CHAPTER 2

LITERATURE REVIEW

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The analysis of elastic wave propagation in thin wall structure gained the attention of researchers in late 1910s. The wave propagation and scattering analysis in structures are the major concern for performance evaluation of civil, mechanical and aerospace engineering structures subjected to aerodynamic and mechanical loadings. The excellent literature surveys by Su *et al.* (2006), Raghavan and Cesnik (2007), Willberg *et al.* (2015), and Mitra and Gopalakrishnan (2016) present a deep insight into the history and development of the analytical, numerical and experimental techniques for the guided wave based SHM and NDT&E applications.

The chapter presents, the state of art review of literature available on the elastic wave, Lamb wave and Rayleigh wave propagation in various structure geometries subjected to various types of loading and boundary conditions. In this chapter, care has been taken to include all the literature available while some of the early work remains unlisted due to unavailability and can be found in the aforementioned references. The review is divided into four sections. The first part presents early developments and fundamentals background of guided waves. Analysis and simulation of elastic waves in the plate and shell structure (analytical, numerical and multiscale analysis) are presented in next part. The third part consists of signal processing and data analysis of Lamb wave (time domain, frequency domain and integrated time and frequency domain analysis). The fourth section describes the error estimation techniques for wave propagation and practical engineering problems. At the end of the chapter, the observations based on the literature review and objectives of the present work are presented.

2.1 Early Developments and Fundamentals

A Large amount of work related to the analysis of elastic wave in the plate and shell-like structures is available in the literature. There are many application areas for elastic waves such as materials inspection, material characterization, seismology etc., and therefore they have been a subject of study (Graff (1975), Achenbach (1984), and Auld (1990)). The first work addressing elastic wave, Rayleigh wave equations, was developed by Lord Rayleigh (1887). Based on this study, Horace Lamb predicted mathematically and described the existence of plate waves now known as Lamb wave. The difference between Rayleigh waves and Lamb waves are their propagation characteristic because of the boundary conditions. Rayleigh waves propagate and transmit close to the free surface of elastic solids; whereas Lamb waves propagate between two free surfaces of a thin-walled structure. The development of Rayleigh and Lamb waves was motivated essentially by its applications in medical industry during World War II. Further, this study was extended by Love (1926), Stoneley (1924), Scholte (1942) and observed various types of waves which are now called as Love, Stoneley and Scholte waves. A comprehensive theory for the guided elastic wave was established by Mindlin (1951). The experimental work for such waves was demonstrated by Frederick (1962). Gazis (1958, 1959) developed and demonstrated the dispersion equations for elastic waves in cylinders. Worlton (1961) was the first person who recognize potential of Lamb waves for NDT and introduced guided Lamb waves as a means of flaw detection. All these fundamental studies established the basics for utilization of the elastic waves for damage identification.

2.1.1 Parameters of Lamb waves for damage detection

Propagation characteristics of guided waves in thin-walled structures are well defined in the literature from Mindlin (1960), Viktorov (1967), and Rose (1999). As mentioned in chapter 1, Lamb waves have various complications which come up in the received signals because of the existence of multimode vibration (symmetric and anti-symmetric) and dispersive behavior of waves. Complications may also be due to the mode conversions that happen if a propagating wave mode interacts with several discontinuities and boundaries (Alleyne (1991), Rose *et al.* (1994), Cawley and Alleyne (1996), Benmeddour *et al.* (2008), and Soleimanpour and Ng (2016)). Very frequently, following parameters of Lamb waves are considered for damage identification:

