Chapter-4

Study of electro-magneto thermoelastic wave propagation

[1]4.1 On electro-magneto-thermoelastic plane waves under Green- Naghdi theory of thermoelasticity-II

4.1.1 Introduction

The present section attempts to investigate the propagation of electro-magneto-thermoelastic plane waves in an unbounded isotropic thermally and electrically conducting media with finite conductivity in the context of type-II theory of thermoelasticity by Green and Naghdi (1992). During 1991-1995, Green and Naghdi have introduced their theory as an alternative way. The propagation of heat has been modeled in a very elegant way to produce a fully consistent theory of thermoelasticity. The theory proposed by Green and Naghdi (1991, 1992, 1993, 1995) has been categorized into three parts which have been labeled as I, II and III. Type I is similar to the classical theory of thermoelasticity having infinite speed of wave propagation. Type II describes the finite speed of wave as a special case of type III i.e. in the heat equation of type III, the heat flux is the combination of type I and II. In addition, type II theory predicts the transmission of heat as thermal waves at finite speeds.

It must be mentioned here that propagation of electromagnetic waves in thermoelastic materials is a very interesting topic for researchers due to its several applications in various fields of science and technology namely thermal power plants, atomic physics, industrial engineering, submarine structures, aerospace etc. Paria (1962)

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and Wilson (1963) have used the heat conduction equation predicted by Fourier law along with the electromagnetic theory to explain harmonically time dependent plane waves of assigned frequency in a homogeneous, isotropic and unbounded solid. Agarwal (1979), Neyfeh and Nemat-Nessar (1972) have investigated electromagnetic plane waves in solids in context of generalized thermoelasticity theory. Roychoudhuri (1984) and Roychoudhuri and Debnath (1983) have presented propagation of magneto-thermoelastic plane waves in rotating thermoelastic media permeated by a primary uniform magnetic field using generalized thermoelasticity of Lord and Shulman (1967). Chandrasekharaiah (1996) has studied propagation of harmonic plane wave in an unbounded medium by applying GN-II theory of thermoelasticity. Roychoudhuri and Banerjee (Chattopadhyay) (2005) have reported magneto-elastic plane waves in rotating media under thermoelasticity of type-II model. Das and Kanoria (2009) also studied magneto-thermo-elastic waves in a medium with perfect conductivity in the context of Green and Naghdi-III theory. Kothari and Mukhopadhyay explained the propagation of thermoelastic plane waves by employing Green and Naghdi-III theory of thermoelasticity. Prasad and Mukhopadhyay (2012) have reported a study of propagation of plane waves in rotating elastic medium under two temperature thermoelasticity with relaxation time parameter.

Green-Naghdi type II theory accounts for a finite speed for thermal signal. However, there is no internal energy dissipation in this model. Hence, it can be identified as an interesting model to investigate the effects of the interactions of thermal and magnetic fields with mechanical strain field. Due to several applications of plane waves in various fields, we have attempted to present a detailed study of electro-magneto-thermoelastic plane waves in the presence of magnetic field in the context of Green and Naghdi type-II theory of thermoelasticity. Specially, we consider that the medium is of finite electrical conductivity and accounts for the Thomson effect. Basic governing equations are modified by employing Green-Naghdi theory of type-II. Problem has been solved by dividing it into two sub systems to extract both longitudinal and transverse waves propagating through the medium. The first system that corresponds to the longitudinal wave is found to be coupled with the thermal field. However, the second system represents transverse wave that is uncoupled with the thermal field. Both waves are observed to be affected with the magnetic field. The nature of all identified waves are investigated in a detailed way by deriving analytical solution for dispersion relations. Asymptotic expansions of dispersion relation solutions and various components of plane waves like, phase velocity, specific loss and penetration depth are derived for high and low frequency values in all cases. In order to observe the nature of waves in a more clear way and illustrate the analytical results, we further carry out numerical solutions of the problem.The limiting behavior of longitudinal and transverse waves are investigated. Several points highlighting the effects of magnetic field on the behavior of different waves propagating through the medium have been presented. The results of present study are compared with the results of thermoelastic case and a detailed analysis on the effects of presence of the magnetic field under this theory has been presented. The present study is believed to enhance the understanding of thermoelasticity without energy dissipation theory for magneto-thermoelastic problems.

4.1.2 Formulation of the problem and governing equations

We consider a problem of an infinite, homogeneous, isotropic, thermally and electrically conducting solid. Assuming a fixed rectangular Cartesian coordinate system x_i, $i = 1,2,3$; we describe our problem with the following governing equations:

Equations of motion:

$$T_{ij,j} + (\overrightarrow{J} \times \overrightarrow{B})_i + \rho_e E_i = \rho \ddot{u}_i \tag{4.1.1}$$

such that i,j = 1,2,3

where $T_{ij} = \lambda e \delta_{ij} + \mu(u_{i,j} + u_{j,i}) - \gamma\theta\delta_{ij}$ defines the stress-strain and temperature relation.

T_{ij} are the Cartesian components of the linear stress tensor. ρ_e and ρ are the charge density and constant mass density, respectively. Comma followed by subscript implies partial derivative with respect to the corresponding coordinate and superposed dot denotes the partial time derivatives. The term $(\overrightarrow{J} \times \overrightarrow{B})_i$ in equation (4.1.1) occurs due to the Lorentz force which arises due to the interaction between electric and magnetic fields; where $\overrightarrow{J}$ is the electric charge density vector and $\overrightarrow{B}$ is the magnetic flux density vector. E_i is component of electric flux density vector.

Equation of heat conduction:

Since we are employing Green and Naghdi type-II theory of thermoelasticity in our problem, the heat conduction equation can be writen as

$$\rho C_v \ddot{\theta} + \gamma \theta_0 \ddot{e} = K^\star \nabla^2 \theta - \pi_0^\star J_{i,i} \tag{4.1.2}$$

where the terms used in the above equations are defined as below:

C_v is the specific heat at constant strain, θ_0 is the reference temperature.$\pi_0^\star$ is the new material parameter characteristics of GN-II theory for magneto-thermoelastic problem.$\pi_0^\star$ may be termed as rate of Peltier coefficient, where Peltier coefficient is the material parameter connecting the charge density with the heat flow density.

Since we have taken the media affected with magnetic field, hence equations (4.1.1) and (4.1.2) have to be supplemented by Maxwell equations of electro-magneto-thermoelasticity which can be stated as follows:

$$\vec{\nabla} \times \vec{H} = \vec{J} + \dot{\vec{D}} \tag{4.1.3}$$

$$\vec{\nabla} \times \vec{E} = -\dot{\vec{B}} \tag{4.1.4}$$

$$\vec{\nabla}.\vec{B} = 0 \tag{4.1.5}$$

$$\vec{\nabla}.\vec{D} = \rho_e \tag{4.1.6}$$

$$\vec{B} = \mu_e \vec{H} \tag{4.1.7}$$

$$\vec{D} = \epsilon \vec{E} \tag{4.1.8}$$

Further, the modified Ohm's law is given by

$$\vec{J} = \sigma[\vec{E} + \dot{\vec{u}} \times \vec{B}] + \rho_e \dot{\vec{u}} - k_0 \vec{\nabla}\theta \tag{4.1.9}$$

where $\vec{H}$ is the magnetic intensity vector, $\vec{D}$ is the electric flux vector, μ_e, ϵ are magnetic permeability and electric permittivity, respectively. σ is the electric conductivity. $\vec{u}$ is the displacement vector and k_0 is the coefficient which connects the electric current and the temperature gradient which is known as Seebeck coefficient.

The combined effect of Peltier coefficient and Seebeck coefficient is recognized as

Thomson effect. This is notable point of our work that we have considered Thomson effect of interaction through Peltier and Seebeck coefficients in the governing equations. In most of the work on magneto-thermoelasticity, this effect is ignored due to simplicity.

Now, if we attempt to eliminate $\rho_e, \overrightarrow{J}, \overrightarrow{B}, \overrightarrow{D}$ from the equations (4.1.3)-(4.1.9) and equations (4.1.1), (4.1.2), we obtain a system of non linear equations. We can linearize them by setting $\overrightarrow{H} = \overrightarrow{H_0} + \overrightarrow{h}$, where $\overrightarrow{h}$ represents the alteration in the basic magnetic field $\overrightarrow{H_0}$. By assuming this, we achieve the following basic equations:

$$(\overrightarrow{\nabla} \times \overrightarrow{h}) = \sigma[\overrightarrow{E} + \mu_e(\dot{\overrightarrow{u}} \times \overrightarrow{H_0})] - k_0 \overrightarrow{\nabla}\theta + \epsilon \dot{\overrightarrow{E}} \tag{4.1.10}$$

$$(\overrightarrow{\nabla} \times \overrightarrow{E}) = -\mu_e \dot{\overrightarrow{h}} \tag{4.1.11}$$

$$\rho \ddot{\overrightarrow{u}} = \overrightarrow{\nabla}.\overrightarrow{T} + \mu_e\sigma(\overrightarrow{E} \times \overrightarrow{H_0}) + \mu_e^2\sigma(\dot{\overrightarrow{u}} \times \overrightarrow{H_0}) \times \overrightarrow{H_0} - \mu_e k_0(\overrightarrow{\nabla}\theta \times \overrightarrow{H_0}) \tag{4.1.12}$$

$$\rho C_v \ddot{\theta} + \gamma\theta_0 \ddot{e} = K^\star \nabla^2\theta + \pi_0^\star \epsilon(\overrightarrow{\nabla}.\dot{E}) \tag{4.1.13}$$

Here, we have neglected the products of $\overrightarrow{h}, \overrightarrow{u}, \overrightarrow{E}, \theta$and their derivatives for considering the problem as linear.

After eliminating $\overrightarrow{h}$ from equations (4.1.10), (4.1.11), we obtain the following relation:

$$\nabla^2\overrightarrow{E} - \overrightarrow{\nabla}(\overrightarrow{\nabla}.\overrightarrow{E}) = \mu_e\sigma[\dot{\overrightarrow{E}} + \mu_e(\ddot{\overrightarrow{u}} \times \overrightarrow{H_0})] - \mu_e k_0 \overrightarrow{\nabla}\dot{\theta} + \mu_e\epsilon \ddot{\overrightarrow{E}} \tag{4.1.14}$$

Above equations (4.1.12)-(4.1.14) constitute a system of linearized equations which are related with the displacement, thermal and electric fields.

4.1.3 Non-dimensionalization of basic governing equations

While doing our analysis, in order to simplify and parametrize our problem, we introduce the following non dimensional quantities:

$K^\star = \rho C_v c_0^2;\ c_0^2 = \frac{\lambda+2\mu}{\rho};\ a = \frac{\gamma\theta_0}{\mu};\ g = \frac{\gamma}{\rho C_v};\ \theta = \theta_0\theta';$

$s' = c_0^2\epsilon\mu_e;\quad \alpha = \frac{\lambda+2\mu}{\mu};\ \epsilon_\theta = \frac{ag}{\alpha^2};\ \epsilon_e = \frac{\mu_e H^2}{\rho c_0^2};\ t = \frac{t'}{\omega^\star};\ u = \frac{u' c_0}{\omega^\star};$

$x = \frac{c_0 x'}{w^\star};\ \overrightarrow{E} = \frac{H\mu_e c_0}{g}\ \nu = \frac{1}{\sigma\mu_e};\ \nu' = \frac{\nu\omega^\star}{c_0^2};\ k' = \frac{g k_0 \theta_0}{H};\ \pi^\star = \frac{\pi_0^\star \epsilon H \mu_e}{g\rho C_v \theta_0}$

$\overrightarrow{H_0} = H\overrightarrow{n}$ such that $n = (n_1, n_2, n_3)$ denotes a unit vector in the direction of $\overrightarrow{H_0}$ and H is the magnitude of $\overrightarrow{H_0}$. ν' denotes the measure of magnetic viscosity. ϵ_e and ϵ_θ are the magneto-thermo-elastic and thermoelastic coupling coefficients and $\omega^\star$ is the characteristic frequency of the medium.

Now, introducing above non dimensional quantities in the equations (4.1.12)-(4.1.14), we reach the following dimensionless field equations, respectively

$$\nu'\alpha^2\ddot{u}_i = \nu'(\alpha^2-1)e_{,i}+\nu'\nabla^2 u_i-\nu'\alpha^2\epsilon_\theta\theta_{,i}+\alpha^2\epsilon_e\{(\overrightarrow{E}\times\overrightarrow{n})+(\dot{\overrightarrow{u}}\times\overrightarrow{n})\times\overrightarrow{n}-\nu'k'(\overrightarrow{\nabla}\theta\times\overrightarrow{n})\}_i \tag{4.1.15}$$

$$\ddot{\theta} - \theta_{,ii} + \ddot{e} = \pi^\star(\overrightarrow{\nabla}.\dot{\overrightarrow{E}}) \tag{4.1.16}$$

$$\nu' s'\ddot{\overrightarrow{E}} + \dot{\overrightarrow{E}} + (\ddot{\overrightarrow{u}} \times \overrightarrow{n}) - \nu' k'\overrightarrow{\nabla}\dot{\theta} = \nu'\nabla^2\overrightarrow{E} - \nu'\overrightarrow{\nabla}(\overrightarrow{\nabla}.\overrightarrow{E}) \tag{4.1.17}$$

Primes have been removed from dimensionless quantities u', t', x', θ' for the shake of clarity. We further use above equations (4.1.15)-(4.1.17) for solving our problem.

4.1.4 Dispersion relation and plane wave solutions

As we have taken an isotropic solid, so without any loss in generality, we may assume that waves are propagating in the $x_1$direction only. There are three components of displacement and electric field of wave namely (u_1, u_2,u_3) and (E_1, E_2,E_3), respectively and they are acting in the directions $x_{1,}x_2$,x_3, respectively. Further, we assume that the magnetic field is functioning in the x_1x_3 plane i.e. $\overrightarrow{n}$ =(n_1, 0,n_3). The choice of magnetic field is considered in such a manner that it does not effect the significant characteristics of our problem. We assume that all the field variables are the functions of $x = (x_1)$ and time t.

Applying all assumptions stated above in the equations (4.1.15)-(4.1.17), we obtain the following seven equations:

$$\nu'\ddot{u}_1 = \nu' u_1'' - \nu'\epsilon_\theta\theta' + \epsilon_e n_3(E_2 + n_1\dot{u}_3 - n_3\dot{u}_1) \tag{4.1.18}$$

$$\nu'\alpha^2\ddot{u}_2 = \nu' u_2'' + \epsilon_e\alpha^2(n_1E_3 - n_3E_1 - \dot{u}_2 - \nu'k'n_3\theta') \tag{4.1.19}$$

$$\nu'\alpha^2\ddot{u}_3 = \nu' u_3'' - \epsilon_e\alpha^2 n_1(E_2 + n_1\dot{u}_3 - n_3\dot{u}_1) \tag{4.1.20}$$

$$\ddot{\theta} + \ddot{u}' - \theta'' = \pi^\star\dot{E}_1' \tag{4.1.21}$$

$$\nu's'\ddot{E}_1 + \dot{E}_1 + n_3\ddot{u}_2 - \nu'k'\dot{\theta}' = 0 \tag{4.1.22}$$

$$\nu's'\ddot{E}_2 + \dot{E}_2 + n_1\ddot{u}_3 - n_3\ddot{u}_1 = \nu' E_2'' \tag{4.1.23}$$

$$\nu's'\ddot{E}_3 + \dot{E}_3 - n_1\ddot{u}_2 = \nu' E_3'' \tag{4.1.24}$$

In above equations, the primes represent differentiation with respect to the x_1-

coordinate.

Further, we implement some conditions in above equations as given below.

We assume that magnetic field is directed only towards x_1- direction i.e. $n_3 = 0$ and we have taken $n_1 = 1$. With the help of this assumptions, we achieve the following set of equations:

$$\ddot{u}_1 = u_1'' - \epsilon_\theta \theta' \tag{4.1.25}$$

$$\nu' \alpha^2 \ddot{u}_2 = \nu' u_2'' + \epsilon_e \alpha^2 (E_3 - \dot{u}_2) \tag{4.1.26}$$

$$\nu' \alpha^2 \ddot{u}_3 = \nu' u_3'' - \epsilon_e \alpha^2 (E_2 + \dot{u}_3) \tag{4.1.27}$$

$$\ddot{\theta} + \ddot{u}' - \theta'' = \pi^\star \dot{E}_1' \tag{4.1.28}$$

$$\nu' s' \ddot{E}_1 + \dot{E}_1 - \nu' k' \dot{\theta}' = 0 \tag{4.1.29}$$

$$\nu' s' \ddot{E}_2 + \dot{E}_2 + \ddot{u}_3 = \nu' E_2'' \tag{4.1.30}$$

$$\nu' s' \ddot{E}_3 + \dot{E}_3 - \ddot{u}_2 = \nu' E_3'' \tag{4.1.31}$$

Now, subtracting equation (4.1.30) from equation (4.1.31) and adding equations (4.1.26) and (4.1.27), we obtain

$$\nu' s' (\ddot{E}_2 - \ddot{E}_3) + (\dot{E}_2 - \dot{E}_3) + \ddot{u}_3 + \ddot{u}_2 = \nu' (E_2'' - E_3'') \tag{4.1.32}$$

$$\nu'\alpha^2(\ddot{u}_2 + \ddot{u}_3) = \nu'(u''_2 + u''_3) + \epsilon_e\alpha^2((E_3 - E_2) - (\dot{u}_2 + \dot{u}_3)) \tag{4.1.33}$$

Let us now assume $E_2 - E_3 = M$ and $u_2 + u_3 = N$. Hence, above equations imply

$$\nu' s'\ddot{M} + \dot{M} - \nu' M'' + \ddot{N} = 0 \tag{4.1.34}$$

$$\nu'\alpha^2\ddot{N} - \nu' N'' + \epsilon_e\alpha^2(M + \dot{N}) = 0 \tag{4.1.35}$$

In what follows, we are going to deal with the five equations, namely equations (4.1.25), (4.1.28), (4.1.29) and equations (4.1.34) and (4.1.35).

As we observe that equations (4.1.34) and (4.1.35) are independent with the thermal field and these equations are coupled by two fields mechanical (elastic) field and electrical field. Hence, the solutions of these two equations will constitute two kinds of waves elastic (mode) wave and electrical mode wave. However, equations (4.1.25), (4.1.28) and (4.1.29) are undoubtedly affected with the thermal field. Due to this reason, we subdivide these five equations in two separate cases:

4.1.4.1 Case-I

This subsection contains the case in which system of equations are coupled with the temperature field. The equations for this are:

$$\ddot{u}_1 = u''_1 - \epsilon_\theta\theta' \tag{4.1.36}$$

$$\ddot{\theta} + \ddot{u}' - \theta'' = \pi^\star\dot{E}'_1 \tag{4.1.37}$$

$$\nu' s'\ddot{E}_1 + \dot{E}_1 - \nu' k'\dot{\theta}' = 0 \tag{4.1.38}$$

This system preserves the longitudinal waves in nature. Here, the equations are affected with the x_1-component of electric field i.e. E_1.

Solutions:

For finding the solutions of above equations, we take

$$u = a_1 e^{i(-qx+\omega t)}, \quad \theta = a_2 e^{i(-qx+\omega t)}, \quad E_1 = a_3 e^{i(-qx+\omega t)} \tag{4.1.39}$$

where q is the wave number and ω is the angular frequency of plane waves. Here, we have assumed that ω is real and q may be complex quantity, where $Im(q) \leq 0$ must hold for waves to be physically realistic.

Applying the quantities given by (4.1.39) in the equations given by (4.1.36)-(4.1.38) and solving them for a_1, a_2, a_3 we obtain the following dispersion relation for case-I:

$$z^4[1+\nu'^2\omega^2(s'+\pi k')^2] - z^2[P-iQ] + (1+\nu'^2 s'\omega^2(s'+\pi^\star k')) - i\nu'\omega\pi k' = 0 \tag{4.1.40}$$

where $z = \frac{q}{\omega}$ and $P = \nu'^2\omega^2[\pi^\star k' s' + 2s'^2 + s'^2\epsilon_\theta {\pi^\star}^2 k'^2 + 2s'k'\pi^\star + s^\star k'\pi^\star\epsilon_\theta] + (2 + \epsilon_\theta), \quad Q = \nu'\omega[k'\pi^\star(1+\epsilon_\theta)]$

In order to obtain solution of equation (4.1.40), we can write it in the following form:

$$[1+\nu'^2\omega^2(s'+\pi k')^2]z^2 = P - iQ \pm \sqrt{X+iY} \tag{4.1.41}$$

where $X = \sqrt{P^2 - Q^2 - 4\{(1+\nu'^2\omega^2(s'+\pi^\star k')^2)(1+\nu'^2 s'\omega^2(s'+\pi^\star k'))\}}$

$Y = -2PQ + 4\nu'\omega\pi^\star k'(1+\nu'^2\omega^2(s'+\pi^\star k')^2)$

P and Q are already defined above.

