PREFACE

The subject "Coupled dynamical theory of thermoelasticity" deals with the thermal effects on elastic material. It removes the intrinsic drawback of classical uncoupled theory of thermoelasticity that the elastic changes are independent to the temperature and vice versa. This subject has important applications in various fields of science and engineering and during last few decades, it has progressed extensively through several research works carried out in this area and by applying the principles of mechanics and thermodynamics by eminent researchers. Some advancement has been made very recently by applying the concept of fractional calculus.

A mathematical model can be defined as a description of a system by using mathematical concepts and language. The process of developing a mathematical model is termed mathematical modeling. Mathematical models are of great importance in several areas like physics, biology, meteorology, computer science etc. The present thesis brings a light on different models of thermoelasticity and is aimed at constructing mathematical models of the various unsolved problems of wave propagation in homogeneous, elastic and magneto-thermo-elastic solids. The advanced thermoelastic models like, fractional order thermoelastic model as well as the latest model using memory dependent derivative have been employed to solve some problems concerning mutual interactions between thermal and mechanical fields. For some problems, presence of magnetic field has also been been considered and mutual interactions among thermal, mechanical and magnetic field have been taken into account.

This thesis contains six chapters, out of which the **first chapter** chapter begins with the brief introduction on the development of various theories of thermoelasticity. The recent developement of the coupled thermoelasticity as been illaborated and a detailed literature survey relevant with the present context has been provided at the end of the first chapter. **The second chapter** has been divided into two different sections and both sections are devoted to the fractional order thermoelasticity:

The first section is concerned with the boundary integral equation formulation for the solutions of equations under fractional order thermoelasticity in a three dimensional Euclidean space. A mixed type initial boundary value problem is assumed and the fundamental solutions of the corresponding coupled differential equations are obtained in the Laplace transform domain. Morever one reciprocal relation is established and the boundary integral equations are formulated on the basis of our fundamental solutions. The formulation is illustrated with a suitable example.

The second section of this chapeter presents an investigation on the propagation of harmonic plane wave propagating with assigned frequency by implementing the thermoelasticity theory based on fractional order heat conduction law where the fractional order parameter α satisfies $(0 < \alpha \leq 1)$. After formulating the problem, the exact dispersion relation solutions for the plane wave are determined analytically. Asymptotic expressions of different characterization of the longitudinal waves are analyzed by considering two special cases, namely for high frequency and low frequency values. A detailed analysis is presented to highlight the effects of fractional order parameter, α on the nature of the wave. We consider the case of longitudinal wave which is coupled with the thermal field and we ignore the transverse wave as it is observed to be independent to the thermal field. Two different modes: thermal and elastic mode longitudinal waves are found. Finally we compute various wave characterizations for the intermediate values of frequency and verify our analytical results for the limiting cases of wave frequency. A detailed analysis is presented to highlight the effects of fractional order parameter, α on the wave fields. Several important points are highlighted. The most important point which we have found is that in the case of thermal wave, when α goes beyond $1/2$, the nature of wave changes significantly and as α gets the value nearer to 1 it behaves more similarly like the generalized thermoelastic model with one thermal relaxation parameter.

The third chapter provides an attempt to derive a model of thermoelasticity by applying the methodology of memory dependent time derivatives. Specially, a new concept of ''memory dependent'' derivative has been employed to define a problem of wave propagation in a homogeneous, isotropic and unbounded solid due to a continuous line heat source. Both Laplace and Hankel transform method have been employed for the solution of the problem. Analytical results for the distributions of different fields like temperature, displacement and stresses inside the medium have been derived. The problem is illustrated by computing the numerical values of the field variables for a particular material. We have attempted to exhibit the significance of the kernel function and time-delay parameter that are characteristcis of memory dependent derivative thermoelastic model. The effects in the behavior of temperature, displacement and stresses are analyzed with the help of numerical results. Some comparisons have been made through the numerical results to estimate the effects of the kernals and time-delay parameter on each field variable.

The fourth chapter addresses magneto-thermoelasticity theory and this chapter has been divided in two sections to study two different problems. **The first section** of this chapeter analyzes the propagation of one-dimensional electro-magneto-thermoelastic plane waves in an unbounded isotropic thermally and electrically conducting media with finite conductivity. We employ Green-Naghdi theory of thermoelasticity of type-II, i.e., the theory of thermoelasticity without energy dissipation. Heat conduction equation is affected with Thomson coefficient. Basic governing equations are modified by employing Green-Naghdi theory of type-II and mathematical modeling of the problem derives two different systems. The first system is found to be coupled with the thermal field and represents the longitudinal wave. However, the second system represents transverse wave that is uncoupled with the thermal field. Both waves are observed to be affected with the magnetic field. Asymptotic expansions of dispersion relation solutions and various components of plane waves like, phase velocity, specific loss and penetration depth are derived for high and low frequency values. Analytical results are verified with the numerical results for the limiting behavior of longitudinal and transverse waves. The results of present study are compared with the corresponding results of thermoelastic case and a detailed analysis of the effects of presence of the magnetic field under this theory has been presented.

The second section of the fourth chapter is related with the propagation of electro-magneto-thermoelastic plane waves of assigned frequency in a homogeneous isotropic and finitely conducting elastic medium permeated by a primary uniform external magnetic field when we formulate our problem under the theory of Green and Naghdi of type-III (GN-III) to account for the interactions between the elastic, thermal as well as magnetic fields. A general dispersion relation for coupled waves is deduced to ascertain the nature of waves propagating through the medium. Perturbation technique has been employed to obtain the solution of dispersion relation for small thermoelastic coupling parameter and identify three different types of waves. We specially analyze the nature of the wave components like, phase velocity, specific loss and penetration depth of all three modes of waves. We attempt to compute these wave components numerically to observe their variations with frequency. The effect of presence of magnetic field is analyzed. The results under theories of type GN-I and II have also been exhibited as a special cases in which we have found that the coupled thermoelastic waves are un-attenuated and non-dispersive in case of Green-Naghdi model of type-II. The results in this case are observed to be conpletely in contrast with the theories of type-I and type-III. Some specific features of type-III model are highlighted. We achieve significant variations among the results predicted by all three theories.

The fifth chapter is devoted to the propagation of magneto-thermoelastic disturbances produced by a thermal shock in a finitely conducting elastic half-space in contact with vacuum. A normal load has been applied on the boundary of the existing media that is supposed to be permeated by a primary uniform magnetic field. We employ dual phase-lag heat conduction theory and consider both the parabolic type (dual phase-lag magneto-thermoelasticity of type-I (MTDPL-I)) and hyperbolic type (dual phase-lag magneto-thermoelasticity of type-II (MTDPL-II)) dual phase-lag heat conduction models to account for the interactions among the magnetic, elastic and thermal fields. Integral transform technique is applied to solve the present problem and the analytical results of both the cases have been obtained separately. A detailed analysis of results has been performed in order to understand the nature of waves propagating inside the medium and the effects of the phase-lag parameters. The presence of magnetic field has been highlighted. Numerical results have also been obtained to analyze the effects of magnetic field on the behaviour of the solution more clearly and a detailed analysis of the results predicted by two models has been presented. It has been noted that in some cases, there are significant differences in the solution obtained in the contexts of MTDPL-I and MTDPL-II theories of magneto-thermoelasticity.

The last chapter (chapter-6) incorporates the summary of the present work as well as scope for future work in the relevant area.

The list of references, appeared in the thesis are appended at the end of the thesis.