2.1.1.1 Amplitudes of the waves

The measured response of time domain signals from the healthy structure is used as baseline or reference signal and Lamb wave signal received from unhealthy structure compared with baseline signal for damage detection. To identify damage, many Lamb wave based inspection method employ wave attenuation, dispersion and/or mode conversion (Toyama *et al.* (2003), Benmeddour *et al.* (2008), and Staszewski *et al.* (2009)). Generally, the amplitude of guided Lamb waves is utilized to investigate their interactions with discontinuities (Toyama *et al.* (2003). Reflection and transmission power coefficients for the anti-symmetric and symmetric mode of Lamb wave have been computed by researchers (Cawley *et al.* (2003), Benmeddour *et al.* (2008)). Several researchers have calculated attenuation coefficient of Lamb waves experimentally in composite materials to detect damage and delamination. They identified delamination in Carbon Fiber-Reinforced Polymer (CFRP) laminates and sandwich composites on the basis of attenuation of the transmitted Lamb wave and established that this technique is a reliable and simple to detect damage (Tan *et al.* (1995), Birt (1998), Kessler *et al.* (2002), Prasad *et al.* (2004)). A linear relationship between the attenuation of the fundamental symmetric wave mode and the degree of the impact damage has also been obtained by Birt (1998). It has been observed that antisymmetric Lamb modes have relatively high attenuation property compared with symmetric wave modes (Wilcox *et al.* (2005), Konstantinidis *et al.* (2006), and Kim and Sohn (2007)). In general, it is very complicated to quantitatively evaluate the delamination size (Toyama and Takatsubo (2004)).

2.1.1.2 Phase velocity and group velocity

Lamb waves are dispersive, and their velocities are dependent on plate thickness and wave frequency. This phenomenon is known as dispersion. The plot of phase and group velocities with respect to excitation frequency is referred as dispersion curve and used to predict the relationship among frequency, thickness, phase velocity and group velocity. The basic method to express the propagation of a guided wave in a particular material is with their dispersion curves (Kessler (2002)). Tang and Henneke (1989), and Cawley (1990) might be the initial researchers who have employed the velocimetric method to identify damage. The variation in wave velocities due to change in material properties or geometry is tracked to estimate the flaws and discontinuities. Nayfeh and Chimenti (1989) presented dispersion relations of Lamb waves in a composite lamina. Yuan and Hsieh (1998) found the exact solutions of dispersion of guided waves in composite shells. Mazeika *et al.* (2010) have presented the comparison of modeling and experimental results of the phase velocity measurement of Lamb wave in the isotropic plate. Neau (2003) employed both group velocity dispersions and wave curves in a composite lamina and compared them with experiments. Many researchers utilized guided Lamb wave for the characterization of materials and their usage to reveal mechanical properties of plate like structures (Vollmann *et al.* (1998), and Bermes (2007)). Most of the guided wave based damage identification methods calculate arrival time (or time-of-flight) of scattering waves from discontinuities, then by recognizing the group velocity, the position of damage can be determined.

2.2 Analysis and Simulation of Lamb Wave

Modeling of wave damage interaction can provide an efficient and cost-effective approach to assist the Lamb wave analysis and the development of damage detection algorithms before conducting time-consuming experiments (Rose (1999), Staszewski *et al.* (2004), Giurgiutiu (2008), and Boller *et al.* (2009)). Simulation is also helpful in investigating complex strategies that are difficult to set up experimentally. Fundamental and vital features of Lamb waves such as dispersions and wave mode shapes can be acquired using modeling and simulation. Furthermore, wave propagations and scattering in realistic structures with complicated geometries and characterization of different types of damage can be easily simulated.

This section reviews the state-of-the-art in numerical analysis of guided wave propagation and scattering. A concise introduction to wave mechanics problem is presented. A detailed assessment of capability for simulating elastic wave phenomena of the following methods are compiled: (i) analytical analysis, (ii) numerical analysis (wavelet analysis, miscellaneous methods such as Local Interaction Simulation Approach (LISA), Mass-Spring Lattice Models (MSLMs), Spectral Cell Method (SCM), fictitious domain methods and Boundary Element Methods (BEMs)).

2.2.1 Analytical analysis

The application of analytical scheme is very suitable when simulating guided Lamb waves. Their evaluation is less expensive than numerical techniques. Analytical techniques provide the wave response of the structure without using any type of domain discretization. The analytical methods are based on closed-form solutions of Lamb wave characteristic equations. They are most convenient to solve the dispersion relations for simple geometry structures, such as thin plates and cylinders. The characteristic equations and solutions for simple geometry have been fully developed as given in many articles (Viktorov (1967), Graff (1975), Lowe (1995), Rose (1999), and Giurgiutiu (2008)).