Asymptotic expressions for wave number:

Solution of equation (4.1.41) yields the solution for q. Out of four possible solutions for q, only two solutions ($q_{1,2}$) correspond to $Im(q) \leq 0$. These two solutions represent two different kinds of waves - modified (quasi) magneto-thermal wave and modified (quasi) magneto-elastic wave. Both the waves are longitudinal in nature. Since equations (4.1.36), (4.1.37) and (4.1.38) are coupled with the elastic field, magnetic field as well as temperature field and we are obtaining dispersion relation (equation (4.1.41)) from these three equations. So the waves obtained from equation (4.1.41) are not purely elastic and not purely thermal in nature; they are affected with the magnetic field parameters also. Due to this reason, waves are named as modified (quasi) magneto-thermal wave and modified (quasi) magneto-elastic wave. Now, by using theorem of complex analysis and after a long calculation, we will derive the following approximated solutions for the cases of very high and low frequency values.

High Frequency asymptotes: Assuming ω to be very large and solving equation (4.1.41), we obtain the high frequency asymptotic expressions for q as

$$q_1 = \frac{1}{2\nu'(s' + \pi^\star k')}[\sqrt{A_1 + \sqrt{(A_1^2 - 4\nu'^4 s'(s' + \pi^\star k')^2)}}\omega[1+\{\frac{A_7}{4} - \frac{1}{2(\nu'(s' + \pi^\star k'))^2}\}\frac{1}{\omega^2}] - i$$

$$\sqrt{\frac{A_1 + \sqrt{(A_1^2 - 4\nu'^4 s'(s' + \pi^\star k')^2)}A_8}{2}}[1 + \{\frac{B_8}{2A_8} - \frac{1}{2(\nu'(s' + \pi^\star k'))^2}\}\frac{1}{\omega^2}]], \quad \omega \to \infty \tag{4.1.42}$$

$$q_2 = \frac{1}{\nu'(s'+\pi^\star k')}[\sqrt{a_1}\omega[1+\{\frac{4a_1^2a_2+b_1^2}{8a_1^2} - \frac{1}{2(\nu'(s'+\pi^\star k'))^2}\}\frac{1}{\omega^2}] - i\sqrt{\frac{a_1c_1}{2}}$$

$$[1+\{\frac{c_2}{2c_1} - \frac{1}{2(\nu'(s'+\pi^\star k'))^2}\}\frac{1}{\omega^2}]], \quad \omega \to \infty \tag{4.1.43}$$

Low frequency asymptotes:

Similarly, assuming ω to be very small and solving equation (4.1.41), we obtain the low frequency asymptotic expressions for q as

$$q_1 = \omega\sqrt{\frac{a}{2}}[1+(\frac{m_7}{2}+\frac{b^2}{8a^2} - \frac{(\nu'(s'+\pi^\star k'))^2}{2})\omega^2] - i\omega\sqrt{\frac{b}{a}}$$

$$[1-\{\frac{a^2m_8}{2b^2}+\frac{(\nu'(s'+\pi^\star k'))^2}{2}\}\omega^2\}], \quad \omega \to 0 \tag{4.1.44}$$

$$q_2 = \omega\sqrt{\frac{a'}{2}}[1-\{\frac{2P_2a'^2}{b'^2}+\frac{(\nu'(s'+\pi^\star k'))^2}{2}\}\omega^2] - i\omega^2\sqrt{\frac{b'}{2a'}}$$

$$[1+\{\frac{2P_2a'^2}{b'^2}+\frac{(\nu'(s'+\pi^\star k'))^2}{2}\}\omega^2], \quad \omega \to 0 \tag{4.1.45}$$

where various notations used are defined as

$A_1 = \nu'^2[3\pi^\star k's' + s'\epsilon_\theta(s'+\pi^\star k') + 2s'^2 + {\pi^\star}^2 k'^2]$;

$A_2 = [2A_1^2 - 8\nu'^4 s'(s'+\pi^\star k')^2)(A_1(4+\epsilon_\theta) - B_1^2 - 4\nu'^2(2s'^2 + {\pi^\star}^2 k'^2 + 3s'\pi^\star k')] + (4\nu'^3\pi^\star k'(s'+\pi^\star k')^2 - 2A_1B_1)^2$;

$A_3 = \frac{A_2+2\{A_1(4+\epsilon_\theta)-B_1^2-4\nu'^2(s'(s'+\pi^\star k')+(s'+\pi^\star k')^2)\}}{4(A_1^2-4\nu'^4s'(s'+\pi^\star k')^2)}$

$A_4 = \frac{A_2-2[A_1(4+\epsilon_\theta)-B_1^2-4\nu'^2(s'(s'+\pi^\star k')+(s'+\pi^\star k')^2)]}{2\sqrt{2}(A_1^2-4\nu'^4s'(s'+\pi^\star k')^2)}$;

$A_5 = 1 + \frac{A_1}{\sqrt{(A_1^2-4\nu'^4 s'(s'+\pi^\star k')^2)}}$; $A_6 = 1 + \frac{B_1}{(\sqrt{2(A_1^2-4\nu'^4 s'(s'+\pi^\star k')^2)})}$; $A_7 = \frac{B_5}{A_5} + \frac{A_6^2 A_4 + 2A_5 B_5}{2A_5^2}$;

$A_8 = -\frac{B_5}{A_5} + (\frac{A_6^2 A_4 + 2A_5 B_5}{2A_5^2})^2$; $B_1 = \nu' \pi^\star k'(1+\epsilon_\theta)$;

$B_2 = [A_1(4+\epsilon_\theta) - B_1^2 - 4\nu'^2 s'^2]^2 + 2(4\nu'^3 \pi^\star k'(s'+\pi^\star k')^2 - 2A_1 B_1)(4\pi^\star \nu' k' - B_1(4+2\epsilon_\theta))$;

$B_3 = \frac{4(\epsilon_\theta^2+4\epsilon_\theta)(A_1^2-4\nu'^4 s'(s'+\pi^\star k')^2) - A_2^2 + 2B_2^2}{4(A_1^2-4\nu'^4 s'(s'+\pi^\star k')^2)^2}$, $B_4 = \frac{-4(\epsilon_\theta^2+4\epsilon_\theta)(A_1^2-4\nu'^4 s'(s'+\pi^\star k')^2) - A_2^2 + 2B_2^2}{2\sqrt{2}(A_1^2-4\nu'^4 s'(s'+\pi^\star k')^2)}$;

$B_5 = \frac{1}{2}[A_3 + \frac{2+\epsilon_\theta}{\sqrt{(A_1^2-4\nu'^4 s'(s'+\pi^\star k')^2)}}]$, $B_6 = \frac{B_4}{A_4}$;

$B_8 = \frac{-1}{4}[\frac{A_6^2 A_4 + 2A_5 B_5}{A_5^2}]^2 + \frac{1}{2}[\frac{2A_4 A_6 B_6 + B_5^2 + 2A_5 B_5}{A_5^2}]$, $a_1 = 4\sqrt{2}[A_1 - \sqrt{(A_1^2 - 4\nu'^4 s'(s'+\pi^\star k')^2)}]$;

$a_2 = \frac{(4+2\epsilon_\theta) - A_3\sqrt{(A_1^2-4\nu'^4 s'(s'+\pi^\star k')^2)}}{2}$, $b_1 = \nu'\pi^\star k'(1+\epsilon_\theta) - \sqrt{\{A_4(A_1^2 - 4\nu'^4 s'(s'+\pi^\star k')^2)\}}$;

$b_2 = \frac{B_4\sqrt{\{A_4(A_1^2-4\nu'^4 s'(s'+\pi^\star k')^2)\}}}{A_4}$, $c_1 = \frac{1}{2}[\frac{-a_2}{a_1} + \frac{2a_1 a_2 + b_1^2}{2a_1^2}]$;

$c_2 = \frac{1}{2}[-a_3 + \frac{1}{2}(\frac{2b_1 b_2 + a_2^2 + 2a_1 a_3}{a_1^2}) - \frac{2a_1 a_2 + b_1^2}{4a_1^2},]$, $m_1 = 2(\epsilon_\theta^2 + 4\epsilon_\theta)[A_1(4+\epsilon_\theta) - B_1^2 - 4\nu'^2 s'(s'+\pi^\star k') - 4\nu'(s'+\pi^\star k')^2]$;

$m_2 = [(A_1(4+\epsilon_\theta) - B_1^2 - 4\nu'^2 s'(s'+\pi^\star k') - 4\nu'(s'+\pi^\star k')^2)^2 + 2(\epsilon_\theta^2 + 4\epsilon_\theta)(A_1^2 - 4\nu'^4 s'(s'+\pi^\star k')^2) + 2(4\nu'^3\pi^\star k'(s'+\pi^\star k')^2 - 2A_1 B_1)(4\nu'\pi^\star k' - B_1(4+2\epsilon_\theta))]$;

$m_3 = \frac{1}{2}[\frac{m_1}{2(\epsilon_\theta^2+4\epsilon_\theta)^2} + A_1(4+\epsilon_\theta) - B_1^2 - 4\nu'^2 s'(s'+\pi^\star k') - 4\nu'(s'+\pi^\star k')^2]$;

$m_4 = \frac{1}{2}[A_1^2 - 4\nu'^4 s'(s'+\pi^\star k')^2 + \frac{m_2}{2} - \frac{m_1^2}{4(\epsilon_\theta^2+4\epsilon_\theta)^2}]$, $m_5 = \frac{m_1}{2(\epsilon_\theta^2+4\epsilon_\theta)} - A_1(4+\epsilon_\theta) + B_1^2 + 4\nu'^2 s'(s'+\pi^\star k') + 4\nu'(s'+\pi^\star k')^{2;}$

$m_7 = \frac{1}{(2+\epsilon_\theta)+(\epsilon_\theta^2+4\epsilon_\theta)^2}[A_1 + m_3\frac{\sqrt{\epsilon_\theta^2+4\epsilon_\theta}}{2}]$, $m_8 = \frac{1}{(2+\epsilon_\theta)+(\epsilon_\theta^2+4\epsilon_\theta)^2}[\frac{-m_3^2}{4} + \frac{m_4}{2}]$;

$a' = (2+\epsilon_\theta) + (\epsilon_\theta^2 + 4\epsilon_\theta)^{1/2}$, $b = m_5\sqrt{\frac{\epsilon_\theta^2+4\epsilon_\theta}{2}} + B_1$, $a = (2+\epsilon_\theta) - (\epsilon_\theta^2 + 4\epsilon_\theta)^{1/2}$;

$b' = B_1 - m_5\sqrt{\frac{\epsilon_\theta^2+4\epsilon_\theta}{2}}$, $P_2 = \frac{\sqrt{\epsilon_\theta^2+4\epsilon_\theta}(\frac{-m_3^2}{4}+\frac{m_4}{2})}{(2+\epsilon_\theta)-(\epsilon_\theta^2+4\epsilon_\theta)^{1/2}}$

From the above analytical results, we can now predict the nature of waves by determining the physical components of waves like, phase velocity, specific loss and penetration depth with the help of the following formulae:

Phase velocity $= \frac{\omega}{Re[q]}$

Specific loss $= 4\pi|\frac{Im[q]}{Re[q]}|$

Penetration depth $= \frac{1}{|Im[q]|}$

Out of the two waves represented by $q_{1,2}$ we assume that the first wave corresponding to q_1 represents quasi-magneto-thermal wave and the second wave represented by denotes quasi-magneto-elastic wave.

In the next section, using above formulae, all wave components and their limiting values will be derived for both types of waves (quasi-magneto-thermal wave and quasi-magneto-elastic wave) that we have identified in this case.

Quasi-magneto-thermal wave

(A) High frequency asymptotic expressions for components of quasi-magneto-thermal wave

(a) Phase velocity

For the present case, Phase velocity can be achieved by the following formula:

Phase velocity $=V_t = \frac{\omega}{Re[q_1]}$

From the solution given by (4.1.42), we obtain the asymptotic solution for phase velocity of quasi-magneto-thermal wave for very high frequency values as

Phase velocity $= V_t \sim \frac{2\nu'(s'+\pi^\star k')}{[\sqrt{A_1+\sqrt{(A_1^2-4\nu'^4 s'(s'+\pi^\star k')^2)}}}[1-\{\frac{A_7}{4}-\frac{1}{2(\nu'(s'+\pi^\star k'))^2}\}\frac{1}{\omega^2}]$

Clearly, V_t tends to constant value $\frac{2\nu'(s'+\pi^\star k')}{\sqrt{A_1+\sqrt{(A_1^2-4\nu'^4 s'(s'+\pi^\star k')^2)}}}$ as $\omega \to \infty$

(b) Specific loss

Specific loss can be achieved by the following formula:

Specific loss $= S_t = 4\pi|\frac{Im[q_1]}{Re[q_1]}|$

From equation (4.1.42), we find the solution for specific loss of quasi-magneto-thermal wave for high frequency values as follows:

$S_t \sim 4\pi\frac{\sqrt{A_8}}{\sqrt{2}}\frac{1}{\omega}[1+\{\frac{B_8}{2A_8}-\frac{A_7}{4}\}\frac{1}{\omega^2}]$

Therefore S_t tends to 0 as $\omega \to \infty$.

(c) Penetration depth

Penetration depth can be determined by the following formula:

Penetration depth $= D_t = \frac{1}{|Im[q_1]|}$

Equation (4.1.42) yields, the solution for penetration depth of quasi-magneto-thermal wave for high frequency values as follows:

$D_t \sim \frac{2\nu'(s'+\pi^\star k')}{\sqrt{(A_1+\sqrt{(A_1^2-4\nu'^4 s'(s'+\pi^\star k')^2)})A_8}}[1-\{\frac{B_8}{2A_8}-\frac{1}{2(\nu'(s'+\pi^\star k'))^2}\}\frac{1}{\omega^2}]$

Which implies that D_t tends to constant value $\frac{2\nu'(s'+\pi^\star k')}{\sqrt{(A_1+\sqrt{(A_1^2-4\nu'^4 s'(s'+\pi^\star k')^2)})A_8}}$ as $\omega \to \infty$.

(B) Low frequency asymptotic expressions for components of quasi-magneto-thermal wave

(a) Phase velocity

By using solution given by equation (4.1.44), we obtain the approximated solution of phase velocity of quasi-magneto-thermal wave for low frequency values as

Phase velocity $= V_t \sim \sqrt{\frac{2}{a}}[1-(\frac{m_7}{2}+\frac{b^2}{8a^2}-\frac{(\nu'(s'+\pi^\star k'))^2}{2})\omega^2]$

which implies that V_t tends to constant value equal to $\sqrt{\frac{2}{a}}$ as $\omega \to 0$.

(b) Specific loss $= S_t \sim \frac{\sqrt{2b}}{a}[1-\{\frac{a^2 m_8}{2b^2}+\frac{m_7}{2}+\frac{b^2}{8a^2}\}\omega^2]$

From above S_t tends to constant equal to $\frac{\sqrt{2b}}{a}$ as $\omega \to 0$.

(c) Penetration depth $= D_t \sim \frac{\sqrt{2}}{\omega\sqrt{\frac{b}{a}}}[1+\{\frac{a^2 m_8}{2b^2}+\frac{(\nu'(s'+\pi^\star k'))^2}{2}\}\omega^2\}]$

Clearly D_t becomes ∞ as $\omega \to 0$.

Quasi-magneto-elastic wave

In a similar way as above, asymptotic expressions of all the physical components of second wave, i.e., quasi-magneto-elastic wave are determined from solution for q_2 given by (4.1.43) and (4.1.45) in the following form:

(C) High frequency asymptotic expressions for components of quasi-magneto-elastic wave

(a) Phase velocity $= \frac{\omega}{Re[q_2]}$

Therefore, phase velocity of quasi-magneto-elastic wave $= V_e \sim \frac{\nu'(s'+\pi^\star k')}{\sqrt{a_1}}[1 - \{\frac{4a_1^2a_2+b_1^2}{8a_1^2} - \frac{1}{2(\nu'(s'+\pi^\star k'))^2}\}\frac{1}{\omega^2}]$

Here, we note that V_e tends to constant value $\frac{\nu'(s'+\pi^\star k')}{\sqrt{a_1}}$ as $\omega \to \infty$.

(b) Specific loss $= 4\pi|\frac{Im[q_2]}{Re[q_2]}|$

Using above formula, we achieve

Specific loss $= S_e \sim \sqrt{\frac{c_1}{2}}\frac{1}{\omega}[1 + \{\frac{c_2}{2c_1} - \frac{4a_1^2a_2+b_1^2}{8a_1^2}\}\frac{1}{\omega^2}]$

which shows that $S_e \to 0$ as $\omega \to \infty$.

(c) Penetration depth $= \frac{1}{|Im[q_2]|}$

With the help of formula given above we find

Penetration depth $= D_e \sim \frac{\sqrt{2}\nu'(s'+\pi^\star k')}{\sqrt{a_1c_1}}[1 - \{\frac{c_2}{2c_1} - \frac{1}{2(\nu'(s'+\pi^\star k'))^2}\}\frac{1}{\omega^2}]$

and clearly, $D_e \to \frac{\sqrt{2}\nu'(s'+\pi^\star k')}{\sqrt{a_1c_1}}$ as $\omega \to \infty$.

(D) Low frequency asymptotic expressions for components of quasi-magneto-elastic wave

(a) Phase velocity $= V_e \sim \sqrt{\frac{2}{a'}}[1 + \{\frac{2P_2a'^2}{b'^2} + \frac{(\nu'(s'+\pi^\star k'))^2}{2}\}\omega^2]$

which implies that $V_e \to$constant value equal to $\sqrt{\frac{2}{a'}}$ as $\omega \to 0$.

(b) Specific loss $= S_e \sim \frac{\omega\sqrt{b'}}{a'}[1 + \{\frac{4P_2a'^2}{b'^2} + (\nu'(s' + \pi^\star k'))^2\}\omega^2]$

which indicates that $S_e \to 0$ as $\omega \to 0$.

(c) Penetration depth $= D_e \sim \sqrt{\frac{2a'}{b'}\frac{1}{\omega^2}}[1-\{\frac{2P_2a'^2}{b'^2}+\frac{(\nu'(s'+\pi^\star k'))^2}{2}\}\omega^2]$

Therefore, we observe that $D_e \rightarrow \infty$as $\omega \rightarrow 0$.

4.1.4.2 Case-II

In the present case, as given by equations (4.1.34), (4.1.35) the system of equations is independent of thermal field θ. Equations are given as

$$\nu' s' \ddot{M} + \dot{M} - \nu' M'' + \ddot{N} = 0 \tag{4.1.46}$$

$$\nu' \alpha^2 \ddot{N} - \nu' N'' + \epsilon_e \alpha^2 (M + \dot{N}) = 0 \tag{4.1.47}$$

Dispersion relation solutions for case-II

In order to obtain the solutions of equations (4.1.46) and (4.1.47), we assume

$$M = a_4 e^{i(-qx+\omega t)}, \quad N = a_5 e^{i(-qx+\omega t)} \tag{4.1.48}$$

On solving equations (4.1.46) and (4.1.47) for a_4, a_5, we obtain the following dispersion relation for the present case:

$$\nu' q^4 - q^2[\nu'^2\omega^2(s'+\alpha^2) - i\omega(1+\alpha^2\epsilon_\theta)] + \alpha^2 s'\nu'^2\omega^4 - i\alpha^2\omega^3(1+s'\epsilon_e) = 0 \tag{4.1.49}$$

As we can clearly observe from equation (4.1.49), we can find the four solutions for the wave number q and out of these four possible solutions for q, we take only two solutions ($q_{3,4}$) that correspond to $Im(q) \leq 0$ for the wave to be physically realistic. Hence, in the present case, two different kinds of waves are generated. Since only two different fields: mechanical and electrical fields are effective here, these two solutions represent two different kinds of waves namely, quasi-electro magneto shear wave and quasi-magneto-elastic shear wave. Here we are achieving shear waves in nature. By solving the dispersion relation (4.1.49) and following the similar approach as in Case-I, we derive the high frequency and low frequency asymptotic solutions for wave number, q for the second system as follows:

High frequency asymptotes:

$$q_3 = \omega\frac{\sqrt[4]{2\nu' s'}}{2}[1+\frac{m'_{14}}{2}\frac{1}{\omega^2}]-i\sqrt{\frac{m'_2+m'_4}{2\sqrt{2\nu' s'}}}[1-\frac{(m'_{13}(b'_1+\sqrt{m'_1}))^2}{\omega^2(m'_2+m'_4)^2}],\quad \omega\to\infty \quad (4.1.50)$$

$$q_4 = \omega\frac{\sqrt[4]{2\nu'\alpha^2}}{2\sqrt{2}}[1+\frac{m^\star_6}{2\omega^2}]-im^\star_7\sqrt{2\nu'\alpha^2}[1-\frac{(m^\star_5)^2}{m^\star_7}\frac{1}{\omega^2}],\quad \omega\to\infty \quad (4.1.51)$$

Low frequency asymptotes:

$$q_3 = \sqrt{\frac{\omega}{2}}\sqrt{\{m'_2+\sqrt{m'_2}\}}[1+\frac{m'_7}{2}\omega]-i\sqrt{\frac{\omega}{2}}\sqrt{\{m'_2+\sqrt{m'_2}\}}[1-\frac{m'_7}{2}\omega],\quad \omega\to 0 \quad (4.1.52)$$

$$q_4 = \frac{\sqrt[4]{m'_9}}{4}\omega[1+\frac{m'_{11}}{2}\omega^2]-i'\frac{m'_{13}\omega^2}{\sqrt{2}}[1+\frac{(m'_{10})^2}{\{2m'_2(m'_6)^2+m'_{10}m'_9\}}\omega^2],\quad \omega\to 0 \quad (4.1.53)$$

where, all constants are given by

$b'_1=\nu'(s'+\alpha^2),\ m'_1=(\nu'(s'-\alpha^2))^2,\ m'_2=(1+\alpha^2\epsilon_e),\ m'_4=\frac{m'_1((m'_2)^2+(m'_3+2m'_1m'_2)^2)}{4(m'_2)^2};$

$m'_3=(s'+\alpha^2)+\alpha^2(1+\alpha^2\epsilon_e)-2\alpha^2(1+s'\epsilon_e),\quad m'_6=[(\frac{m'_3}{m'_1})^2-4\frac{m'_2}{m'_1}];$

$m'_7=b'_1+\frac{m'_3}{2\sqrt{m'_2}},\quad m'_9=b'_1-\frac{m'_3}{2\sqrt{m'_2}},\quad m'_{10}=\frac{m'_3m'_5}{2\sqrt{m'_2}};$

$m'_{11}=\frac{-3m'_9m'_{10}+m'_2(m'_6)^2}{(m'_9)^2},\quad m'_{13}=\sqrt{\frac{m'_9m'_{10}+m'_2(m'_6)^2}{(m'_9)^2}};$

$m'_{14}=\frac{(m'_2)^2-4m'_1m'_2}{8\sqrt{m'_1}}+\frac{(m'_2+m'_4)^2}{4(2\nu's')^2};$

$m_5^\star = (m_2' - m_4^\star)^2 - (2\nu'\alpha^2)\frac{((m_2')^2 - 4m_1'm_2')}{4\sqrt{m_1'}}$;
$m_6^\star = \frac{m_5^\star}{4} - \frac{((m_2')^2 - 4m_1'm_2')}{16\sqrt{m_1'}}$, $\quad m_7^\star = \frac{m_5^\star}{4} + \frac{((m_2')^2 - 4m_1'm_2')}{16\sqrt{m_1'}}$.

