

### CONCLUSIONS AND FUTURE WORK

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#### 7.1 Conclusions

The thesis provides several significant contributions to the analysis and modeling of guided elastic waves through wavelet based multiscale method, which lays the foundation for using wavelets for the simulation of waves for structural health monitoring. Here we highlight the most significant aspects of these contributions.

We presented a wavelet-based framework to reduce the size of finite element stiffness matrix which is becoming too large in the case of nonlinear wave propagation problem. Although the tone burst signals have very localized nature yet they require very fine grid in the whole domain to simulate higher harmonics. Without disturbing higher harmonics, the proposed method is able to reduce the computational cost significantly. In the thesis, B-splines and Daubechies wavelets are used, but other wavelets can also be used. Some attenuation of the waves are not affecting the higher harmonics, so it will not disturb the analysis. This approach has many simple and advantageous features drawing from both, conventional finite element and wavelet methods. Without disturbing the programming advantages of FEM regarding the implementation of boundary conditions and efficient numerical integration of interpolation functions, wavelet based methods are able to reduce the size of the matrix as much as one by sixteenth of original FE matrix. These fundamental characteristics show that the wavelet based method can be utilized for more complex wave propagation problems.

We have extended the proposed technique with NS wavelet transform to enhance the computational performance for the simulation of wave propagation. The localization properties and multilevel structure of the wavelets in the physical space with the NS operator are proved to be an efficient tool in developing the adaptive computational approach. The present adaptive grid algorithm, while capturing full generality, is independent of domain dimension. As a concluding remarks, inherent hierarchical and adaptive nature of wavelets possess several appealing advantages in computational modeling. Wavelet methods are well suited for multiscale problems occurring in several areas of mathematical physics.

Accuracy, reliability, and computational cost are crucial issues of any efficient error estimator to design adaptive algorithms for the numerical solution of partial differential equations. The wavelet based *a posteriori* error indicator is proposed to identify the location of grid refinement at a low computational cost to achieve error bound solution for engineering problems. In this approach, the wavelet transform is used for error indicators that are easier and much simpler to implement than those available in the literature. Furthermore, it is locally efficient and can be used in complex engineering analysis with any finite element subspaces. Comparisons of the proposed estimator with exact error and two scale error with the help of numerical experiments show that it is as good as other error estimators. The key feature of the method is an automatic algorithm for any finite element grid structure. In situations where it might be impossible to directly verify the hypothesis, the findings of this study facilitate future researchers to advance the error estimation techniques.

The task of signal processing and analyzing the Lamb wave propagation/scattering in any thin-walled structures for detection, localization and visualization of damage is

really challenging from both numerical and experimental viewpoints. In order to identify highly localized discontinuities, the present thesis considers the suppression of white Gaussian noise along with coherent noise in the transient Lamb wave signals as received from the SHM and NDT&E application. In this analysis, the performance of three different filtering methods, namely wavelet transform method, matched filter technique and wavelet matched filter method are tested on FE simulated signals. Initially, this study distinctly explores the effectiveness of mean square error based and Shannon entropy-based criterion for choosing wavelet function and the decomposition level, appropriately. Using the optimal wavelet function, the contaminated signal is processed via the wavelet transform where acceptable responses are not received for low SNR cases. It should be noted that the performance of the soft-thresholding is better to reduce unwanted noise interference than the hard thresholding. In identical conditions when the signals are processed through MFT, there is sufficient improvement in the final output as well as it is easy to detect the damage. Some further significant enhancement in the output signal is received when the WMFM is used. This makes the technique flexible and attractive for practical usage.

## **7.2 Future Research Directions**

The work provides several opportunities for continuing research in the area of numerical modeling and analysis. We developed and demonstrated the validity of several concepts of wavelets for adaptive solution schemes, error estimation, as well as signal processing. While our results are promising, many of these concepts are yet to be incorporated into full scale analyses of practical engineering problems. The wavelet

extrapolation approach leads to the intriguing possibility of discretizing the temporal dimension at multiple scales. Again, this is an area for further investigation.

A general methodology is demonstrated to incorporate wavelet based *a posteriori* error estimates which gives confidence in the validity of design updates on approximation models. Further work is necessary to apply the proposed wavelet based tools to practical, large-scale problems. In the area of error estimation and mesh adaptation for functional outputs, more work is necessary to study the quality of discretization error estimates in the presence of nonlinearity. Although finite element discretizations of linear parabolic and elliptic systems have been shown to form consistent approximations, the same is not known for nonlinear problems relevant to many applications. This includes the selection of the most suitable wavelets for complex geometries with the large class of practical applications.

The WMFM which is better than MFT and wavelet technique for filtering various types of noise is developed and discussed in chapter 6. It could not be used on field data due to nonavailability of experimental setup for testing highly complex structure (such as bridges, dam, etc.) under running conditions. Denoising of SHM signal from such structures using WMFM is the future goal.