The early development of the analytical studies concerning Lamb waves date back to 1911, when Love extended the work of Lord Rayleigh (1987) and first modeled the horizontally polarized waves, i.e., shear-horizontal waves. Later on, Horace Lamb studied waves in an elastic plate in 1917. The main motivation behind his pioneering work was associated to seismic problems. Lamb established the theoretical fundamental principles to model elastic waves. In the abovementioned works, only a single frequency was studied to present the dispersion relations between the wave number and the excitation frequency. Onoe (1955) has analyzed the qualitative behavior of the Lamb wave dispersion curves with propagating Lamb modes. Thereafter, Mindlin (1960) presented a comprehensive analysis for the dispersion equation for each Lamb wave mode. Viktorov (1967) and Achenbach (1984) solved the problem of forced motion in the two-dimensional case and also calculated dispersion relation very accurately. All these analyses are for two-dimensional and plane strain problem. Graff (1975) presented a three-dimensional analysis of circular crested waves.

Later on, the Fourier transform and the Cauchy's theorem have been employed for analysis of wave response in general three-dimensional plates with different load distributions (Gomilko *et al.* (1991), Shi *et al.* (2003), Raghavan and Cesnik (2005), Giurgiutiu (2005, 2008), Von *et al.* (2007), and Von and Lammering (2007, 2009)). Modal analysis has also been employed to present analytical solution (Jin (2003)). Higher order plate theories have also been utilized in order to compute a good approximation of the dispersion equation (Yang and Yuan (2005)).

Major analytical tools for the purpose of dispersion relation for composite laminate are transfer matrix method (Thomson–Haskell method), global matrix approach (Lowe (1995)), and some other more specific methods, such as effective elastic constant method which is based on homogenization (Habeger *et al.* (1979)). Nayfeh (1991, 1995) has given dispersion relations for generally anisotropic layered composites. During the last few years, global matrix method is widely used and this technique remains numerically robust for any range of frequency-thickness values but has a relatively slow convergence. Many other specific analytical techniques for the propagation characteristics of guided Lamb waves in the composite structure are comprehensively reported by Chien *et al.* (1994), and Ghoshal (2002).

The modeling of elastic waves in shell structures such as pipes, cylinder, curved geometry have been studied. Wave propagation in plates with curved geometry have been studied by Towfighi *et al.* (2002), Demma (2005), Gridin and Craster (2004), Ratassepp and Klauson (2006), and Vivar-Perez *et al.* (2014). Different boundary conditions and body forces have been considered in shell structures (Harris (2002), Gridin and Craster (2004), and Vivar-Perez (2012)). Barshinger and Rose (2004) explored the Lamb wave propagation in an elastic hollow cylinder coated with a viscoelastic material. The Wiener-

Hopf technique and other analytical methods have been employed for some specific scattering problems (Rokhlin (1981), McKeon and Hinders (1999), and Diligent *et al.* (2002)).

These analytical methods have many advantages such as (i) the influence of each wave mode can be analyzed individually, (ii) the qualitative performance of guided wave propagation can be expressed easily by analytical expressions, and (iii) infinite wave numbers for a given frequency can be calculated. However, the major limitation of this method is that it is applicable for simple geometries and specific distributions of load only. There have been several attempts to explain the propagation of Lamb waves in complex geometries using analytical methods (Harris (2002), Towfighi *et al.* (2002), Gridin and Craster (2004), and Ratassepp and Klauson (2006)). However, analytical methods are still incompetent to simulate arbitrary complex domain. Therefore, to overcome the intrinsic limitation of analytical methods, several techniques have been developed and applied to simulate specific aspects of elastic waves propagation with the variety of problems. The development of semi analytical approaches for analysis of the wave field moves toward hybrid formulation, which is based on the coupling of analytical and numerical technique. Another approach to solve this problem is a numerical method. The main advantage of this method is that the difficulties associated with complicated geometries and interactions of damage are easy to handle.

2.2.2 Numerical analysis

The elastic, acoustic and electromagnetic waves equations, which express the propagation and scattering, are hyperbolic Partial Differential Equations (PDEs). The Finite difference (Delsanto (1992), boundary element (Tadeu *et al.* (2007)), and finite element (Kuhlemeyer (1973), Jianga (1995), and Bathe (1996)) based methods have been used for the simulation of guided Lamb waves. Majority of the past research, which was conducted during the 90's was based on transient analysis in the time domain and was dedicated to the study of the basic physics associated with the excitation and propagation of guided waves in plate like structures (Alleyne and Cawley (1992), Verdict *et al.* (1992), Balasubramanyam *et al.* (1996), Zgonc and Achenbach (1996), and Alleyne *et al.* (1998)). More recently, numerical simulations have been used for (1) handling complicated guided wave propagation and interaction problems, (2) the characterization of different types of damage, (3) nonlinear guided wave propagation, and (4) study of the behavior of piezoelectric transducers.