Quasi-electro-magnetic-shear wave

From the analytical solutions given by equations (4.1.50) and (4.1.52), we can predict the nature of quasi-electro-magnetic shear wave by determining the physical components of this wave. With the help of suitable formulae stated above, we find the asymptotic expressions of various wave components and their limiting values for this wave as follows:

Asymptotic expansions for phase velocity, specific loss and penetration depth for quasi-magneto-elastic shear wave

From equations (4.1.50) and (4.1.52) and with the help of desired formulae of phase velocity, specific loss and penetration depth, we derive the asymptotic expressions of the wave components and their limiting values for quasi-electro-magnetic-shear wave in the following form:

(E) High frequency asymptotes for components of quasi-electro-magnetic shear wave

(a) Phase velocity $= V_{el} = \frac{\omega}{Re[q_3]} \sim \frac{2}{\sqrt[4]{2*\nu'*s'}}[1 - \frac{m_{14}'}{2}\frac{1}{\omega^2}]$

We observe that V_{el} becomes constant value $\frac{2}{\sqrt[4]{2\nu's'}}$ as $\omega \to \infty$.

(b) Specific loss $= S_{el} = 4\pi|\frac{Im[q_3]}{Re[q_3]}| \sim \frac{\sqrt{\frac{m_2'+m_4'}{2\sqrt{2\nu's'}}}}{\frac{\sqrt[4]{2\nu's'}}{2}}\frac{1}{\omega}[1 - \{\frac{(m_{13}'(b_1' + \sqrt{m_1'}))^2}{\omega^2(m_2'+m_4')^2} + \frac{m_{14}'}{2}\}\frac{1}{\omega^2}]$

Hence, S_{el} has limiting value 0 as $\omega \to \infty$.

(c) Penetration depth $= D_{el} = \frac{1}{|Im[q_3]|} \sim \frac{1}{\sqrt{\frac{m_2'+m_4'}{2\sqrt{2\nu's'}}}}[1 + \frac{(2\nu's'm_{13}'))^2}{\omega^2(m_2'+m_4')^2}]$

Clearly, D_{el} tends to constant value $\frac{1}{\sqrt{\frac{m_2'+m_4'}{2\sqrt{2\nu's'}}}}$ as $\omega \to \infty$.

(F) Low frequency asymptotes for components of quasi-electro-magnetic shear wave

(a) Phase velocity $=V_{el} \sim \frac{\sqrt{2\omega}}{\sqrt{\{m_2'+\sqrt{m_2'}\}}}[1-\frac{m_7'}{2}\omega]$

Hence,V_{el} tends to 0 as $\omega \to 0$.

(b) Specific loss $= S_{el} \sim 4\pi\frac{\sqrt{\{m_2'+\sqrt{m_2'}\}}}{\sqrt{\{m_2'+\sqrt{m_2'}\}}}[1-m_7'\omega]$

which indicates that S_{el} tends to constant value 4π as $\omega \to 0$.

(c) Penetration depth

Penetration depth $= D_{el} \sim \frac{\sqrt{2}}{\sqrt{\omega}\sqrt{\{m_2'+\sqrt{m_2'}\}}}[1+\frac{m_7'}{2}\omega]$

We note that D_{el} tends to ∞as $\omega \to 0$.

Quasi-magneto-elastic shear wave

From equations (4.1.51) and (4.1.53) and with the help of desired formulae of phase velocity, specific loss and penetration depth, we derive the asymptotic expressions of the wave components and their limiting values for quasi-magneto-elastic-shear wave as follows:

(G) High frequency asymptotes for quasi-magneto-elastic shear wave

(a) Phase velocity $= V_s = \frac{\omega}{Re[q_4]} \sim \frac{1}{\frac{\sqrt[4]{2\nu'\alpha^2}}{2\sqrt{2}}}[1-\frac{m_6^\star}{2\omega^2}]$

V_s tends to constant value $\frac{2\sqrt{2}}{\sqrt[4]{2\nu'\alpha^2}}$ as $\omega \to \infty$.

(b) Specific loss $= S_s = 4\pi|\frac{Im[q_4]}{Re[q_4]}|\sim\mathbf{4}\pi\frac{2\sqrt{2}m_7^\star\sqrt{2\nu'\alpha^2}}{\sqrt[4]{2\nu'\alpha^2}}\frac{1}{\omega}[1-\{\frac{(m_5^\star)^2}{m_7^\star}+\frac{m_6^\star}{2}\frac{1}{\omega^2}]$

which implies that S_s becomes 0 when $\omega \to \infty$.

(c) Penetration depth $= D_s = \frac{1}{|Im[q_4]|} \sim \frac{1}{m_7^\star\sqrt{2\nu'\alpha 2}}[1 + \frac{(m_5^\star)^2}{m_7^\star}\frac{1}{\omega^2}]$

which indicates that D_s tends to $\frac{1}{m_7^\star\sqrt{2\nu'\alpha^2}}$ when $\omega \to \infty$.

(H) Low frequency asymptotes for quasi-magneto-elastic shear wave

(a) Phase velocity $= V_s \sim \frac{4}{\sqrt[4]{m_9'}}[1 - \frac{m_{11}'}{2}\omega^2]$, and V_s tends to constant $\frac{4}{\sqrt[4]{m_9'}}$ as $\omega \to 0$.

(b) Specific loss $= S_s \sim 4\pi\frac{\frac{m_{13}'}{\sqrt{2}}}{\frac{\sqrt[4]{m_9'}}{4}}\omega[1 + \{\frac{(m_{10}')^2}{\{2m_2'(m_6')^2+m_{10}'m_9'\}} - \frac{m_{11}'}{2}\}\omega^2]$

and S_s tends to 0 as $\omega \to 0$.

(c) Penetration depth $= D_s \sim \frac{\sqrt{2}}{m_{13}'}\frac{1}{\omega^2}[1 - \frac{(m_{10}')^2}{\{2m_2'(m_6')^2+m_{10}'m_9'\}}\omega^2]$, implying that D_s tends to ∞as $\omega \to 0$.

4.1.5 Numerical results

In the previous section, we observed that we can divide our problem into two different cases that correspond to two different types of waves. One wave is longitudinal in nature and coupled with the thermal field. For this case, we identified two different modes of longitudinal wave: quasi-magneto-thermal wave and quasi-magneto-elastic wave. The second case corresponds to transverse mode wave that is uncoupled with the thermal field. Two different modes are identified for this case too. One is quasi-magneto-elastic shear wave and other one is quasi-electro-magnetic shear wave. We derived the asymptotic expressions of various important wave components for all types of identified waves and found the limiting behavior for very high frequency and low frequency values.

In this section, we make an effort to understand the nature of waves more explicitly for intermediate values of frequency and verify our analytical results of physical components of waves like, phase velocity, specific loss and penetration depth for all cases. For this, we compute the numerical values of different wave characterization for the intermediate values of wave frequency by direct solution of the corresponding dispersion relations. We assume the following values of parameters for a copper like material:

$\epsilon_\theta = 0.0168$, $\lambda = 7.76 \times 10^{10} Nm^{-2}$, $\mu = 3.86 \times 10^{10} Nm^{-2}$, $\mu_e = 4\pi \times 10^{-7} NA^{-2}$; $\sigma = 5.7 \times 10^{7} Sm^{-1}$, $\omega * = 1.72 \times 10^{11} sec^{-1}$, $\rho = 8954 kg/m^3$, $H = 1000 C/ms$

We assume following data for dimensionless parameters:

$k' = 1$; $\pi^\star = 1$

We have used software Mathematica (version 6) and by using the formulae of various components of waves like, phase velocity, specific loss and attenuation coefficient or penetration depth, we compute the numerical values of the wave characterizations for the different values of frequency and present them in different Figures. Description of the nature of waves as we observe are presented below:

4.1.6 Combined analysis of analytical and numerical results

4.1.6.1 Analysis of phase velocity

Case-I

Figures 4.1.1 and 4.1.2 represent the variation of phase velocity of quasi-magneto-thermal mode longitudinal wave for low ad high frequency values, respectively in the context of Green and Naghdi-II theory of thermoelasticity. Figure 4.1.1 represents that the phase velocity of quasi-magneto-thermal mode wave starts from a constant value, then increases for a very small range and attains a constant value near 1.08. Here the notable point is that the phase velocity of quasi-magneto thermal wave is constant in both the cases when $\omega \to 0$ as well as when $\omega \to \infty$. These results are in complete agreement with our analytical results. We further note that the limiting value of phase velocity of this wave as $\omega \to \infty$is effected with the magnetic parameter and in the contrary, the limiting value as $\omega \to 0$ is not affected with the magnetic parameter. This is the important feature of present study in presence of magnetic field. In contrary to this, we find that in absence of magnetic field, we obtain a constant speed of thermal mode wave under GN-II theory all the time and the wave speed is independent of frequency (see Chandrasekharaiah (1996)).

Figures 4.1.3 and 4.1.4 show the nature of phase velocity of quasi-magneto-elastic mode longitudinal wave. The behavior of phase velocity of quasi-magneto elastic mode longitudinal wave resembles with the nature of phase velocity of quasi magneto thermal mode wave i.e. it also starts increasing from a constant value with the increase of frequency and finally goes towards a constant limiting value. But its constant speed is less than the phase velocity of quasi magneto thermal mode wave and achieves the value 0.95 (Figures 4.1.3 and 4.1.4). This result is also in agreement with our analytical results and we noted that the elastic mode wave propagates with the speed equal to $\frac{\sqrt{2}\nu'(s'+\pi^\star k')}{\sqrt{a_1}}$ as frequency tends to infinity.

We observe one more important point from the analytical results that phase velocities of both longitudinal waves are only dependent on thermoelastic coupling constant ϵ_θ for low values of frequency but for high values of frequency, phase velocities of both waves are affected by magnetic parameters.

Case-II

Figures 4.1.5 and 4.1.6 exhibit the nature of phase velocity of quasi-electro-magnetic shear wave for low and high frequency values, respectively. In the present case, the phase velocity of wave is slowly dependent on the frequency. It starts increasing with the frequency and becomes constant very soon; However its constant value is very less and equal to is 0.2. Also from the analytical results we have reached at the conclusion that initially phase velocity is 0 and it starts increasing against frequency and its limiting value is equal to $\frac{2}{\sqrt[4]{2\nu' s'}}$ as frequency tends to infinity.

Figure 4.1.7 depicts the behavior of phase velocity of quasi-magneto-elastic mode shear wave and here we observe that phase velocity is always constant i.e. it is independent with the frequency and this result is proved by our analytical result. This constant value is equal to 0.5 which is clearly less than 1. Here again we reach at the conclusion that quasi-magneto-elastic mode shear waves propagates slowly in comparison to quasi-magneto-elastic mode longitudinal wave. As shear waves moves slowly in comparison to longitudinal waves, hence we are getting shear waves in the present case having lower speed in comparison to case-I. We found from the analytical results that its value is equal to $\frac{2\sqrt{2}}{\sqrt[4]{2\nu'\alpha^2}}$.

4.1.6.2 Analysis of specific loss

Case-I

The objective of the present study is to understand the nature of waves predicted under the theory of thermoelasticity of Green and Naghdi type-II , also called as thermoelasticity without energy dissipation. The effect of the theory in presence of magnetic field on specific loss of waves is identified in the present study. Figures 4.1.8 and 4.1.9 display the variations of specific loss for quasi-magneto thermal mode wave for low and high values of frequency, respectively. From the numerical results it is clear that initially the value of specific loss is 0 but it increases as frequency increases and after reaching a maximum value it starts decreasing and goes to 0. Although, this maximum value is small but we can conclude here that the behavior of specific loss is dependent on frequency, ω in the presence of magnetic field. This fact is also in complete agreement with our analytical results. On the contrary, as reported by Chandrasekharaiah (1996), we note that when magnetic parameters are absent, we obtain that there is no specific loss in waves. This is a distinct feature observed in the present study.

Figures 4.1.10, 4.1.11 depict the nature of specific loss for quasi magneto elastic mode wave for low and high frequencies, respectively. Here, we obtain that the behavior of specific loss of quasi-magneto elastic wave is similar in nature as of quasi-magneto thermal wave.

Case-II

This case corresponds to electrical and elastic mode shear waves. Figures 4.1.12 and 4.1.13 represent the variation of specific loss for quasi-electro-magnetic shear wave for low and high values of frequency, respectively. Here we achieve that the loss is in decreasing trend with the frequency, it starts from a constant value and then decreases as frequency increases and ultimately goes towards 0. The behavior of specific loss for quasi magneto elastic mode shear wave is observed to be very small value near zero for all values of ω. Which is also evident from Figures 4.1.7 and 4.1.11 showing that phase velocity is constant and penetration depth is of very high value. Here, we conclude that quasi-magneto elastic shear wave has very negligible specific loss as compared to the quasi-electro-magnetic shear waves and this fact is in agreement with our analytical results.

4.1.6.3 Analysis of penetration depth

Case-I

Figures 4.1.14 and 4.1.15 depict the nature of penetration depth for quasi-magneto-thermal mode longitudinal wave and quasi-magneto-elastic mode longitudinal wave, respectively. In the present case, the trend of variation of penetration depth is similar in nature for both the waves. It starts from infinity and goes on decreasing and becomes constant. But their constant limiting value (minimum value) is very high. However, comparatively the value of depth is less for thermal mode wave in comparison with elastic mode wave. This is also in complete agreement with our analytical results and we observed that at very low frequency we achieve the infinite value of depth but their limiting values when $\omega \to \infty$ are $\frac{2\sqrt{2}\nu'(s'+\pi^\star k')}{\sqrt{\frac{A_1+\sqrt{(A_1^2-4\nu'^4 s'(s'+\pi^\star k')^2)}A_8}{2}}}$ and $\frac{\sqrt{2}\nu'(s'+\pi k')}{\sqrt{a_1 c_1}}$ for quasi-magneto-thermal mode and quasi-magneto-elastic mode longitudinal wave, respectively.

Case-II

From the analytical results, we can say that the nature of penetration depth for the present case is in such a manner that initially it starts from infinity and then decreases as the frequency increases and ultimately it becomes constant. This constant has a very high value. This fact is also verified by our numerical results presented in Figures 4.1.16 and 4.1.17.

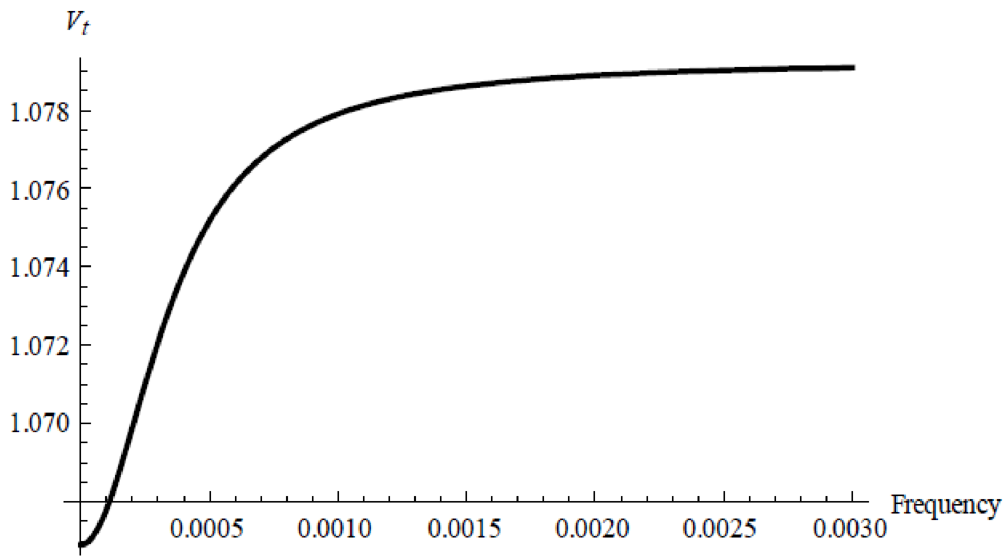

Figure 4.1.1 Phase velocity for quasi-magneto thermal wave (low frequency)

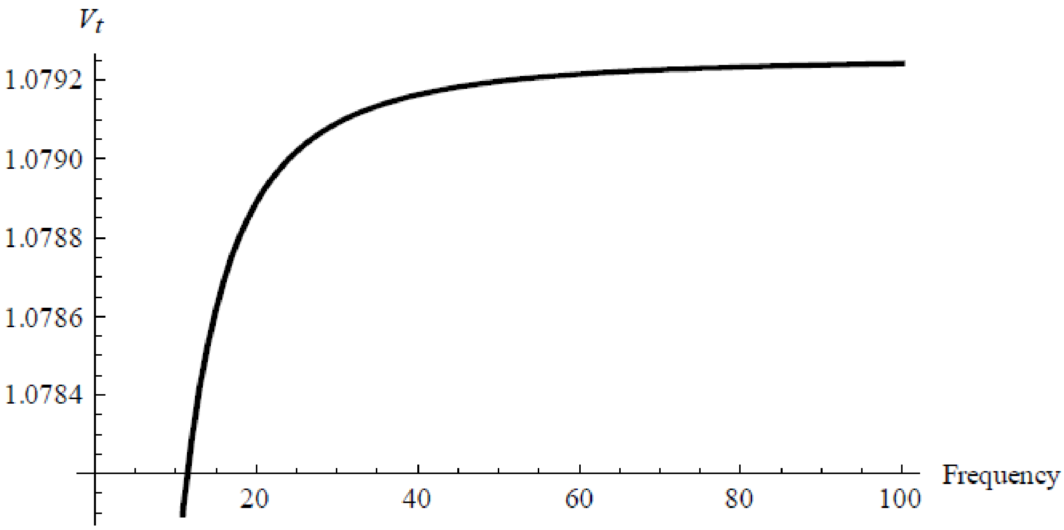

Figure 4.1.2 Phase velocity for quasi-magneto thermal wave (high frequency)

