaining its highest absolute value for the case of linear kernel function.

(vi) Region of influence for each physical field is observed to be finite and increases as the time increases.

(vii) We can conclude from the results that present theory of thermoelasicity with memory dependent time derivatives supports for the finite speed of waves propagating through the medium for thermoelastic interaction. Hence, this theory supports second sound effect and may be considered as a generalized theory of thermoelasticity.

6.2 Future scope

In recent years, increasing attention is being devoted to understand the thermoelastic responses in case of rapid heating of metallic films through short-pulse laser techniques which is required for advance applications in modern micro-fabrication technologies. Phase-lag models have been supported by the successful correlation to many microscopic models as well as by a rigorous derivation in the framework of non-equilibrium and irreversible thermodynamics. There is a scope for more investigation on thermoelastic interactions inside complex structural materials under rapid heating by employing such models. The interest in the coupled field of magneto-thermoelasticity are increasing among the scientists and researchers due to its application in such practical situations as physics, geophysics, acoustics, damping of acoustic waves in magnetic fields, optics etc. There is an ample scope of further research to address magneto-themo-elastic interactions for various situations involving rapid heating.

Nowadays, fractional calculus has been playing a very crucial role among researchers as fractional calculus is the generalization of the ordinary differential and integration to non-integer order. Theory of thermoelasticity with fractional order time derivatives is a new branch of research. It appears that the fractional operators in describing the transition between distinct differential equations and different models are attracting the attention of scientists and researchers in several fields, like economics, mathematical physics or engineering. Although, a limited work has been reported in this area, but they are few in number to consider the coupled system of equations in the study of thermoelastic deformations in solid mechanics. Moreover, non-standard formulations are essential for a constant progress in the development of new materials. The fractional order strain problem may also be applicable in the fields of biomechanics, biomedical problems and skin tissues where knowledge of such changes would enable early diagnostic monitoring for the onset of disease and better assessment of the effectiveness of new drugs or therapies. Therefore, it is benifitial to persue the research in this area. The concept of memory dependent derivative is very new for researchers. It has come in the world before 5 years. Hence, it is obvious that this concept has a vast scope for future research.

It is believed that further research will be essential to study various models of thermoelasticity for the purpose of arriving at the mathematical consistency in the models as well as at the realistic prediction of thermoelastic responses while employing advanced technologies.