A popular numerical method used to simulate wave propagation problem is the Finite Element Method (FEM) (Reddy (1985), Burnett (1987), Zienkiewicz and Taylor (1989), and Cook *et al.* (2001)). The FEM is employed in the time or frequency domain and the method is being continuously improved. Fromme (2001) has investigated numerically and experimentally the scattering of the fundamental antisymmetric wave mode at a circular hole in isotropic plates. Lowe and Diligent (2002) presented the interaction between the fundamental Lamb modes with defects in an isotropic homogenous plate using two-dimensional FE models. Gresil *et al.* (2012) studied the influence of corrosion using Lamb waves. Greve *et al.* (2008) presented the transition from Lamb waves to longitudinal waves in thick panels, numerically and experimentally. Bijudas *et al.* (2013) explored numerical and experimental studies of damage detection in a stiffened plate by time-reversed Lamb waves.

Nowadays, FEM-based modelling and simulation are the most cost effective with commercial software such as ABAQUS, ANSYS, COMSOL, and PATRAN. The commercial softwares have many advantages, such as (i) user graphics interface, (ii) several element types in the element library, (iii) meshing tools with mesh size control, (iv) computationally efficient matrix solvers, and (v) effective post-processing and analysis tools, etc. Guo and Cawley (1992), Alleyne and Cawley (1992), Guo and Cawley (1993), Percival and Birt (1997), Birt (1998), Luo (2000), Olson *et al.* (2007), Diamanti *et al.* (2007), Chang *et al.* (2007), Soni *et al.* (2009), Song *et al.* (2009), Ahmad *et al.* (2009) Pistone *et al.* (2013), and Liu *et al.* (2013) used the FEA software for the Lamb wave simulation. They were able to obtain detailed waveforms, and visualize Lamb waves using post-processing tools. For the quality of the simulation, at least 15–20 nodes per wavelength are usually recommended for the proper solution accuracy (Gresil *et al.* (2012)). Vanli and Jung (2013) explored h-version FEs in conjunction with statistical updating techniques to enhance the damage prediction capability.

FEM has also been extensively employed to study wave propagation in laminated composites because the analytical solution of such problems is not possible due to the anisotropy, layered structure, etc (Yang *et al.* (2006)). More recently, research in SHM and NDT&E field is more focused on 3-D modeling and simulation of composite and sandwich structures (Mustapha *et al.* (2011), Hosseini *et al.* (2013), Hosseini and Gabbert (2013)). Luchinsky *et al.* (2013) investigated impact damage in sandwich panels using Lamb wave scattering. Maio *et al.* (2015) presented the comparative study of Lamb wave modeling of the laminated composite by means of classical plate theory using explicit FEA code and software ABAQUS, and simulation was validated with experimental study. Sause *et al.* (2013) presented the FEM model for guided wave propagation in cylindrical composite pressure vessels. Zhao *et al.* (2014) applied a new third-order shear deformation theory for dispersion curves of the guided wave in the laminated composite. Similar dispersion curves were also computed by using ABAQUS (Zhao, (2013)).

Another promising numerical technique is the Semi-Analytical Finite Element (SAFE) method that has also been cited as spectral or waveguide FEM (Bartoli *et al.* (2006), Maess *et al.* (2007), Predoi *et al.* (2007), and Fan (2010)). SAFE is suitable for handling the variation of the material properties in the thickness direction of a thin structure. It holds the merits known from analytical approaches. For the structure with a uniform arbitrary domain, the SAFE method simply needs the finite element discretization of the domain and takes harmonic motion along the wave propagation direction. Thus, the problem can be changed into an eigenvalue problem which can be easily solved. Fan and Lowe (2009) employed SAFE to study the guided wave dispersion problem of a welded joint structure. Explicit time-integration has been employed by Bartoli *et al.* (2005) to solve dispersion relation and simulate the propagation of Lamb waves in railroad tracks. The SAFE method has also been considered to study the dispersion relation for 1-D and 2-D periodical structures that have varying cross sections (Ruzzene *et al.* (2003), Maess *et al.* (2007)).