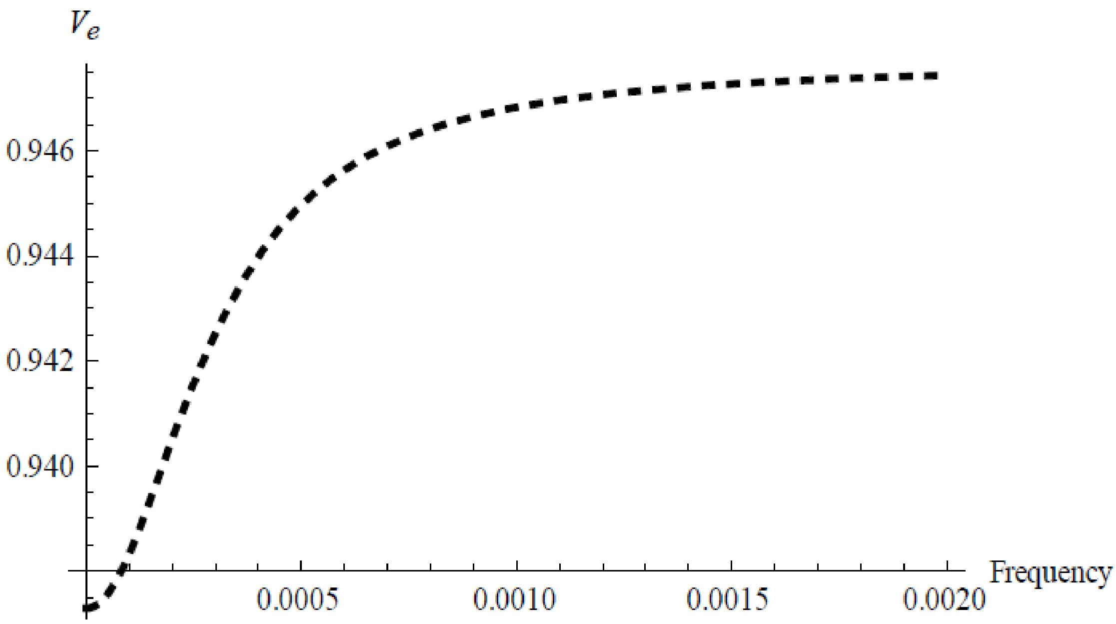

Figure 4.1.3 Phase velocity for quasi-magneto elastic wave (low frequency)

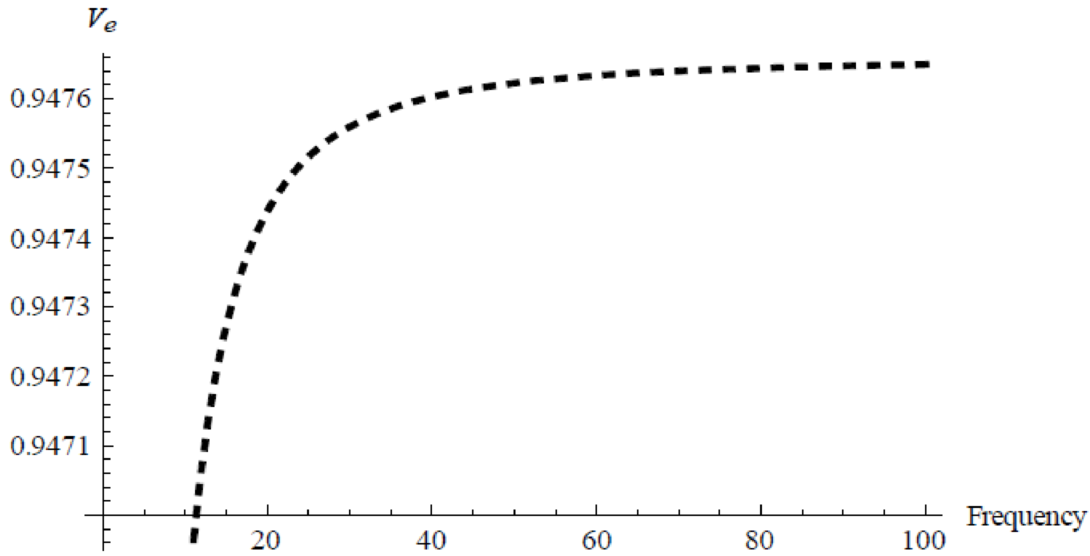

Figure 4.1.4 Phase velocity for quasi-magneto elastic wave (high frequency)

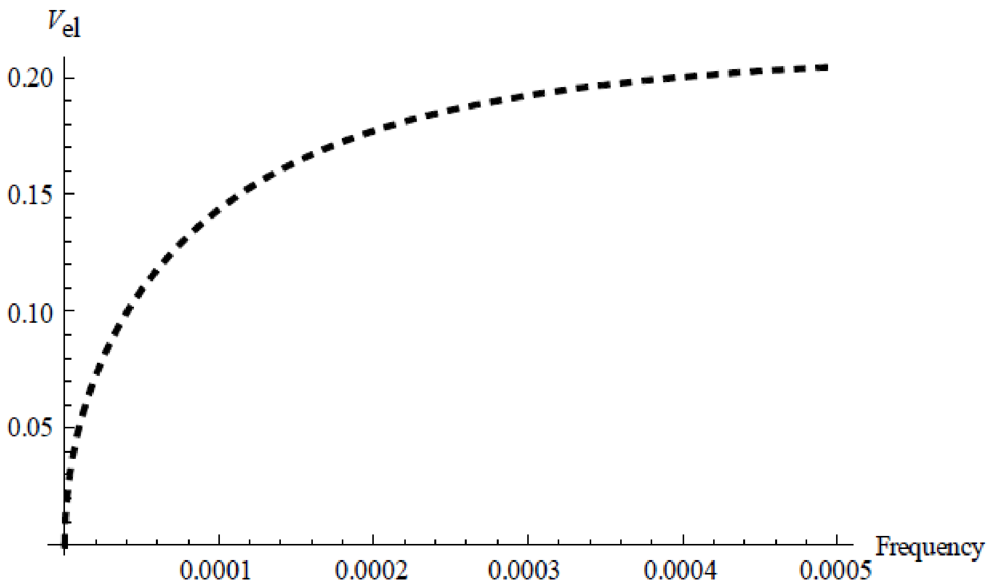

Figure 4.1.5 Phase velocity for quasi-electro-magnetic shear wave (low frequency)

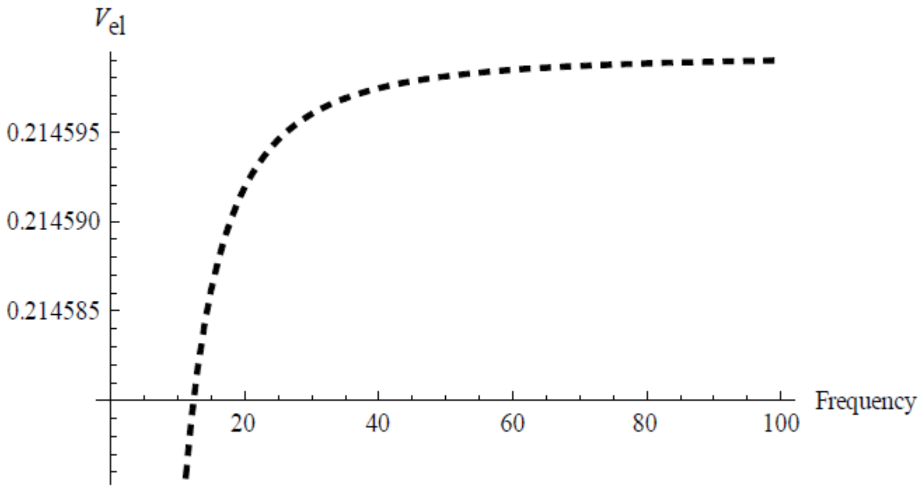

Figure 4.1.6 Phase velocity for quasi-electro-magnetic shear wave (high frequency)

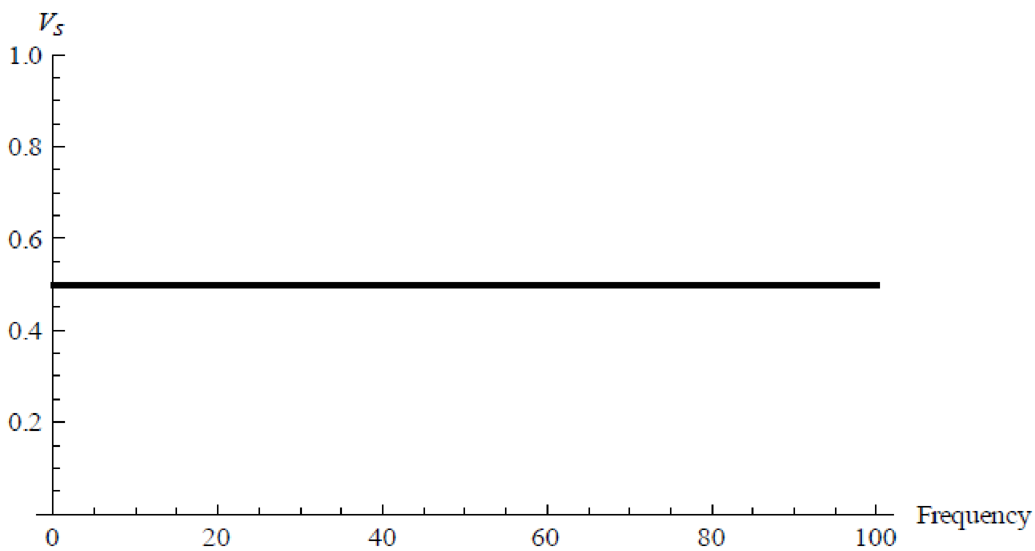

Figure 4.1.7 Phase velocity for quasi-magneto elastic shear wave

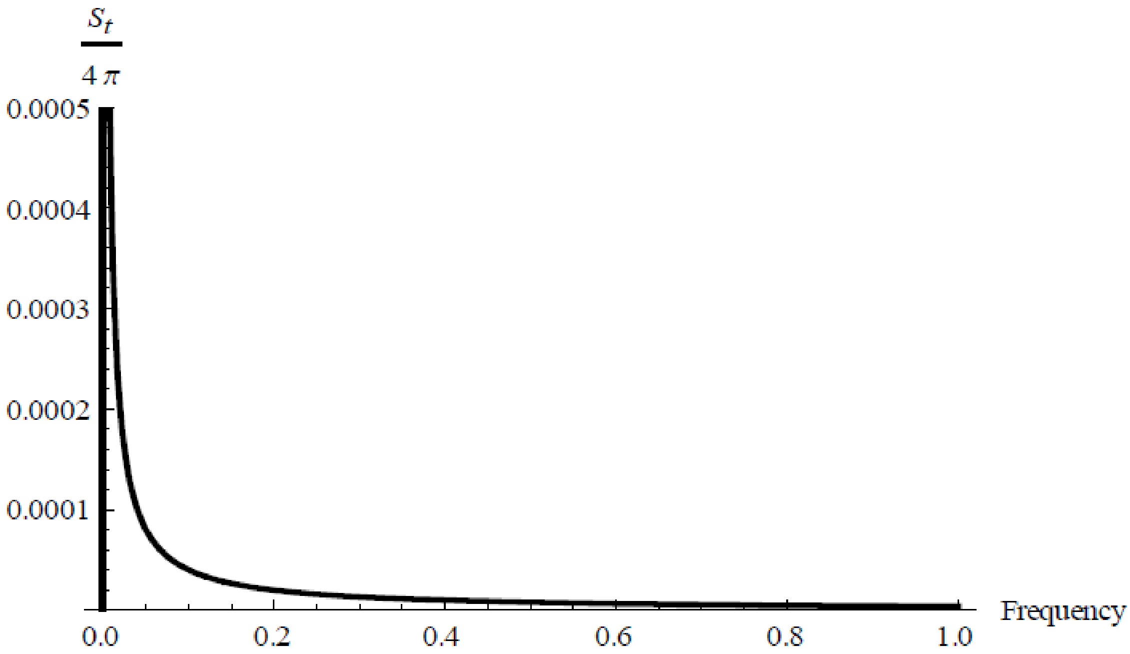

Figure 4.1.8 Specific loss for quasi-magneto thermal wave (low frequency)

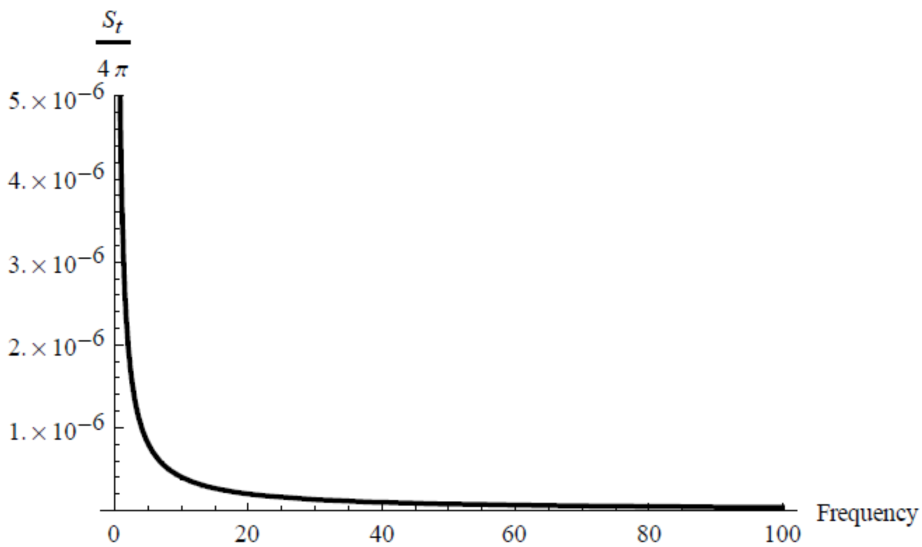

Figure 4.1.9 Specific loss for quasi-magneto thermal wave (high frequency)

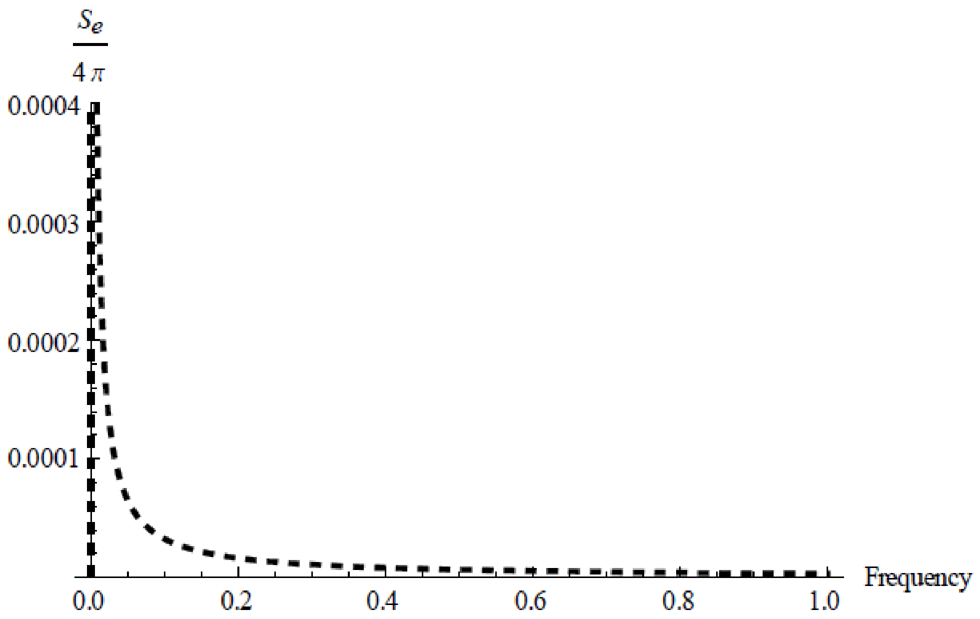

Figure 4.1.10 Specific loss for quasi-magneto elastic wave (low frequency)

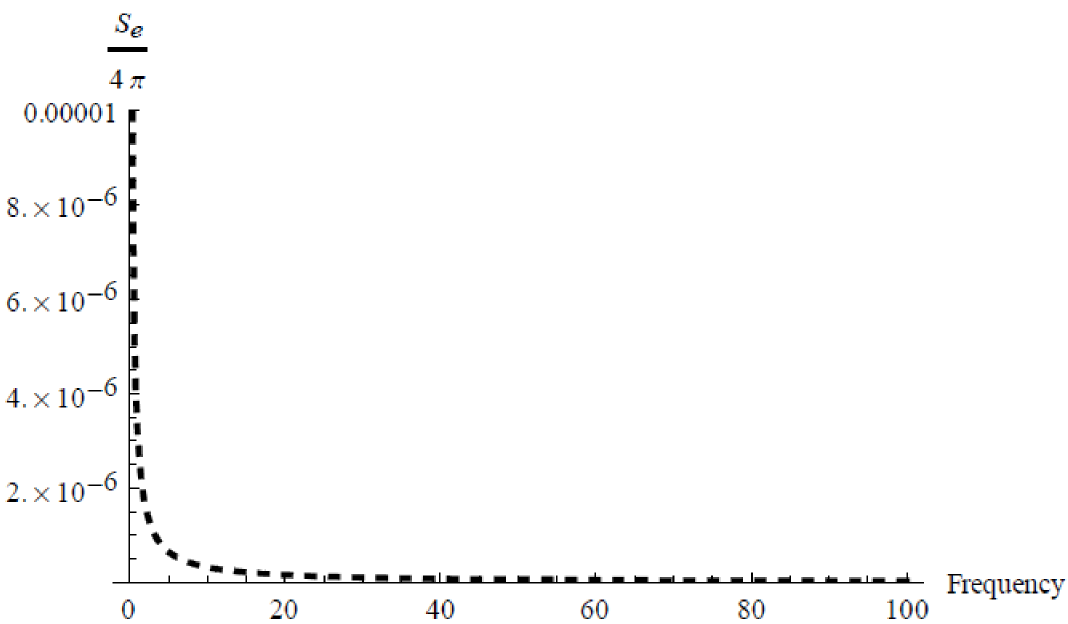

Figure 4.1.11 Specific loss for quasi-magneto elastic wave (high frequency)

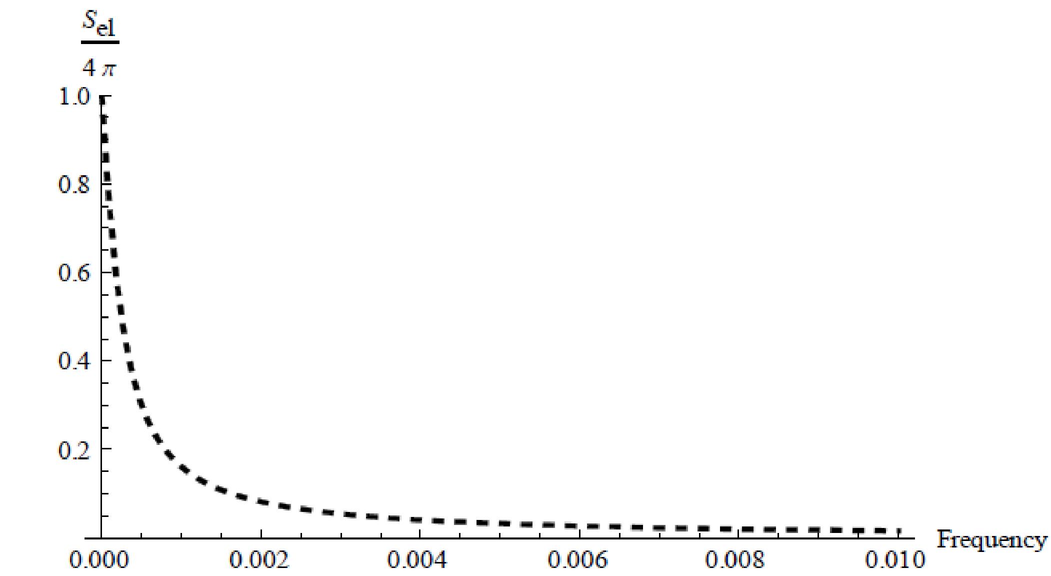

Figure 4.1.12 Specific loss for quasi-electro-magnetic shear wave (low frequency)

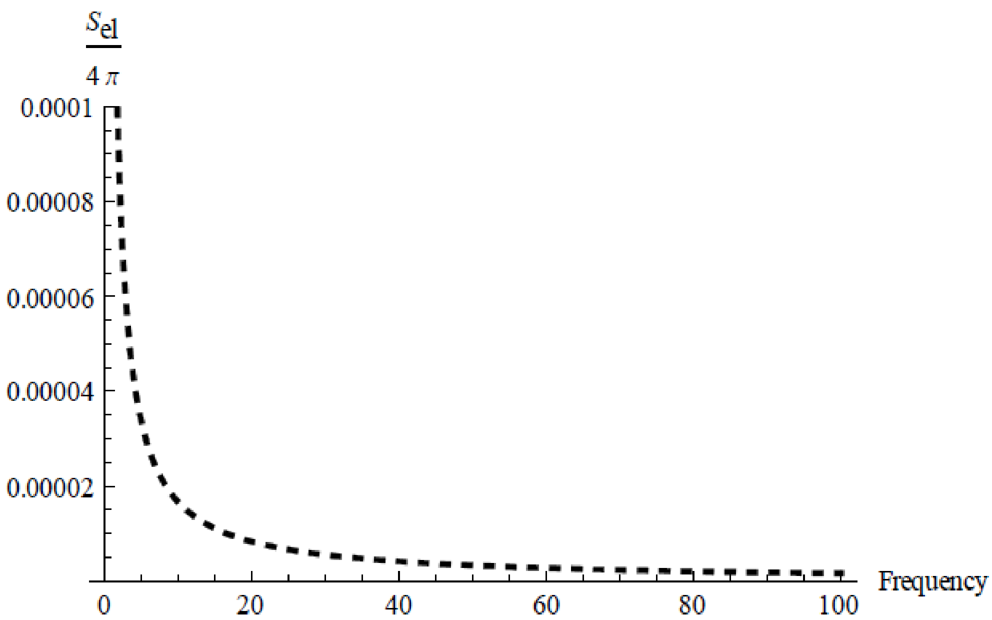

Figure 4.1.13 Specific loss for quasi-electro-magnetic shear wave (high frequency)