The Finite Difference Method (FDM) has also been used for the study of wave simulation and damage interaction by several researchers (Delsanto (1992), Delsanto (1994), Moulin (2006)). FDM is generally based on Taylor expansions and its direct substitution into the governing differential equations of wave motion. In this numerical approach, the field variables are defined at the nodal intersections of the structured grid. FDM uses second or higher order approximations to space derivatives. FDM based techniques are computationally expensive for large models and complex domain. An overview of FDM based techniques for wave propagation is presented by Bond (1990).

The major limitation of the FD schemes is that stiffness jumps due to continuously changing physical properties which causes stability problems (Yu (2013)). Furthermore, boundaries as well as discontinuities between different types of media lead to inaccurate solutions and generate severe errors (Delsanto (1992)). With this in mind, more recently, Delsanto has proposed the LISA in combination with the Sharp Interface Model (SIM) (Delsanto (1992), Delsanto (1994), Moulin (2006)). While the approach is formally similar to the FDM scheme, it does not utilize directly any finite difference equations but the associated iteration equations are acquired directly from heuristic considerations. The LISA is extremely efficient from a numerical point of view and precise to model complex wave phenomena and diffusion problems in complex media (Delsanto (1997), Lee and Staszewski (2002)). The algorithm was next extended to simulate Lamb wave interaction with a rectangular silt in an isotropic plate and identify discontinuities in the orthotropic plate (Lee and Staszewski (2007) Sundararaman and Adams (2008), Obenchain and Cesnik (2014)). Packo *et al.* (2012) has utilized the parallelizability of the LISA for Lamb wave simulation in complex structures and significantly reduced computational time using a graphical processing unit and a computer unified device architecture. Kluska *et al.* (2012) have employed cellular automata to enhance the precision of the geometrical description. LISA has also been employed as an authentication tool to verify several experimental techniques and their finding (Mallet *et al.* (2004), Staszewski *et al.* (2004)).

Other established numerical methods have also been utilized by several researchers to model the complex propagation behavior of Lamb wave and their interaction with damage such as the Elastodynamic Finite Integration Technique (EFIT) (Schubert *et al.* (1998), Schubert (2004)), Boundary Element Method (BEM) (Cho and Rose (1996), Pérez-Gavilán and Aliabadi (2000), Rose *et al.* (2000) Fedelinski (2004)), combination of FEM and BEM (Galán and Abascal (2005, 2005)), Scaled Boundary Finite Element Method (SBFEM) (Chen *et al.* (2012)), SCM (Joulaian (2014)), Finite Cell Method (FCM) (Vivar-Perez *et al.* (2014)) and enriched FEM (Han and Bathe (2012)).

Adamou and Craster (2004), Selezov et al. (2018) have used spectral methods due to the high accuracy and less computational cost for the simulation of wave propagation in elastic media. Mitra and Gopalakrishnan proposed wavelet-based spectral method for simulation of elastic wave propagation in one dimensional and two dimensional (Mitra and Gopalakrishnan (2006)). They utilized the compact support and orthogonal property of wavelets in the Galerkin method for the simulation of waves. However, discretization of the domain is necessary for complex geometry. Application of wavelets in such problems will be a challenging task but it will be very useful.

In recent years, wavelet-based numerical methods gained attention for solving partial differential equations. The major advantage of this approach is that it allows one to examine a problem in multiscale. In addition, wavelet based schemes are efficient in a problem comprising singularities and sharp transitions in solutions in limited zones of a computational domain. The detail descriptions of this approach will be presented in detail in Chapter 3 of the thesis.

2.3 Processing of Lamb Wave Signal

Guided wave-based damage identification process is dependent on signals captured by a sensor or sensor network. The Lamb wave responses obtained from the sensors are often contaminated due to the presence of random noise, multiple reflections from the damage as well as boundaries and interference from natural structural vibration.

The accuracy and precision of guided wave based damage detection technique are essentially subject to the processing and interpretation of the captured signals. Therefore, a signal processing technique should be able to extract the signal feature from the measured response to distinguish between the healthy and damaged structure. Several time domain analysis, frequency analysis, and integrated time-frequency analysis have been introduced for Lamb wave based SHM and NDT&E.

2.3.1 Time domain analysis

The direct time domain analysis of a signal has been used by many researchers to identify damage both globally as well as locally. The responses of the structure received from sensors have been analyzed using different time series modeling approaches. The time series method could be but not limited to Moving Average (MA), Regression, Autoregressive Integrated Moving Average (ARIMA) and Autoregressive (AR). Valdes and Soutis (2001) have investigated the delamination in a composite beam by computing the Time of Flight (ToF) in the received guided wave signal. Sohn *et al.* (2000) have presented the use of control charts to locate the damage on a concrete column by means of AR model as a feature extraction method. Variations in the AR coefficients were analyzed to predict whether the signals are coming from the defective or healthy structure. Kullaa (2003) has investigated the working bridges to monitor the integrity of the structure based on the modal parameters.

Zang *et al.* (2004) have demonstrated a hybrid approach to capture the essential features from measured vibration signals. Their damage detection approach was based on the coupling independent component analysis in the time domain and artificial neural networks. Omenzetter *et al.* (2006) have investigated damage using changes in the coefficients of ARIMA based on time series analysis. Wang *et al.* (2009) have demonstrated weighted moving average control charts based on time series approach to detect damage from numerically simulated case study data. Except for a small number of successful applications in identifying damage, time-series analysis is normally incompetent of properly separating defect-scattered information from noise. In addition, a reference signal is compulsory for comparison.

2.3.2 Frequency domain

To reveal discontinuities induced by structural damage, which may not be clearly observed in the time domain, the Lamb wave signal is often transformed into frequency domain via Fourier Transform (FT). It gives the information regarding the frequency of the waves and their corresponding strengths. Fast Fourier transform (FFT) and its 2D form (2D-FFT) are generally used for guided wave signal analysis (Heller (2000), Veidt *et al.* (2002), Koh *et al.* (2002), and Youbi *et al.* (2004)). Some researchers (Alleyne and Cawley (1991), Gao *et al.* (2003)) have successfully implemented this approach for separating multi-mode Lamb waves. Since both 2D-FT and 2D-FFT require a considerable volume of signals received from various locations, a large number of sensors must be ascertained to scan the whole structural surface. This technique does not deliver information associating the location of that frequency.

2.3.3 Integrated time frequency domain analysis

To overcome the shortcoming of either time or frequency domain analysis of the guided wave signal, the combination of time and frequency domains is introduced to avoid any potential loss of information contained in a Lamb wave signal. The integrated time-frequency domain analysis is illustrated by the Short Time Fourier Transform (STFT), Wigner-Ville Distribution (WVD), Hilbert Transform (HT), Hilbert-Huang Transforms (HHT) and wavelet transform.

In 1946, Dennis Gabor presented STFT, which is the simplest form of time-frequency analysis. STFT has been used to obtain the frequency and spatial information of a wave. With its facility to simultaneously unveil features of the time and frequency of a signal, it is well suited for analyzing non-stationary signals, such as guided wave signals (Ihn and Chang (2004, 2004), Kim and Kim (2001), Sung *et al.* (2000), and Chang *et al.* (2007)). Kim and Kim (2001) have implemented this approach to detect damage in a structural beam. However, the major drawback of STFT is that the time window is same for all frequencies therefore satisfactory precision cannot be received at the time- and frequency-axes simultaneously. Hence STFT may not be the optimal choice for analyzing Lamb signals. This deficiency can be overcome by using other transforms such as HT, HHT, and wavelet transform.