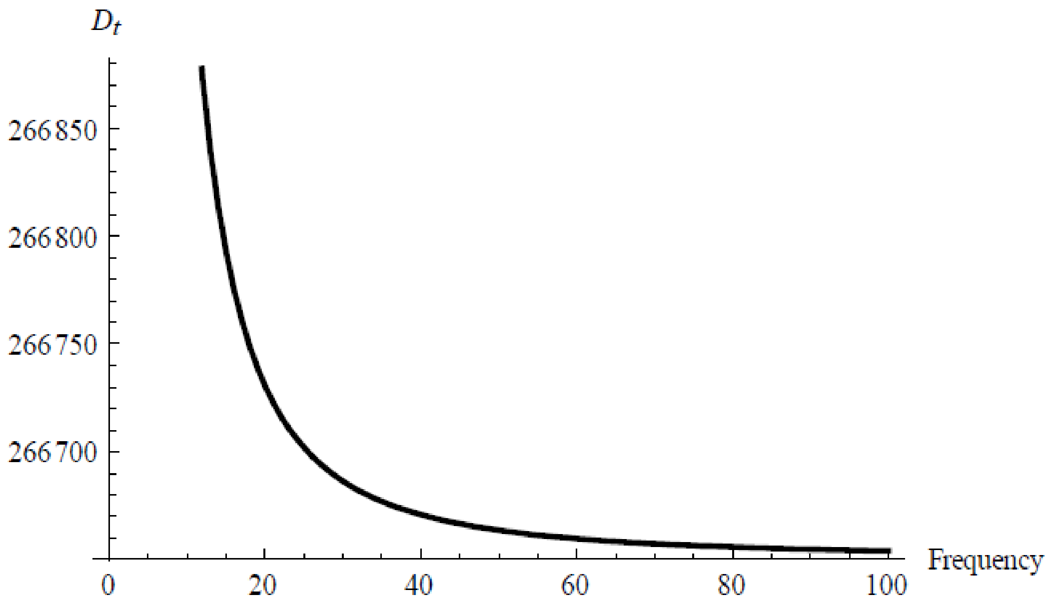

Figure 4.1.14 Penetration depth for quasi-magneto thermal wave

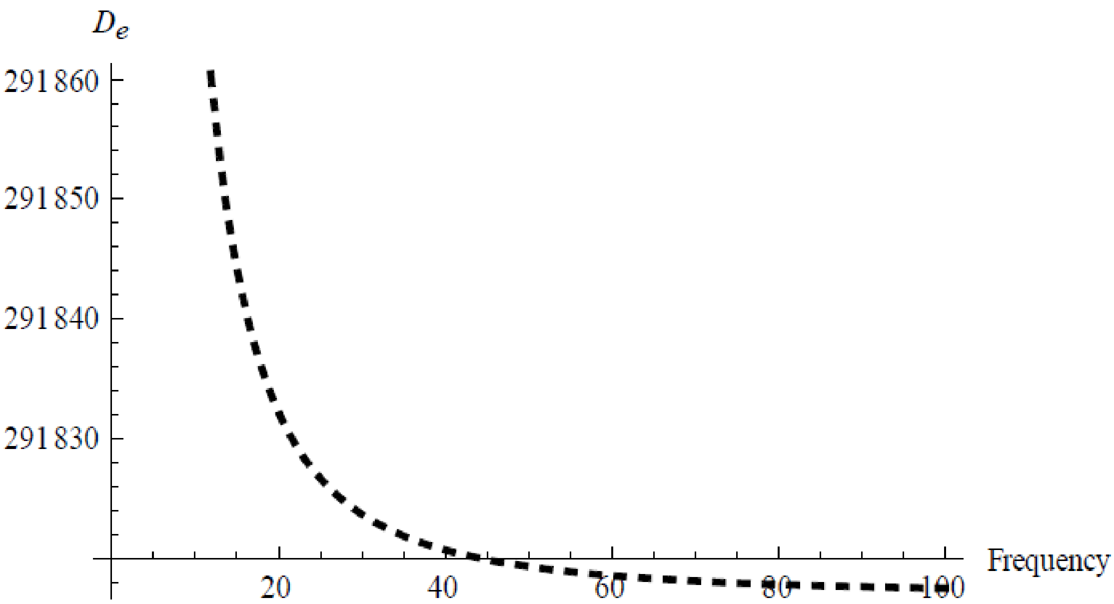

Figure 4.1.15 Penetration depth for quasi-magneto elastic wave

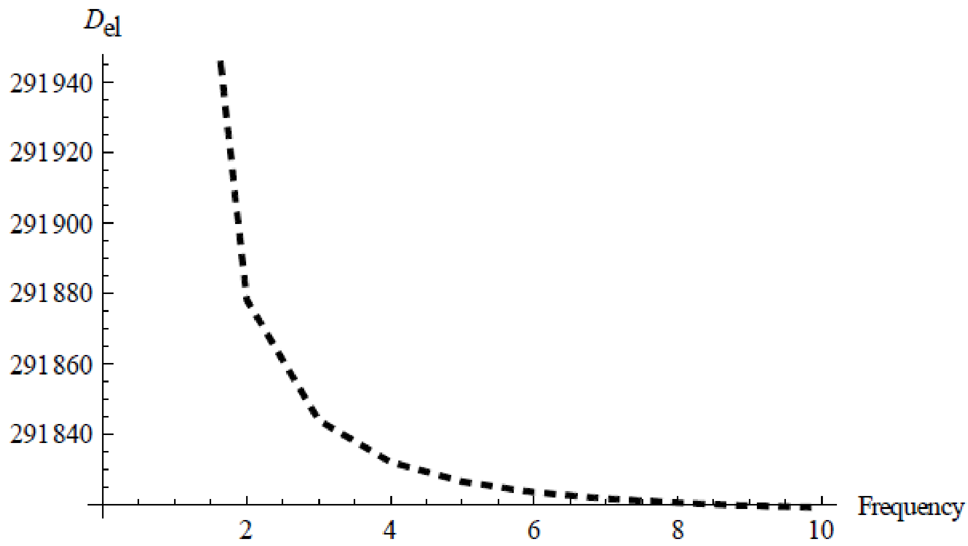

Figure 4.1.16 Penetration depth for quasi-electro- magnetic shear wave

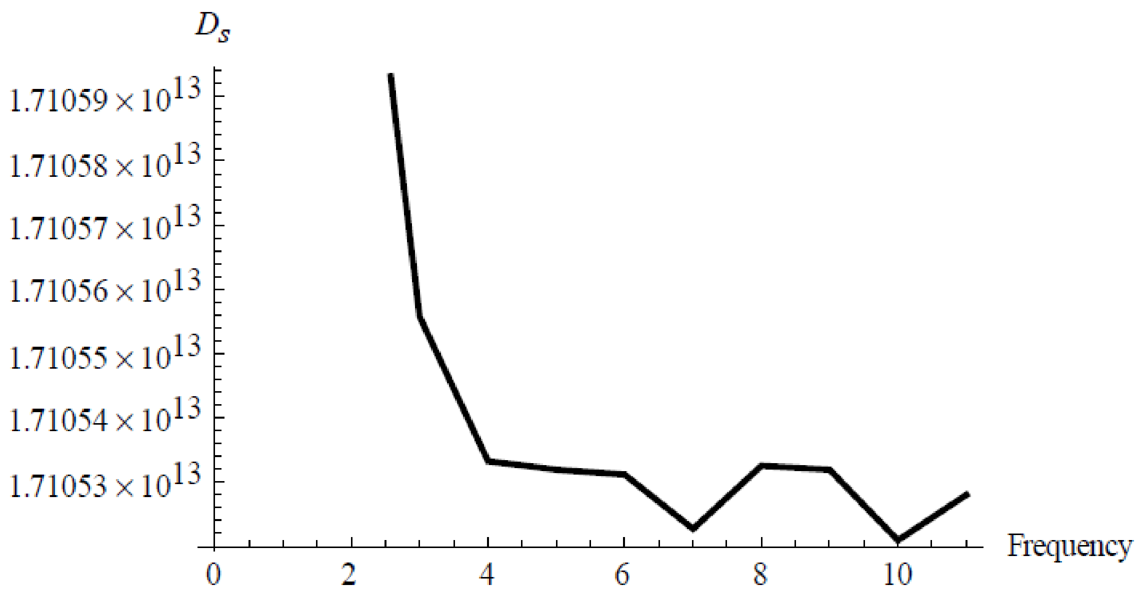

Figure 4.1.17 Penetration depth for quasi-magneto-elastic shear wave

4.1.7 Conclusions

In the present study, we deal with a detailed investigation of electro-magneto-thermoelastic plane waves in the presence of magnetic field in the context of Green and Naghdi type-II theory of thermoelasticity. We have subdivided our problem in two sub cases. For the analytical results, small and high frequency asymptotic approximation technique have been employed. We identify different mode waves and analyze each type by deriving asymptotic expansions of various wave components. Numerical results have also been presented. From analytical and numerical results in two cases, we can conclude the following features:

1. We obtain longitudinal and transverse mode waves corresponding to two separate cases in which the transverse mode wave is completely independent of the thermal field, but mechanical and electrical fields are coupled together. Hence, we identify quasi-magneto-elastic shear wave and quasi-electro-magnetic shear wave.

2. The longitudinal wave is coupled with thermal field. We identify quasi-magneto elastic mode longitudinal wave and quasi-magneto-thermal mode longitudinal wave. Both waves are almost similar in nature. They are dispersive in nature and show constant limiting speed as frequency increases to high value. However, both waves propagate with negligible specific loss and very high penetration depth.

3. Presence of magnetic field plays a very important role in the results of longitudinal waves which can be understood from the following discussion:

when we put all magnetic field parameters equal to 0 in equation (4.1.41), we obtain the dispersive relation coupled with purely elastic and thermal fields. The dispersion relation equation is given by

$$q^4 - \{\omega^2(2 + \epsilon_\theta)\}q^2 + \omega^4 = 0 \tag{4.1.54}$$

Solution of above equation gives four real roots. From all the roots of above equation, we take two positive roots of q i.e. $q_{1,2}$ and with the help of them we can calculate phase velocity of purely elastic mode and purely thermal mode longitudinal waves as

$$V_{1,2} = \frac{\sqrt{2}}{\sqrt{(2 + \epsilon_\theta) \pm \sqrt{\epsilon_\theta^2 + 4\epsilon_\theta}}} \tag{4.1.55}$$

It is clear from above results that phase velocity of elastic and thermal waves are constant and are frequency independent. They have no imaginary parts in the absence of magnetic field. However from our analytical and numerical results we achieve that phase velocity is dependent on the frequency in the presence of magnetic field. We can again say that since imaginary parts are absent in the roots of dispersive relation (equation (4.1.55)), hence specific loss will be completely zero and penetration depth will be infinite that means there is no decay of waves and waves will penetrate the materials till an infinite depth in materials. Similar results are reported by Chandrasekharaiah (1996) for the thermoelastic case. In the present study, we considered the presence of magnetic field and due to that we observe that phase velocity of elastic mode wave and thermal mode wave is slowly dependent on frequency ω. It starts from a constant value and then increases as the frequency increases but it finally becomes constant. Further more, we note a small value of specific loss and very high value of penetration depth in presence of magnetic field.

4. We obtain quasi-electro-magnetic shear wave and quasi-magneto-elastic shear wave in which phase velocity of quasi-magneto elastic mode shear wave is always constant, i.e. frequency independent and its value is 0.5, however quasi-electro-magnetic shear wave is slowly dependent on frequency. Like the case of longitudinal waves, for the transverse mode waves, the specific loss shows very less value and penetration depth has a very high constant limiting value, although they are dispersive in nature. It is believed that the results of the present investigation highlight several specific features of magneto-thermoelastic interactions under thermoelasticity without energy dissipation which has not been investigated by any researcher in this direction.

[1]4.2 On magneto-thermo-elastic wave propagation in a finitely conducting medium under thermoelasticity of type- I, II and III

4.2.1 Introduction

In the present section, we investigate the propagation of electro-magneto-thermoelastic plane waves of assigned frequency in a homogeneous, isotropic and finitely conducting elastic medium permeated by a primary uniform external magnetic field in the context of Green and Naghdi of type-III (GN-III). We formulate our problem to account for the interactions between the elastic, thermal as well as magnetic fields. A general dispersion relation for coupled waves is deduced to ascertain the nature of waves propagating through the medium. Perturbation technique has been employed to obtain the solution of dispersion relation for small thermo-elastic coupling parameter and identify three different types of waves. We specially analyze the nature of important wave components like, phase velocity, specific loss and penetration depth of all three modes of waves. We attempt to compute these wave components numerically to observe their variations with frequency. The effect of presence of magnetic field is analyzed. The results under theories of type GN-I and II have also been exhibited as a special cases in which we have found that the coupled thermoelastic waves are un-attenuated and non-dispersive in case of Green-Naghdi-II model which is completely in contrast with the theories of type-I and type-III. Some specific features of type-III model are highlighted. We achieve significant variations among the results predicted by all three theories.

[1] **This work is in 'under review' in Journal "Archives of Mechanics".**

4.2.2 Problem formulation and basic governing equations

For our present study, an infinite, homogeneous, isotropic, thermally and electrically conducting solid permeated by a primary magnetic field $\vec{B_0} = (B_1, B_2, B_3)$ is considered. The media is characterized by the density ρ and Lame elastic constants λ and μ.

Using a fixed rectangular Cartesian coordinate system *(x,y,z)* and employing the thermoelasticity theory of Green and Naghdi (1991-1993), the equations of motion and the equation of heat conduction in the presence of magnetic field in the absence of external body force (mechanical) and heat sources can be represented in the following manner:

Equation of motion:

$$\mu\nabla^2\vec{u} + (\lambda+\mu)\vec{\nabla}(\vec{\nabla}.\vec{u}) + \vec{J}\times\vec{B} - \gamma\vec{\nabla}\theta = \rho\ddot{\vec{u}} \tag{4.2.1}$$

Equation of heat conduction (1991):

$$K^{\star}\nabla^2\theta + K\nabla^2\dot{\theta} = \rho C_v\ddot{\theta} + \gamma T_0\ddot{u}_{i,i} \tag{4.2.2}$$

where $\vec{J}$ is the current density vector and $\vec{J}\times\vec{B}$ is the electromagnetic body force (Lorentz force).

$\vec{B} = \vec{B_0} + \vec{b}$ is the total magnetic field which is assumed to be small so that the products with $\vec{b}$ and $\dot{\vec{u}}$ and their derivatives can be neglected for the linearization of the field equations. $\vec{b} = (b_x, b_y, b_z)$ is the perturbed magnetic field. Dots denote the derivatives with respect to the time t.

Notice that if we put $K^{\star} = 0$ in equation (4.2.2), then the equation is acknowledged by the heat conduction equation for GN-I theory of thermoelasticity and if we substitute $K = 0$ in equation (4.2.2), we obtain the heat conduction equation of GN-II theory of thermoelasticity.

Since magnetic field has been applied in the medium, hence the equation of motion needs to be supplemented by generalized Ohm's law in a continuous medium with Maxwell's electromagnetic field equations.
Maxwell equations (where the displacement current and charge density are neglected) are given by

$$\vec{\nabla} \times \vec{H} = \vec{J} \tag{4.2.3}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \overrightarrow{B}}{\partial t} \tag{4.2.4}$$

where $\vec{B} = \mu_e \vec{H}$ and μ_e is the magnetic permeability.

$$\vec{\nabla}.\vec{B} = 0 \tag{4.2.5}$$

Generalized Ohm's law is given by

$$\vec{J} = \sigma[\vec{E} + \frac{\partial u}{\partial t} \times \vec{B}] \tag{4.2.6}$$

Here, $\frac{\partial u}{\partial t}$ is the particle velocity of the medium. Small effect of temperature gradient on $\vec{J}$ is neglected.

4.2.3 Dispersion relation and its analytical solution

We assume that plane waves are propagating towards x-direction. Due to this all the field quantities are proportional to $e^{i(kx-\omega t)}$, where k is the wave number and ω is the angular frequency of plane waves. Here we have assumed that ω is real and

k may be complex quantity, where $Im(k) \leq 0$ must hold for waves to be physically realistic. In the context of above consideration, we can write all the quantities in the following manner:

$\vec{u} = (u, v, w) = (u_0, v_0, w_0)e^{i(kx-\omega t)}$

$\theta = \theta_0 e^{i(kx-\omega t)}$, $\vec{E} = (E_x, E_y, E_z)$, $\overrightarrow{J} = (j_1, j_2, j_3)e^{i(kx-\omega t)}$, $\vec{b} = (b_x, b_y, b_z) = (b_1, b_2, b_3)e^{i(kx-\omega t)}$

where u_0, v_0, w_0; j_1, j_2, j_3, b_1, b_2, b_3, θ_0 are constants.

With the help of Maxwell relations, we achieve the following relation:

$$div \overrightarrow{b} = 0 \tag{4.2.7}$$

Above relation implies that $b_x = 0$. Using equation (4.2.3) we obtain

$$\mu_e \overrightarrow{J} = \vec{\nabla} \times \overrightarrow{b} \tag{4.2.8}$$

which is in agreement with the following value of $\overrightarrow{J}$

$$\vec{J} = [0, -\frac{ikb_z}{\mu_e}, \frac{ikb_y}{\mu_e}] \tag{4.2.9}$$

and

$$\vec{J} \times \vec{B_0} = [-\frac{ik(b_z B_3 + b_y B_2)}{\mu_e}, \frac{ikb_y B_1}{\mu_e}, \frac{ikb_z B_1}{\mu_e}] \tag{4.2.10}$$

Thus the term $\overrightarrow{J} \times \overrightarrow{B}$ can be replaced by the term $\overrightarrow{J} \times \overrightarrow{B_0}$.

Substituting values of the quantities $\vec{u}$ and θ in the equation of heat conduction , we achieve the following relation:

$$\theta = \alpha u_0 \tag{4.2.11}$$

where

$$\alpha = \frac{i\nu\theta_0 k\omega^2}{K^\star k^2 - iKk^2\omega - \rho C_v \omega^2} \tag{4.2.12}$$

We obtain $\vec{E} = (E_x, E_y, E_z) = (E_x, \frac{\omega b_z}{k}, -\frac{\omega b_y}{k})$

Now, making use of the field quantities $\vec{u}$, $\vec{J}$ and $\vec{E}$ and comparing both sides of the modified Ohm's law (in which $\vec{B}$ is replaced by $\vec{B_0}$), we achieve the following three relations:

$$\sigma[E_x - i\omega(qB_3 - rB_2)] = 0 \tag{4.2.13}$$

$$\sigma[\frac{\omega b_z}{k} - i\omega(rB_1 - pB_3)] = -\frac{ikb_z}{\mu_e} \tag{4.2.14}$$

$$\sigma[-\frac{\omega b_y}{k} - i\omega(pB_2 - qB_1)] = \frac{ikb_y}{\mu_e} \tag{4.2.15}$$

Further, substituting the values of field quantities $\overrightarrow{u}$,$\overrightarrow{J}$,$\vec{B_0}$,$\vec{\theta}$ in the equation of motion, we achieve the following relations:

$$u_0(-\rho\omega^2 + (\lambda + 2\mu)k^2 + i\nu\alpha k) + \frac{ik}{\mu_e}(b_3B_3 + b_2B_2) = 0 \tag{4.2.16}$$

$$v_0(-\rho\omega^2 + \mu k^2) - \frac{ik}{\mu_e}(b_2B_1) = 0 \tag{4.2.17}$$

$$w_0(-\rho\omega^2 + \mu k^2) - \frac{ik}{\mu_e}(b_3B_1) = 0 \tag{4.2.18}$$

Equations (4.2.14) and (4.2.15) can be rewritten as

$$u_0\sigma(i\omega B_3) + w_0(-\sigma i\omega B_1) + b_3[\frac{ik}{\mu_e} - \frac{\sigma\omega}{k}] = 0 \tag{4.2.19}$$

$$u_0\sigma(-i\omega B_2) + v_0(\sigma i\omega B_1) - b_2[\frac{ik}{\mu_e} + \frac{\sigma\omega}{k}] = 0 \tag{4.2.20}$$

Equations [(4.2.16)-(4.2.20)] constitute a system of five equations with five unknowns namely u_0, v_0, w_0, b_2, b_3.

We can make further assumptions: we have taken that $\vec{b}$ is directed along y-axis and we consider $w_0 = 0$ provided that $(\mu k^2 - \rho\omega^2 \neq 0)$ so that $b_3 = 0$ (equation (4.2.18)). Hence, applied and perturbed magnetic field are taken as $(\overrightarrow{B_1}, \overrightarrow{B_2}, 0)$ and $(0, \overrightarrow{b_2}, 0)$, respectively. Applying the above assumptions in the equations [(4.2.16)-(4.2.20)], all above five equations reduce into three following homogeneous equations having three unknowns u_0, v_0 and b_2 as

$$u_0[-\rho\omega^2 + (\lambda + 2\mu)k^2 + i\nu\alpha k] + \frac{ikB_2 b_2}{\mu_e} = 0 \tag{4.2.21}$$

$$v_0(-\rho\omega^2 + \mu k^2) - \frac{ikB_1 b_2}{\mu_e} = 0 \tag{4.2.22}$$

$$u_0(\sigma i\omega B_2) - v_0\sigma(i\omega B_1) + b_2(\frac{ik}{\mu_e} + \frac{\sigma\omega}{k}) = 0 \tag{4.2.23}$$

In order to have a solution for u_0, v_0 and b_2 of the above three equations, we must have

$$\begin{vmatrix} -\rho\omega^2 + (\lambda + 2\mu)k^2 + i\nu\alpha k & 0 & \frac{ikB_2}{\mu_e} \\ 0 & \mu k^2 - \rho\omega^2 & \frac{-ikB_1}{\mu_e} \\ i\sigma\omega B_2 & 0 & (\frac{ik}{\mu_e} + \frac{\sigma\omega}{k}) \end{vmatrix} = 0$$

Now, we assume that the initial magnetic field is directed towards y axis i.e. $\vec{B_0} = (0, B_2, 0)$ i.e. $B_1 = 0$ in the above equation (4.2.23). For simplifying the expansion of above determinant, we introduce above the following non dimensional quantities:

$\chi = \frac{\omega}{\omega^\star}$, $\xi = \frac{kc_1}{\omega^\star}$, $\epsilon_H = \frac{\omega^\star \nu_H}{c_1^2}$

$\epsilon_\theta = \frac{T_0 \nu^2}{\rho^2 C_v c_1^2}$, $\nu_H = \frac{1}{\mu_e \sigma}$,

$k_1 = \frac{K^\star}{\rho C_v c_1^2}$, $k_2 = \frac{K \omega^\star}{\rho C_v c_1^2}$

$c_1 = \sqrt{\frac{(\lambda+2\mu)}{\rho}}$, $c_2 = \sqrt{\frac{\mu}{\rho}}$

where c_1 is the longitudinal elastic wave velocity and c_2 is the transverse elastic wave velocity. We further assume that

$s = \frac{c_2}{c_1}$, $R_H = \frac{B_2^2}{\rho c_1^2 \mu_e}$.

where R_H is the magnetic pressure number. Substituting the value of α from eqn. (4.2.12) and employing all dimensionless quantities in the expansion of the above determinant, we find the following dispersion relation:

$$(s^2\xi^2 - \chi^2)[\{(\xi^2 - \chi^2)(\xi^2 k_1 - \chi^2) - ik_2\xi^2\chi(\xi^2 - \chi^2) - \epsilon_\theta \xi^2 \chi^2\}(\chi + i\xi^2\epsilon_H)$$
$$+R_H\chi\xi^2\{(\xi^2 k_1 - \chi^2) - ik_2\xi^2\chi\}] = 0 \quad (4.2.24)$$

The first part $(s^2\xi^2 - \chi^2) = 0$ in (4.2.24) corresponds to a transverse elastic wave which is clearly found to be uncoupled by thermal and magnetic field. Hence, we take the second part of (4.2.24) as

$$\{(\xi^2-\chi^2)(\xi^2k_1-\chi^2)-ik_2\xi^2\chi(\xi^2-\chi^2)-\epsilon_\theta\xi^2\chi^2\}(\chi+i\xi^2\epsilon_H)+R_H\chi\xi^2\{(\xi^2k_1-\chi^2)-ik_2\xi^2\chi\} = 0 \quad (4.2.25)$$

Above equation is clearly identified as the dispersion relation for coupled thermal-dilatational-electrical waves propagating in the medium in the present context. (If $k_2 = 0$ in equation (4.2.24) (i.e. for Green-Naghdi-II); we will achieve the equation which matches completely with equation (3.29) of Roychoudhuri and Banerjee (2005)). With the help of equation (4.2.25), we will characterize the behaviour of plane waves propagating in the present context. We will specially concentrate on the analysis of the important wave characterizations like, phase velocity, specific loss and penetration depth. For solving equation (4.2.25), we attempt to find the perturbation solution of the dispersion equation for small values of ϵ_θ. Therefore, substituting ϵ_θ= 0 in the dispersion relation, we achieve the following solutions:

$$\xi^2 = A\chi^2 \tag{4.2.26}$$

where $A = \frac{k_1+ik_2\chi}{k_1^2+k_2^2\chi^2} = a_1 + ib_1$

$$[(\xi^2 - \chi^2)(\chi + i\xi^2\epsilon_H) + R_H\chi\xi^2] = 0 \tag{4.2.27}$$

Equation (4.2.26) being an equation of degree 4, we consider the roots of above equation (4.2.27) as $\pm\alpha_1$and $\pm\alpha_2$.

where

$$\alpha_{1,2}^2 = M_{1,2}^2 \tag{4.2.28}$$

where

$$M_{2,1}^2 = \frac{i\chi^2\epsilon_H - \chi(1+R_H) \pm \{\chi^2(1+R_H)^2 - \chi^4\epsilon_H^2 - 2\chi^3(1+R_H)\epsilon_H i + 4i\epsilon_H\chi^3\}^{\frac{1}{2}}}{2i\epsilon_H} \tag{4.2.29}$$

On simplifying above equation, we obtain its more simplified form given by

$$M_2^2 = a_4 + ib_4 \tag{4.2.30}$$

and

$$M_1^2 = a_5 + ib_5 \tag{4.2.31}$$

where $a_4 = \frac{\chi^2\epsilon_H + b_3}{2\epsilon_H}$, $b_4 = \frac{\chi(1+R_H)-a_3}{2\epsilon_H}$;

$a_5 = \frac{\chi^2\epsilon_H - b_3}{2\epsilon_H}$, $b_5 = \frac{\chi(1+R_H)+a_3}{2\epsilon_H}$;

$a_3 = \sqrt{\frac{a_2+\sqrt{a_2^2+b_2^2}}{2}}$, $b_3 = \sqrt{\frac{-a_2+\sqrt{a_2^2+b_2^2}}{2}}$;

$a_2 = \chi^2(1+R_H)^2 - \chi^4\epsilon_H^2, \quad b_2 = 2\chi^3\epsilon_H(1-R_H)$

Since we are trying to find the perturbation solution for small values of thermoelastic coupling constant ϵ_θ. Then we write ξ^2 in the following forms:

$$\xi_1^2 = \alpha_1^2 + n_1\epsilon_\theta + O(\epsilon_\theta^2) \tag{4.2.32}$$

$$\xi_2^2 = \alpha_2^2 + n_2\epsilon_\theta + O(\epsilon_\theta^2) \tag{4.2.33}$$

$$\xi_3^2 = A\chi^2 + n_3\epsilon_\theta + O(\epsilon_\theta^2) \tag{4.2.34}$$

Substituting equations (4.2.32), (4.2.33) and (4.2.34) in equation (4.2.25) and comparing the lowest power of ϵ_θ on both sides of equation and neglecting the terms of $O(\epsilon_\theta^2)$, we obtain the following solution:

$$\xi_1^2 = \alpha_1^2[1 + A\epsilon_\theta\chi^2\frac{(\chi + i\alpha_1^2\epsilon_H)}{G_1}] \tag{4.2.35}$$

$$\xi_2^2 = \alpha_2^2[1 + A\epsilon_\theta\chi^2\frac{(\chi + i\alpha_2^2\epsilon_H)}{G_2}] \tag{4.2.36}$$

$$\xi_3^2 = A\chi^2[1 + \frac{A\epsilon_\theta\chi^3(1 + iA\chi\epsilon_H)}{(A-1)(\chi^3 + iA\epsilon_H\chi^4) + R_H A\chi^3}] \tag{4.2.37}$$

where

$$G_{1,2} = ((\chi + i\alpha_{1,2}^2\epsilon_H)(\alpha_{1,2}^2 - \chi^2) + \chi R_H \alpha_{1,2}^2) + (\alpha_{1,2}^2 - \chi^2)((1+R_H)\chi + 2i\alpha_{1,2}^2\epsilon_H - i\epsilon_H\chi^2) \tag{4.2.38}$$

Further simplifying equation (4.2.38), we obtain

$$G_1 = a_6 + ib_6 \tag{4.2.39}$$

$$G_2 = a_7 + ib_7 \tag{4.2.40}$$

where $a_6 = (\chi - b_5\epsilon_H)(a_5 - \chi^2) - a_5 b_5 \epsilon_H + \chi R_H a_5 + (a_5 - \chi^2)((1+R_H)\chi - 2b_5\epsilon_H) - 2a_5 b_5 \epsilon_H + b_5 \epsilon_H \chi^2$

$b_6 = b_5(\chi - b_5\epsilon_H) + a_5\epsilon_H(a_5 - \chi^2) + \chi R_H b_5 + b_5(1 + R_H)\chi - 2b_5^2\epsilon_H + (2a_5\epsilon_H - \epsilon_H\chi^2)(a_5 - \chi^2)$

$a_7 = (\chi - b_4\epsilon_H)(a_4 - \chi^2) - a_4 b_4 \epsilon_H + \chi R_H a_4 + (a_4 - \chi^2)((1 + R_H)\chi - 2b_4\epsilon_H) - 2a_4 b_4 \epsilon_H + b_4 \epsilon_H \chi^2$

$b_7 = b_4(\chi - b_4\epsilon_H) + a_4\epsilon_H(a_4 - \chi^2) + \chi R_H b_4 + b_4(1 + R_H)\chi - 2b_4^2\epsilon_H + (2a_4\epsilon_H - \epsilon_H\chi^2)(a_4 - \chi^2)$

Clearly, the above expressions for $\xi_i, i = 1, 2, 3$ (with $Im(\xi_i \leq 0)$ correspond to three modes of plane waves propagating inside the medium.

4.2.4 Analytical expressions of various components of magneto-thermo-elastic plane wave

In order to calculate several components of waves like, phase velocity, specific loss and penetration depth, we use the following formulae:

Phase velocity:

$$V_{1,2,3} = \frac{\chi}{Re[\xi_{1,2,3}]} \tag{4.2.41}$$

Specific loss:

$$S_{1,2,3} = 4\pi|\frac{Im[\xi_{1,2,3}]}{Re[\xi_{1,2,3}]}| \tag{4.2.42}$$

Penetration depth:

$$D_{1,2,3} = \frac{1}{|Im[\xi_{1,2,3}]|} \tag{4.2.43}$$

Now, substituting expressions of α_1^2, α_2^2, G_1, G_2 and A in equations (4.2.35), (4.2.36) and (4.2.37) and further solving them, we obtain their more simplified form in the following manner:

$$\xi_1^2 = a_5 + \epsilon_\theta[\frac{a_5\chi^2(P_1a_6 + Q_1b_6) - b_5\chi^2(a_6Q_1 - b_6P_1)}{a_6^2 + b_6^2}]$$

$$+i[b_5 + \epsilon_\theta[\frac{b_5\chi^2(P_1a_6 + Q_1b_6) + a_5\chi^2(a_6Q_1 - b_6P_1)}{a_6^2 + b_6^2}]] \tag{4.2.44}$$

$$\xi_2^2 = a_4 + \epsilon_\theta[\frac{a_4\chi^2(P_2a_7 + Q_2b_7) - b_4\chi^2(a_7Q_2 - b_7P_2)}{a_7^2 + b_7^2}]$$

$$+i[b_4 + \epsilon_\theta[\frac{b_4\chi^2(P_2a_7 + Q_2b_7) + a_4\chi^2(a_7Q_2 - b_7P_2)}{a_7^2 + b_7^2}]] \tag{4.2.45}$$

$$\xi_3^2 = \chi^2[a_1 + C\epsilon_\theta + i(b_1 + D\epsilon_\theta)] \tag{4.2.46}$$

where $P_1 = a_1\chi - a_1b_5\epsilon_H - b_1a_5\epsilon_H, \quad Q_1 = b_1\chi - b_1b_5\epsilon_H + a_5a_1\epsilon_H;$

$P_2 = a_1\chi - a_1b_4\epsilon_H - b_1a_4\epsilon_H, \quad Q_2 = b_1\chi - b_1b_4\epsilon_H + a_4a_1\epsilon_H;$

$C = \frac{a_1a_8P_3 + a_1b_8Q_3 - b_1a_8Q_3 + b_1b_8P_3}{a_8^2 + b_8^2}, \quad D = \frac{b_1a_8P_3 + b_1b_8Q_3 + a_1a_8Q_3 - a_1b_8P_3}{a_8^2 + b_8^2};$

$P_3 = a_1\chi^3 - 2a_1b_1\epsilon_H\chi^4, \quad Q_3 = b_1\chi^3 - b_1^2\epsilon_H\chi^4;$

$a_8 = a_1\chi^3(1 + R_H) - 2a_1b_1\epsilon_H\chi^4$, $b_8 = b_1\chi^3(1 + R_H) - b_1\epsilon_H\chi^4$

With the help of theorem of complex analysis (Ponnusami (2001)), we obtain real and imaginary parts of ξ_1, ξ_2, ξ_3 in the following manner:

$$Re[\xi_1] = \frac{\sqrt{a_5 + \sqrt{a_5^2 + b_5^2}}}{\sqrt{2}} + \frac{\epsilon_\theta}{2\sqrt{2}(\sqrt{a_5 + \sqrt{a_5^2 + b_5^2}})}[\chi^2 S + (Sa_5 + S'b_5)\frac{\chi^4}{(a_5^2 + b_5^2)}] \tag{4.2.47}$$

$$Im[\xi_1] = \frac{\sqrt{-a_5 + \sqrt{a_5^2 + b_5^2}}}{\sqrt{2}} + \frac{\epsilon_\theta}{2\sqrt{2}(\sqrt{-a_5 + \sqrt{a_5^2 + b_5^2}})}[-\chi^2 S + (-Sa_5 + S'b_5)\frac{\chi^4}{(a_5^2 + b_5^2)})] \tag{4.2.48}$$

where $S = [\frac{a_5(P_1a_6+Q_1b_6)-b_5(a_6Q_1-b_6P_1)}{a_6^2+b_6^2}]$,$S' = [\frac{b_5(P_1a_6+Q_1b_6)+a_5(a_6Q_1-b_6P_1)}{a_6^2+b_6^2}]$

Similarly,

$$Re[\xi_2] = \frac{\sqrt{a_4 + \sqrt{a_4^2 + b_4^2}}}{\sqrt{2}} + \frac{\epsilon_\theta}{2\sqrt{2}(\sqrt{a_4 + \sqrt{a_4^2 + b_4^2}})}[\chi^2 T + (Ta_4 + T'b_4)\frac{\chi^4}{(a_4^2 + b_4^2)})] \tag{4.2.49}$$

$$Im[\xi_2] = \frac{\sqrt{-a_4 + \sqrt{a_4^2 + b_4^2}}}{\sqrt{2}} + \frac{\epsilon_\theta}{2\sqrt{2}(\sqrt{-a_4 + \sqrt{a_4^2 + b_4^2}})}[-\chi^2 T + (-Ta_4 + T'b_4)\frac{\chi^4}{(a_4^2 + b_4^2)})] \tag{4.2.50}$$

where $T = [\frac{a_4(P_2a_7+Q_2b_7)-b_4(a_7Q_2-b_7P_2)}{a_7^2+b_7^2}]$,$T' = [\frac{b_4(P_2a_7+Q_2b_7)+a_4(a_7Q_2-b_7P_2)}{a_7^2+b_7^2}]$

$$Re[\xi_3] = \frac{\chi\sqrt{a_1 + \sqrt{a_1^2 + b_1^2}}}{\sqrt{2}} + \frac{\chi\epsilon_\theta}{2\sqrt{2}\sqrt{a_1 + \sqrt{a_1^2 + b_1^2}}}[C + \frac{Ca_1 + Db_1}{\sqrt{a_1^2 + b_1^2}}] \tag{4.2.51}$$

$$Im[\xi_3] = \frac{\chi\sqrt{-a_1+\sqrt{a_1^2+b_1^2}}}{\sqrt{2}} + \frac{\chi\epsilon_\theta}{2\sqrt{2}\sqrt{-a_1+\sqrt{a_1^2+b_1^2}}}[-C+\frac{-Ca_1+Db_1}{\sqrt{a_1^2+b_1^2}}] \tag{4.2.52}$$

From the solutions obtained as above, we can clearly observe that there are three modes of waves which are dissimilar to each other. We denote the first wave as modified (quasi-magneto) elastic (dilatational) wave, the second one as modified (quasi-magneto) thermal wave and the third one as (quasi-magneto) electrical wave. It is observed that both the elastic and thermal mode wave are influenced by the thermoelastic coupling constants ϵ_θ, magneto-elastic coupling constant ϵ_H, as well as magnetic pressure number R_H. We will specially concentrate on these two modes. Using equations [(4.2.47)-(4.2.52)] in the formulae given by [(4.2.41)-(4.2.43)], we can obtain the various components of magneto-thermoelastic plane wave like, phase velocity, specific loss and penetration depth for all three kinds of waves regarding GN-III theory of thermoelasticity. For all the waves considered here, we consider the cases when $Im(\xi_i) \leq 0$.

For special cases, if we assume $k_1 = 0$ in the above solutions then obtained solution is acknowledged by solutions for GN-I theory of thermoelasticity and similarly, if we substitute $k_2 = 0$ in above solutions, then we obtain the case of GN-II theory of thermoelasticity.

4.2.5 Numerical results

From the analytical results obtained above, we conclude that three various modes of waves have been extracted from the coupled dispersion relation (equation (4.2.25)) which are named as modified (quasi-magneto) elastic (dilatational) wave, modified (quasi-magneto) thermal wave and (quasi-magneto) electrical wave. In order to illustrate the analytical solution and to have a critical analysis of the nature of waves with the variation of frequency, we will now make an attempt to show the variations of various wave components of the identified waves with the help of numerical results. We will also assess the limiting behavior of wave components for very high and low frequency values.

We make an attempt to represent the plane wave characterizations numerically with the help of computational work by using programming on mathematica. Copper material has been chosen for the purpose of numerical evaluations. The physical data for our problem are taken as follows Ezzat (2004):

$\epsilon_\theta = 0.0168$

$\lambda = 7.76 \times 10^{10} Nm^{-2}$

$\mu = 3.86 \times 10^{10} Nm^{-2}$

$\mu_e = 4 * 10^{-7} Nms^2/C^2$

$\sigma = 5.7 * 10^7 Sm^{-1}$

$\omega^\star = 1.72 * 10^{11} sec^{-1}$

$\rho = 8954 Kgm^{-3}$

We assume the non-dimensional values of $k_2 = 1$.

We will specially analyze the physical parameters of waves like, phase velocity, specific loss and penetration depth. Using the formulae given by equations (4.2.40)-(4.2.42), we compute these components of waves of different modes and display our results in various Figures. Our analytical work is devoted to GN-III theory of magneto thermoelasticity for a finitely conducting medium. Furthermore, in order to make a comparison between GN-I, GN-II & GN-III models, we carry out our computational work for these two special cases too. Due to this reason, we have considered five different cases in each figure. These cases are differentiated as we have taken five values of the non dimensional thermal conductivity rate i.e. $k_1 = 0, 0.5, 1, 2, 3$. $k_1 = 0$ represents the case of GN-I model and other four values of

$k_1$represent the case of GN-III model of thermoelasticity. In each Figure, the thin solid line is used for $k_1 = 0$(GN-I), thick dotted line is for $k_1 = 0.5$, thin dashed line is for $k_1 = 1$, thick solid line is for $k_1 = 2$ and thick dashed line is used for $k_1 = 3$. We find that the trend of all the wave components of the third mode wave, i.e., electric mode wave are almost similar in the contexts of GN-I and GN-III model. However, a prominent difference in the results under GN-I model and GN-III model is indicated for the quasi-magneto elastic wave and quasi-magneto thermal mode wave. It is further observed that for the two cases when $k_1 = 0$ and $k_1 = 0.5$, i.e., when $k_1 < 1$, the nature of waves under GN-III model are very similar to the nature of waves under GN-I model. However, when the value of dimensionless thermal conductivity-rate is greater than 1, the nature of waves are changed i.e. the cases of $k_1 > 1$ give more prominently different results of GN-III theory of thermoelasticity as compared to GN-I model. Figures for the case of GN-II model have been presented separately. We highlight several specific features arisen out of the numerical results in the following sections:

4.2.5.1 Analysis of phase velocity

Using the formula given by (4.2.40), we compute the phase velocity of all three modes of waves. Figures 4.2.1 and 4.2.2 display the variation of phase velocity of modified (quasi-magneto) elastic (dilatational) wave for the high and low frequency values, respectively. The important observation from Figure 4.2.1 is that the nature of plots for the cases $k_1 = 0, 0.5$ are almost similar to each other, but other three plots are different from them. However, in other three cases there is a significant difference in variations of phase velocity of this mode, although surprisingly in all cases each figure goes towards a constant limiting value which is nearer to 1. It is further noted that for the first two cases (when $k_1 = 0, 0.5$), the phase velocity of magneto-elastic wave starts increasing from 0 value with the increase of frequency and after achieving a local maximum value starts decreasing and become constant value 1. But in the cases when $k_1 \geq 1$, phase velocity continuously increases to reach to its constant limiting value 1 without showing any local maxima. This is more clearly understandable from Figure 4.2.2. Hence from the above description, we can conclude that modified elastic wave behaves differently in the context of GN-III theory of thermoelasticity as compared to GN-I theory.