With a flexible choice of window size, the WVD transform can provide a superior resolution (Wang and McFadden (1993), Prosser *et al.* (1999), Niethammer *et al.* (2001), Kim and Kim (2001), Ge *et al.* (2002)). The effectiveness of WVD in the processing of guided waves was reviewed and discussed with other time-frequency representations, namely, the reassigned spectrogram, the smoothed WVD, Hilbert spectrum and HHT (Niethammer *et al.* (2001), Oseguda *et al.* (2003), Quek *et al.* (2003), Salvino *et al.* (2005), Wang *et al.* (2011), Chun *et al.* (2014), Pai *et al.* (2015)). Although each method was found to have its strengths and weaknesses, it was concluded that reassigned spectrogram emerged as the superior technique for analyzing Lamb wave mode. In general, a WVD transform requires a high computational cost. For avoiding mathematical aliasing, very high sampling rate, at least four times as high as the number of the sampling points of the signal is required (Wang (2001)). Additionally, WVD may lose sensitivity when applied directly to noisy wave signals. Furthermore, alterations of a short duration or low magnitude in a wave signal may not be observed properly.

In the last two decades, the wavelet transform has emerged as a signal-processing tool for denoising, filtering, data compression, feature extraction and feature selection. In general, CWT is mainly efficient for analysis and visualization of Lamb wave signal, while discrete wavelet transform is more effective for signal denoising and data compression. The descriptions of this approach and limitations will be discussed in detail in Chapter 6 of this thesis. Other signal processing techniques that can be employed for studying Lamb wave signals are Matching Pursuit (MP) algorithms (Mallet and Zhang (1993)), Monte-Carlo based MP decomposition (Das *et al.* (2009)), blind deconvolution (Zheng (2001)), cyclostationary processes (Rubini and Sidahmed (1997)), or statistical modelling (Martin *et al.* (1990)) and probability analysis (Achenbach *et al.* (1997)). However, there is a lack of suitable filter for processing of low SNR signal and suppression of coherent noise in Lamb wave propagation problems.

2.4 Error Estimation

This section gives a brief review of a posteriori error estimators. Computational approximations of numerical problems always contain some numerical errors. A mathematical theory for estimating such errors has become very important to computational science. The first use of error estimates and adaptive meshing approaches for computational solutions started some 30 years ago (Babuška and Rheinboldt (1978, 1978, 1979, 1979)). A concise history of the subject is given in the book by Ainsworth and Oden (2011). Also, the books by Oden and Demkowicz (1989), Verfurth (1996), and

Babuška and Strouboulis (2001) present an excellent overview of the methods developed in the late nineties.

Most of the *a posteriori* error estimators are based on energy norms. *A posteriori* error estimates can be classified into residual type (explicit and implicit) and recovery-type; which are presented in detail by Demkowicz et al. (1984, 1985), Bank (1985, 1986). Recovery based error estimations are based on so-called recovered gradient fields. Brauchli and Oden (1971) have proposed L2-Projection (L2-PR) technique and computed the recovered stress field by a global least square fitting of the stresses. A less specific procedure was introduced by Zienkiewicz and Zhu (1987, 1992, 1992), who proposed simple recovery based error estimator algorithm. Later, the accuracy of the method was significantly improved by modifying the recovery method known as super convergent patch recovery (SPR) method (Zienkiewicz and Zhu (1992, 1992)). Many researchers have utilized similar recovery schemes based on interpolation error estimates (Peraire *et al.* (1987), Warren *et al.* (1991), Castro-Diaz *et al.* (1997), Habashi *et al.* (2000)). The, recovery based techniques have many limitations. Ainsworth and Oden (2000) demonstrated a second-order ODE case where the error estimate is zero while the actual error can be arbitrarily large.

Another commonly used error estimator is element residual method, which is computed by substituting the approximate result into the underlying partial differential equation. The residual methods can be classified into explicit and implicit. Explicit residual schemes do not involve solving any auxiliary problems. They only require direct computations using available data. While, implicit methods demand the solution of local or global problems, using the residuals indirectly. A residual method in error estimation has been demonstrated and applied by Demkowicz *et al.* (1984), Irimie and Bouillard

(2001) have used an explicit residual method to compute error estimates in the context of the Helmholtz equation. Bank (1986), and Bank and Weiser (1985) also used residual methods. Zhang *et al.* (2001) have demonstrated adaptive results using discretization error and residual indicators for the Euler equations. Stewart and Hughes (1996, 1997) have introduced residual error estimators for the classical Galerkin as well as Galerkin least-squares finite element methods. Babuška *et al.* (1997) have also presented a one-dimensional analysis of a Dirichlet element error estimator for the Helmholtz problem.