Figures 4.2.3 and 4.2.4 show the variations of modified (quasi-magneto) thermal wave in the contexts of GN-I and GN-III models for high and low frequencies, respectively. In the present context, waves are propagating from a constant limiting value greater than 0 when frequency tends to zero value and then starts increasing with respect to frequency and goes towards infinity. The differences of results under GN-I and GN-III is more significant for low frequency values (see Figure (4.2.4)) and as frequency increases the differences decrease. We observe that wave propagates with faster speed in GN-I case in comparison to GN-III model of thermoelasticity, although the case k_1 =0.5 of GN-III model shows very similar behaviour like the case of GN-I. Further we notice that in three cases when k_1 =0, 0.5 and 1, we locate a minimum value of speed of modified (quasi-magneto) thermal wave nearer to 1 for a critical value of frequency and thereafter speed of wave starts increasing with respect to frequency. When $k_1 > 1$,wave moves with increasing speed and goes towards infinity (without showing any local minimum value) with the increase of frequency.

Figures 4.2.5 and 4.2.6 present the behavior of phase velocity of (quasi-magneto) electrical wave in GN-I and GN-III models of thermoelasticity for high and low frequency, respectively. Here, phase velocity of waves increases rapidly with respect to frequency in both Figures. In the present case, wave is unaffected with the values of k_1 and the predictions of GN-I and GN-III model is almost similar in nature.

4.2.5.2 Analysis of specific loss

Specific loss is defined as the loss of energy per stress cycle as defined by formula (4.2.41). Figures 4.2.7 and 4.2.8 exhibit the variations of specific loss of modified (quasi-magneto) dilatational mode wave for high and low frequencies, respectively. In high frequency case, initially specific loss is 0 but it starts increasing with respect to frequency and giving a maximum value of specific loss it starts decreasing and goes towards 0 value. This maximum value is different for different values of $k_1$and this difference is more clear from Figure 4.2.8. It is further evident from Figure 4.2.8. that the maximum value of loss for GN-III model is less than that of GN-I model and results are very close in nature for $k_1 = 0$ and $k_1 = 0.5$. There is no prominent difference in results for small values of k_1, specially where k_1 is less than 1 but for the cases when the value of k_1 is greater than 1, difference among the results of specific loss in GN-I and GN-III is more prominent. Figures 4.2.9 and 4.2.10 depict the variation of specific loss of modified (quasi-magneto) thermal mode wave. Here, we obtain significant differences between GN-I and GN-III for both high and low frequency values, although the specific loss of magneto-thermal wave in all cases show a constant limiting value nearer to 4π, ultimately. However, the trends of variation of plots in various cases are different. When $k_1 = 0$ i.e. in case of GN-I model, specific loss suddenly increases and reaches to its constant value 4π but as thermal conductivity rate increases from the value 1, the specific loss slowly increases to reach to the same constant value 4π which is realistic in physical point of view. Differences among plots are more prominent in low frequency cases (see Figure 4.2.10). The specific loss of (quasi-magneto) electrical wave is observed to to be constant in all cases and it is unaffected with the values of thermal conductivity rate, k_1(see Figures 4.2.11, 4.2.12)

4.2.5.3 Analysis of penetration depth

The behaviour of penetration depth of modified (quasi-magneto) elastic dilatational wave can be seen from Figures 4.2.13, 4.2.14 which show that it decreases from infinite value as the frequency increases and reaches to constant limiting value 119. But we observe significant differences among the plots of this field for different thermal conductivity rate. We see that this difference is more prominent for lower frequency values, although in all cases, it finally reaches constant value 119. Further more, it is noted that their mode of variation with respect to frequency is different. When $k_1$is small, specially when it is less than 1, the trend of plots is almost similar. It starts decreasing from infinity as frequency increases but its mode of variation is not smooth when $k_1$is less than 1. While deceasing, penetration depth shows a corner point before it becomes constant and this peak value decreases as thermal conductivity rate increases. When $k_1>1$, depth decreases smoothly and becomes constant value 119 and no minimum peak value is observed.

From Figures 4.2.15 and 4.2.16, we obtain the behaviour of penetration depth of modified (quasi-magneto) thermal mode wave for high and low frequencies, respectively. Difference among plots for various values of $k_1$is not prominent for very high frequency values, but the trend of variation in all cases is decreasing from infinity to a very low constant value nearer to 0. When we concentrate on Figure 4.2.16 which shows the variation for low frequency cases, we note prominent disagreement of GN-I theory with GN-III theory. Although, this wave field shows a decreasing trend from infinity in all five cases but their mode of variation is different. In the plot of GN-I, behaviour of plot is not smooth for all frequency values and it gives a corner point for a critical value of frequency. For the case when $k_1 = 0.5$, the plot is similar in nature to that of GN-I. Here the mode of variation of wave is also not smooth and gives a corner point. However, in this case, the value of penetration depth at the corner point is higher than that in case of GN-I model. However, for the cases when $k_1 \geq 1$, the trend of variation of plots is smooth and decreases smoothly from infinity to a very less value nearer to 0.

Penetration depth of quasi-magneto electrical wave exhibits rapidly decreasing behaviour with respect to frequency and becomes constant with the finite value in both the GN-I and GN-III models of thermoelasticity (see Figures 4.2.17 and 4.2.18) and there is no significant differences in the results predicted by GN-I and GN-III models.

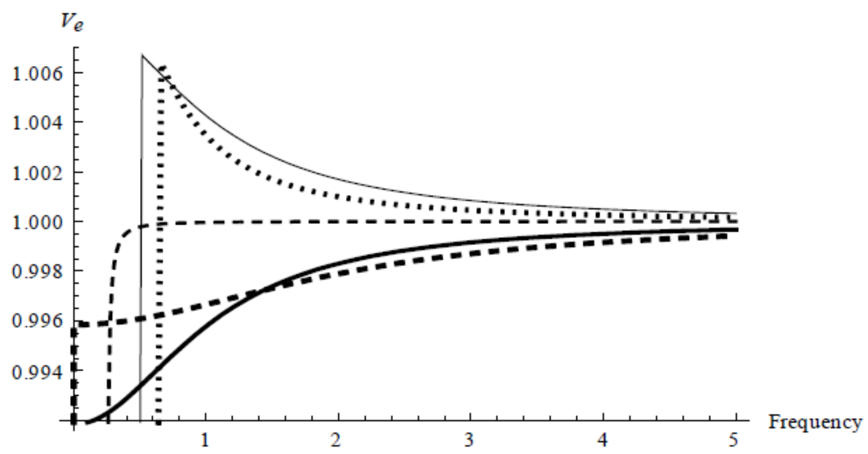

Figure 4.2.1 Phase velocity of quasi-magneto-dilatational wave (low frequency): Thin line: $k_1 = 0$ (GN-I), Thick dotted line: $k_1 = 0.5$, Thin dashed line: $k_1 = 1$, Thick line: $k_1 = 2$, Thick dashed line: $k_1 = 3$

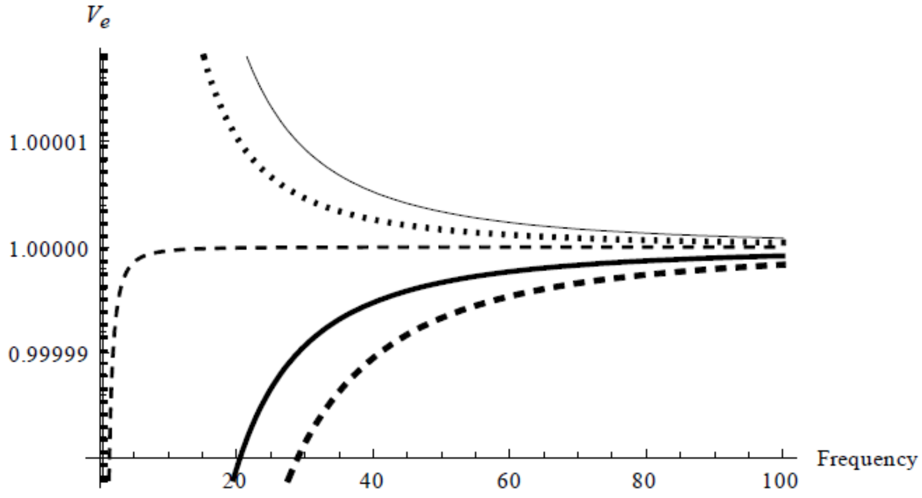

Figure 4.2.2 Phase velocity of quasi-magneto-dilatational wave (high frequency): Thin line: $k_1 = 0$ (GN-I), Thick dotted line: $k_1 = 0.5$, Thin dashed line: $k_1 = 1$, Thick line: $k_1 = 2$, Thick dashed line: $k_1 = 3$

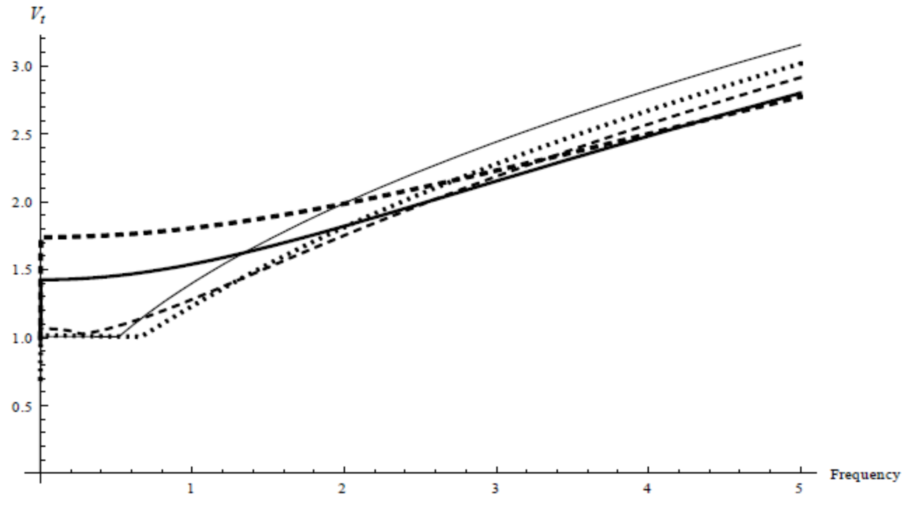

Figure 4.2.3 Phase velocity of quasi-magneto-thermal wave (low frequency): Thin line: $k_1 = 0$ (GN-I), Thick dotted line: $k_1 = 0.5$, Thin dashed line: $k_1 = 1$, Thick line: $k_1 = 2$, Thick dashed line: $k_1 = 3$

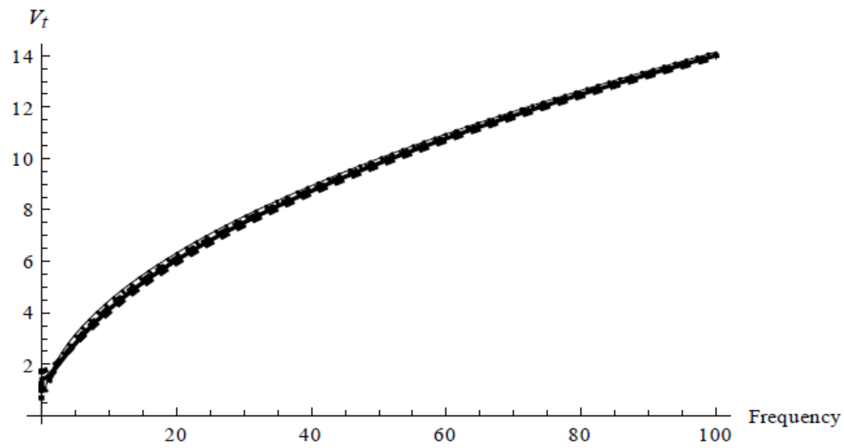

Figure 4.2.4 Phase velocity of quasi-magneto-thermal wave (high frequency): Thin line: $k_1 = 0$ (GN-I), Thick dotted line: $k_1 = 0.5$, Thin dashed line: $k_1 = 1$, Thick line: $k_1 = 2$, Thick dashed line: $k_1 = 3$

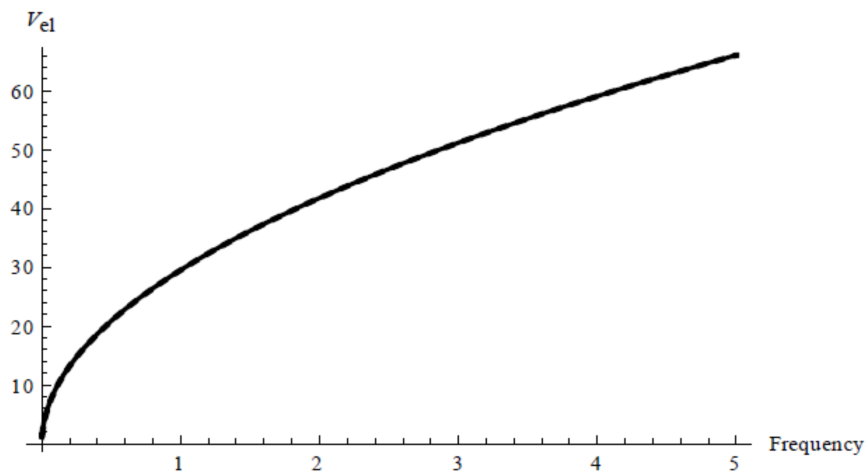

Figure 4.2.5 Phase velocity of quasi-magneto-electrical wave (low frequency): Thin line: $k_1 = 0$ (GN-I), Thick dotted line: $k_1 = 0.5$, Thin dashed line: $k_1 = 1$, Thick line: $k_1 = 2$, Thick dashed line: $k_1 = 3$

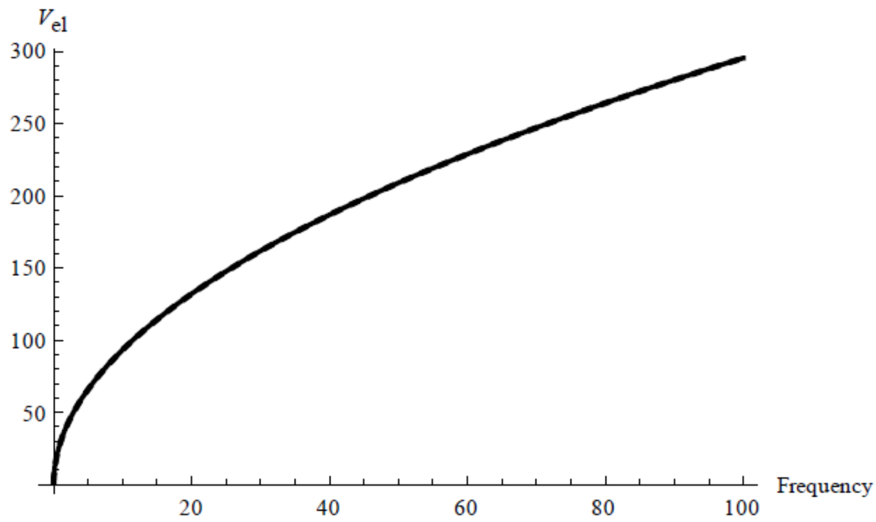

Figure 4.2.6 Phase velocity of quasi-magneto-electrical wave (high frequency): Thin line: $k_1 = 0$ (GN-I), Thick dotted line: $k_1 = 0.5$, Thin dashed line: $k_1 = 1$, Thick line: $k_1 = 2$, Thick dashed line: $k_1 = 3$

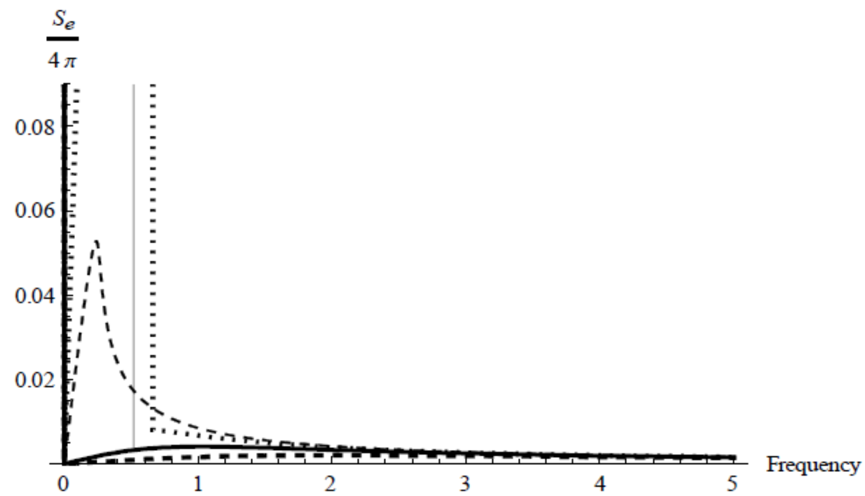

Figure 4.2.7 Specific loss of quasi-magneto-dilatational wave (low frequency): Thin line: $k_1 = 0$ (GN-I), Thick dotted line: $k_1 = 0.5$, Thin dashed line: $k_1 = 1$, Thick line: $k_1 = 2$, Thick dashed line: $k_1 = 3$

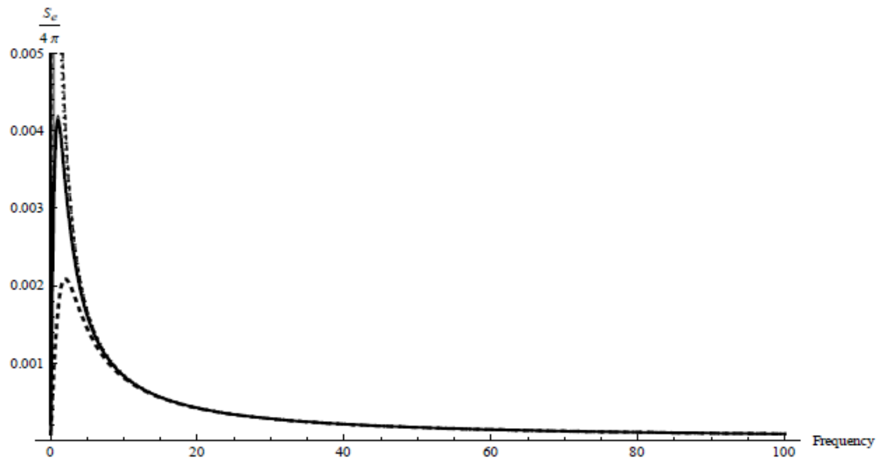

Figure 4.2.8 Specific loss of quasi-magneto-dilatational wave (high frequency): Thin line: $k_1 = 0$ (GN-I), Thick dotted line: $k_1 = 0.5$, Thin dashed line: $k_1 = 1$, Thick line: $k_1 = 2$, Thick dashed line: $k_1 = 3$

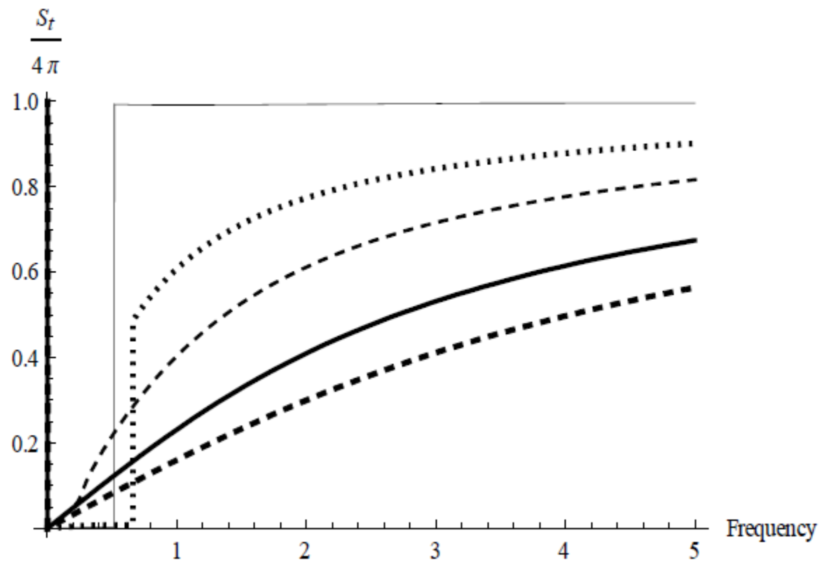

Figure 4.2.9 Specific loss of quasi-magneto-thermal wave (low frequency): Thin line: $k_1 = 0$ (GN-I), Thick dotted line: $k_1 = 0.5$, Thin dashed line: $k_1 = 1$, Thick line: $k_1 = 2$, Thick dashed line: $k_1 = 3$

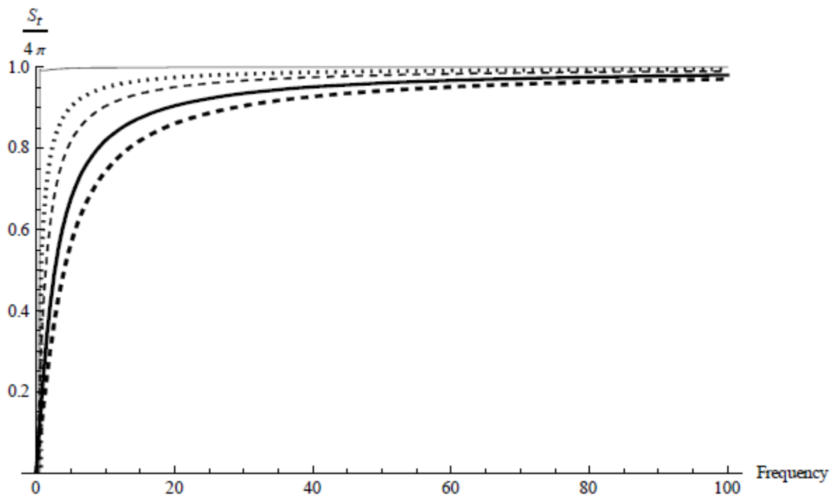

Figure 4.2.10 Specific loss of quasi-magneto-thermal wave (high frequency): Thin line: $k_1 = 0$ (GN-I), Thick dotted line: $k_1 = 0.5$, Thin dashed line: $k_1 = 1$, Thick line: $k_1 = 2$, Thick dashed line: $k_1 = 3$

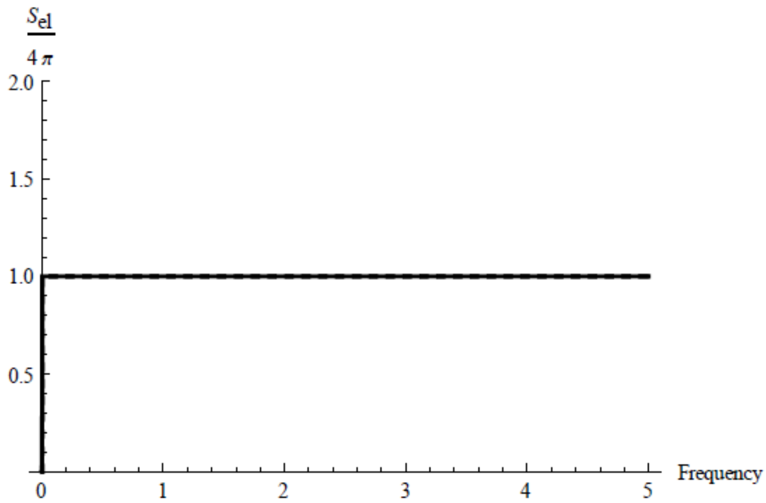

Fig.4.2.11 Specific loss of quasi-magneto-electrical wave (low frequency): Thin line: $k_1 = 0$ (GN-I), Thick dotted line: $k_1 = 0.5$, Thin dashed line: $k_1 = 1$, Thick line: $k_1 = 2$, Thick dashed line: $k_1 = 3$

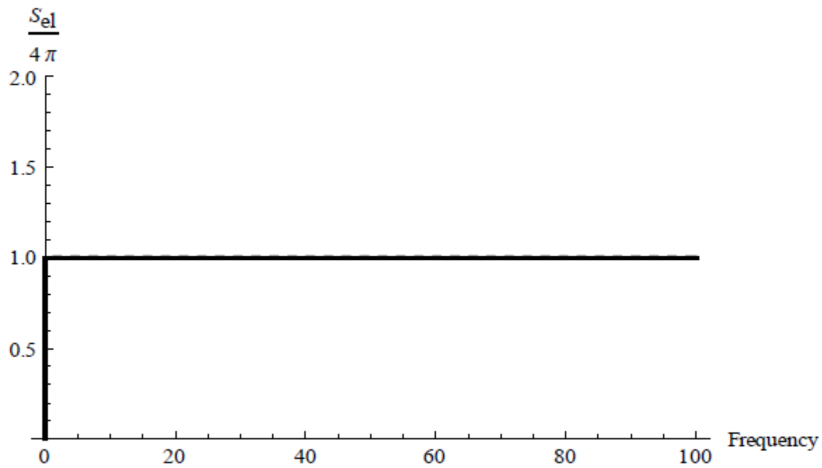

Fig.4.2.12 Specific loss of quasi-magneto-electrical wave (high frequency): Thin line: $k_1 = 0$ (GN-I), Thick dotted line: $k_1 = 0.5$, Thin dashed line: $k_1 = 1$, Thick line: $k_1 = 2$, Thick dashed line: $k_1 = 3$

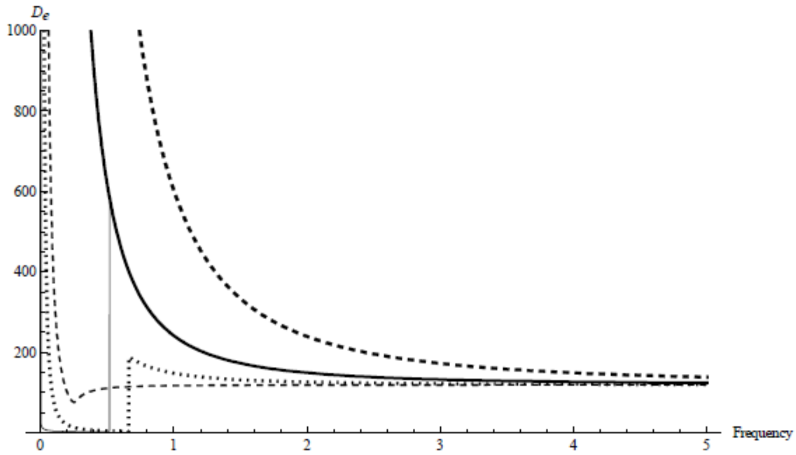

Figure 4.2.13 Penetration depth of quasi-magneto-dilatational wave (low frequency): Thin line: $k_1 = 0$ (GN-I), Thick dotted line: $k_1 = 0.5$, Thin dashed line: $k_1 = 1$, Thick line: $k_1 = 2$, Thick dashed line: $k_1 = 3$

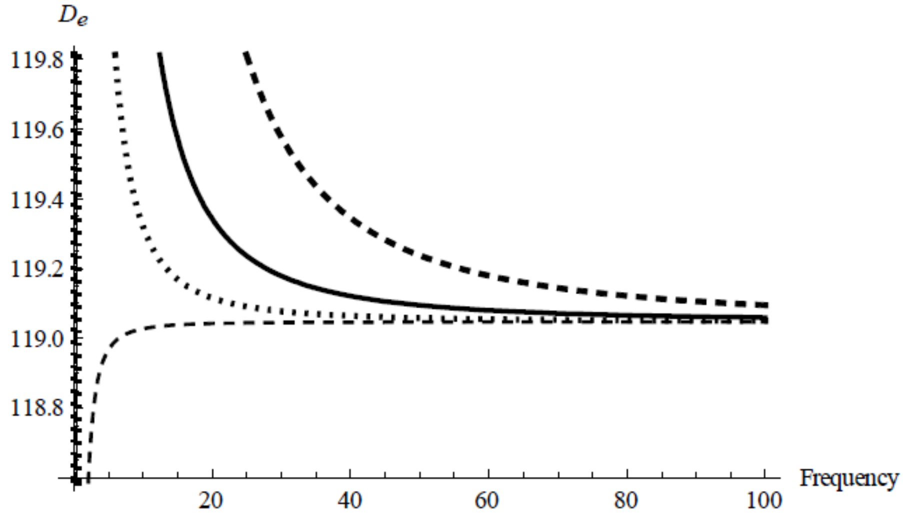

Figure 4.2.14 Penetration depth of quasi-magneto-dilatational wave (high frequency): Thin line: $k_1 = 0$ (GN-I), Thick dotted line: $k_1 = 0.5$, Thin dashed line: $k_1 = 1$, Thick line: $k_1 = 2$, Thick dashed line: $k_1 = 3$

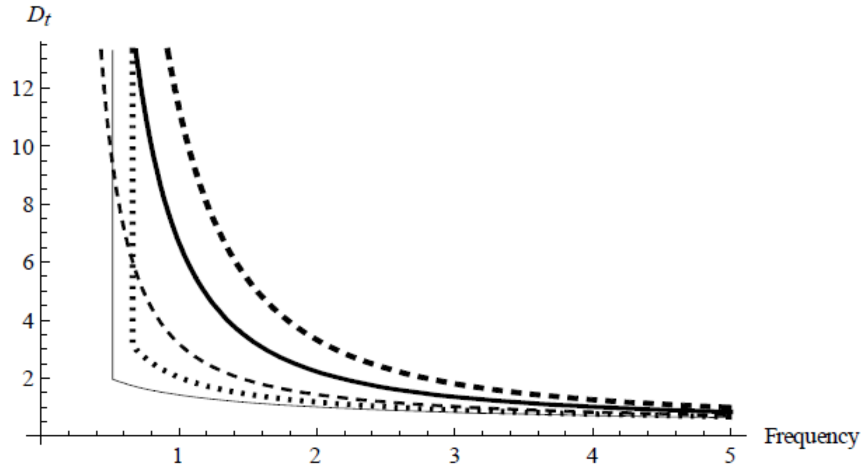

Figure 4.2.15 Penetration depth of quasi-magneto-thermal wave (low frequency): Thin line: $k_1 = 0$ (GN-I), Thick dotted line: $k_1 = 0.5$, Thin dashed line: $k_1 = 1$, Thick line: $k_1 = 2$, Thick dashed line: $k_1 = 3$

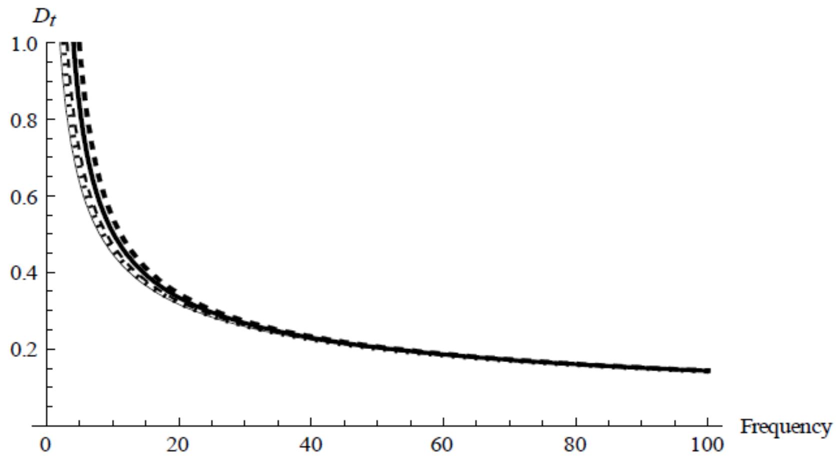

Figure 4.2.16 Penetration depth of quasi-magneto-thermal wave (high frequency): Thin line: $k_1 = 0$ (GN-I), Thick dotted line: $k_1 = 0.5$, Thin dashed line: $k_1 = 1$, Thick line: $k_1 = 2$, Thick dashed line: $k_1 = 3$

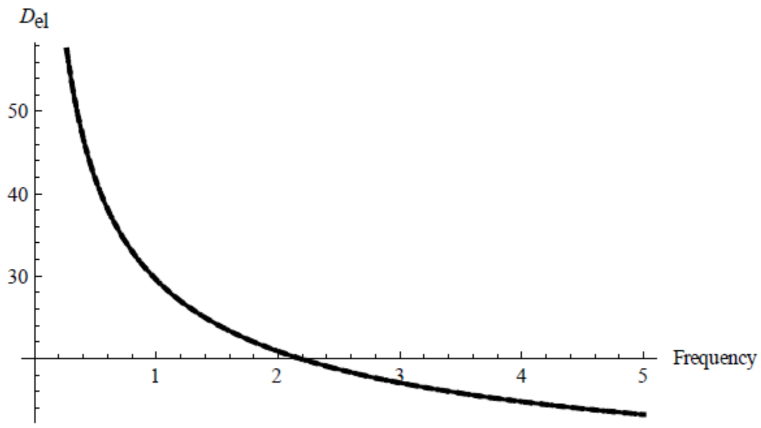

Figure 4.2.17 Penetration depth of quasi-magneto-electrical wave (low frequency): Thin line: $k_1 = 0$ (GN-I), Thick dotted line: $k_1 = 0.5$, Thin dashed line: $k_1 = 1$, Thick line: $k_1 = 2$, Thick dashed line: $k_1 = 3$

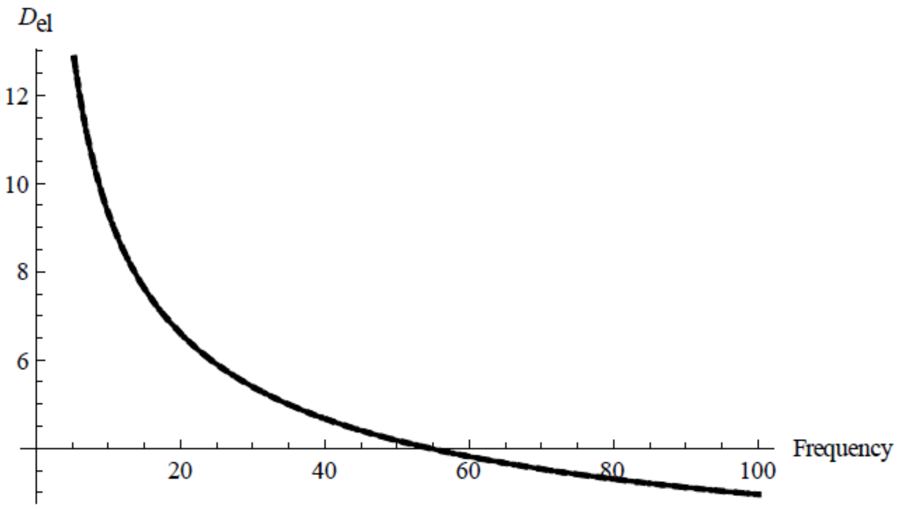

Figure 4.2.18 Penetration depth of quasi-magneto-electrical wave (high frequency): Thin line: $k_1 = 0$ (GN-I), Thick dotted line: $k_1 = 0.5$, Thin dashed line: $k_1 = 1$, Thick line: $k_1 = 2$, Thick dashed line: $k_1 = 3$

4.2.6 Special case: (GN-II model)

As we have already stated above that when $k_2 = 0$, we obtain the equations for Green and Naghdi-II (GN-II) model of magneto-thermoelasticity. We have presented the numerical results in context of this theory too. Figures 4.2.19 and 4.2.20 display the nature of propagation of modified (quasi-magneto) elastic dilatational wave and modified (quasi-magneto) thermal wave, respectively. In this case, we achieve completely different nature of thermal wave in comparison to the results predicted by two models GN-I and GN-III. Here we obtain that phase velocity of quasi-magneto dilatational wave as well as of quasi-magneto thermal mode wave show almost similar trend of variation and have constant limiting value. Phase-velocity of thermal wave is greater that the phase velocity of elastic mode wave and both of them reaches to constant value as we increase the frequency. We have already identified that the speed of modified (quasi magneto) thermal wave increases with the increases of frequency in cases of GN-I and GN-III models and goes towards infinity, but it is constant in GN-II model and this is the special feature of GN-II model of magneto-thermoelasticity that the speed of magneto-thermal wave is always finite in this case.

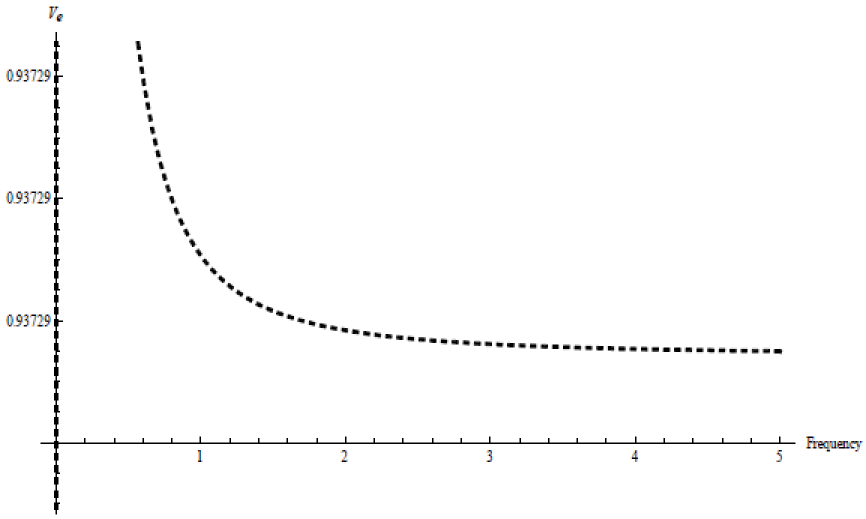

Figure 4.2.19 Phase velocity of Quasi-magneto-dilatational wave for GN-II

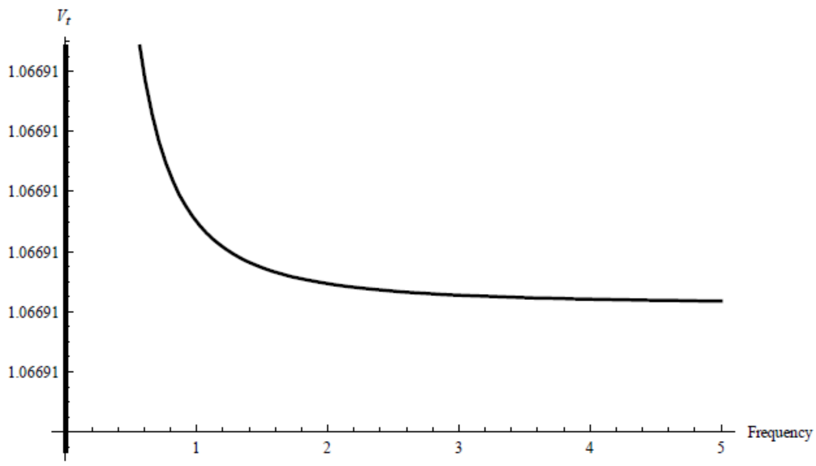

Figure 4.2.20 Phase velocity of Quasi-magneto-thermal wave for GN-II

4.2.7 Conclusions

In the present work, dispersion relation solutions for the plane wave propagating in a magneto-thermoelastic media with finite electrical conductivity have been determined by employing Green and Naghdi theory of thermoelasticity of type-III. We have made a comparative study of GN-I and GN-III theory of thermoelasticity in presence of an external magnetic field. From the derived dispersion relation solution, transverse and longitudinal plane waves are investigated. We find that transverse mode elastic wave is uncoupled from the thermal and magnetic field; Further a general dispersion relation associated to the coupled dilatational-thermal and electrical wave is identified and we make attempt to extract three different modes of waves from this coupled dispersion relation. These waves are identified as quasi-magneto-elastic wave (dilatational wave), quasi-magneto-thermal wave and quasi-magneto-electrical wave. The quasi-magneto-electrical wave is found to have similar variation under GN-I and GN-II theory. However, significant differences are obtained in other two modes, namely modified (quasi-magneto) elastic and modified (quasi-magneto) thermal mode wave predicted by three different models. Hence, we pay attention to these two modes and analyze various wave components like, phase velocity, specific loss and penetration depth. The behavior of the wave components in limiting cases of frequency values have been investigated with the help of graphical plots. Various features are highlighted. The results under GN-II theory of thermoelasticity have also been presented numerically as a special case. It is believed that this study would be useful due to its various applications in different areas of physics, geophysics etc.

The most highlighted features of the present investigation can be summarized as follows:

1. Significant resemblance and non- resemblance among the results under GN-I, GN-II and GN-III theory of thermoelasticity have been identified.

2. The phase velocity of thermal mode wave is found to be an increasing function of frequency under GN-I and GN-III models.

3. Quasi-magneto dilatational and thermal mode waves propagate faster in the theory of type GN-I in comparison to GN-III theory of thermoelasticity. However, phase velocity of quasi magneto-electric wave is unaffected whether we employ GN-I theory or GN-III theory. Quasi-magneto dilatational and thermal wave is found to be non-dispersive, i.e., propagate with constant speed and there is no significant variation on the phase velocity of both the modified elastic wave and modified thermal wave with respect to frequency in the context of GN-II theory of thermoelasiticity.

4. Penetration depth has a less finite value in the case of GN-I and GN-III theory of thermoelasticity. However in GN-II, we see that penetration depth for both waves namely, quasi-magneto dilatational wave and quasi-magneto thermal wave is infinite since waves are propagating with constant speed and there is no specific energy loss of waves. This is a very distinct feature of GN-II model.

5. In view of above points, we can conclude that for coupled magneto-thermoalstic problem, GN-II model exhibits realistic behaviour in comparison to GN-I and GN-III models w.r.t. phase velocity of thermal wave, but when we analyze the behaviour of penetration depth, we find that predictions of GN-I and GN-III theory is more realistic as compared to GN-II model as we obtain in this case

an infinite penetration depth which is also physically unrealistic prediction by GN-II model.

6. It is observed that when thermal conductivity rate, $k_1 < 1$, the plots of all wave fields in the context of GN-III theory shows much resemblance with the plots of GN-I. This implies that the results of GN-III model of thermoelasticity are more prominently different as compared to GN-I model when the thermal conductivity rate is greater than 1.