However, both recovery and residual based error estimator have the significant drawback such as: (1) estimators are not adequate for two dimensions, (2) they estimate some global solution error over the entire domain, and (3) highly complex in the analysis. Hence, another class of methods based on duality techniques have been developed to compute error estimate in terms of quantities of interest, directly. Such algorithms are known as goal-oriented error estimation schemes. In output based or goal-oriented error estimation, practitioners specify the goal of their computations by directly identifying a quantity of interest, where this quantity of interest is manifested themselves as functionals on the solutions of boundary- and initial-value problems. Gartland (1984) has proposed such schemes for one-dimensional elliptic problems, but the expansion of a general setting with two and three-dimensional applications came later. These methods have been demonstrated by several authors (Becker and Rannacher (1996, 2001), Paraschivoiu (1997, 1998, 2000), Prudhomme and Oden (1999), Oden and Prudhomme (2001), Strouboulis and Babuška (2000), Suli and Houston (2003), and Rabizadeh *et al.* (2015, 2016)). These methods are quite complex and often not suitable for the engineering problem. Simple efficient and accurate error estimator is still to be developed.

2.5 Observations Based on The Literature Review

The main observations on the basis of the literature review are presented herein.

- From the literature review on guided wave SHM, one can see that NDT&E using Lamb wave has a lot of potential and has several advantages such as excellent propagation capability, high sensitivity to damage, and excellent performance in providing high precision damage detection in thin walled structures. While the relevant guided wave theory for NDE is fairly developed, the Lamb waves are not in a mature stage particularly for the understanding of wave damage interaction and there has been only some preliminary progress regarding their use. Thus, there is a need for further investigation.
- Several analytical and semi-analytical methods have been proposed mainly for the computation of dispersion curves and the wave propagation behavior in the frequency domain. Analytical models are commonly used for isotropic structures in simple configurations, while semi-analytical approaches are widely considered for laminates. Due to their inadequacy to simulate more realistic structural configurations, the use of these methods is limited to structures with relatively simple geometries. There have been some efforts directed towards the application of these methods in the modeling of wave propagation in the complex geometries. There is considerable scope for guided wave based quantitative damage detection for the engineering practices.
- An immense number of numerical schemes exist for the identification and location of damage, which includes finite element, spectral element, finite difference, LISA, spectral cell, finite cell and other methods. These methods have been

presented in the context of wave propagation and their interaction with discontinuities. Depending on the intended application, each particular technique has its advantages and disadvantages. The major obstacle has been the high mesh density and small time step requirements for analyzing Lamb wave propagation. The primary factors that can be defining the suitability of a numerical technique are the computational cost along with convergence properties and efficiency to handle complexities. Intensive investigation and improvement are matters for further research.

- ❖ Due to simplicity, linear analysis of Lamb wave for damage detection has got more attention than the nonlinear analysis. The nonlinear Lamb wave shows sensitivity to micro/nano scale damage and also has great potential for characterization of material nonlinearity. Nonlinear and higher harmonics Lamb wave analysis is required in order to assess the phenomenon associated with the characteristics of damage.
- ❖ Residual type and recovery type *a posteriori* error estimators are reported in general to analyze the solution accuracy obtained from numerical scheme. The model uncertainty arises from the use of coarse discretization of the computational domain. These error estimation techniques are very complex to compute the error particularly for two dimensional domain. To ensure the reliability of computation, development of an *a posteriori* error estimation technique with the capability to deal complex geometry at involving lower computational cost will be very useful.
- ❖ The majority of the reviewed literature shows the significant difficulties and challenges in signal processing of wave signals for the purpose of damage identification and evaluation. Several feature extraction and signal processing techniques have been developed such as STFT, WVD, HHT, and wavelet

transform. Most of them have not considered coherent noise and very low SNR signals. Therefore, a genuine requirement of proper filtering technique which can resolve the above issues, should be developed.

2.6 Research Context

The first task of the thesis work is to present the wavelet based framework to the simulation of linear and nonlinear wave propagation problems. The second task of the work is to present wavelet-based a posteriori error estimation scheme to estimate the error in the wave propagation problems and practical engineering problems. The third task of the work is to present wavelet matched filter method to extract signal features that can be correlated to the change of structural status.