CHAPTER 1

INTRODUCTION AND THEORETICAL BACKGROUND

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Structural Health Monitoring (SHM) refers to the process of implementing nondestructive autonomous *in situ* damage detection in engineering structures to evaluate the condition of existing structures to ensure the safety of users. The changes in the geometric and/or structural properties affect structural dynamic response, which can be reviewed and analyzed for condition assessment. SHM provides safety of users because of the capability of early detection of damage. This chapter serves as the introduction to the field of SHM, starting with necessary background and basic concepts. It is followed by guided wave based SHM. Sec. 1.2. provides theory and mathematical formulation of elastic wave propagation along with excitation techniques of Lamb waves. The motivation for this research problem is presented in Sec. 1.3. The contribution of this research is summarized in Sec. 1.4. Finally, Sec. 1.5 provides the organization of the remaining chapters.

1.1 Structural Health Monitoring

Routine maintenance of large structures, like storage tanks, bridges, dams, aerospace structures in service usually requires implementation of Non-Destructive Testing and Evaluation (NDT&E) techniques. Conventional assessment is based on either visual inspection or one of the NDT&E techniques developed in the 70s or earlier such as eddy current, radiography, dye penetrants, magnetic flux, and bulk wave ultrasound etc. The overview of most commonly used NDT&E techniques and their characteristics are presented in Table 1.1. NDT&E testing and monitoring are generally carried out off-line where actuators and sensors are temporarily mounted on these structures to interrogate local defects at discrete points which are the very time consuming process. In addition, there are many structural members which are not easily accessible and where conventional NDT&E methods are not practicable. SHM is an emerging new approach for damage detection compared to NDT&E. SHM systems provide state of the structure based on the fact that damage changes the electromechanical or mechanical properties, geometric parameters and boundary conditions of the structures. SHM provides the potentiality of global structural monitoring if actuators and sensors are distributed. For SHM, actuators and sensors are permanently coupled on or embedded in the specimen so that online *in situ* inspection is possible. The main perceptible features of the SHM over NDT&E are shown in Figure 1.1.

Table 1.1 An overview of the most commonly employed NDT&E techniques

Technique	Merits	Demerits	Ref.	Structure accessibility
Visual inspection	Inexpensive equipment, Simple procedure, Simple to implement.	Only surface defects, Only large areas, No data analysis.	(Razi (1996), Worden and Dulieu (2004))	Required
Ultrasonics	Portable, Sensitive to small damage, Quick scan of large area.	Very expensive equipment, Complex results, Specialized system for operation.	(Worden and Dulieu (2004))	Required
Electro-mechanical impedance	Low cost and simple for implementation, Effective for detecting damage in planner structures.	Unable to detect defects distant from sensors, Not highly accurate.	(Dalton *et al.* (2001), Boller (2013))	Not required

Technique	Merits	Demerits	Ref.	Structure accessibility
Infrared thermography	Compact, lightweight, and easy to use, Provides visual picture of the condition.	Prone to corrupt by environmental noise, Relatively insensitive to small damage.	(Worden and Dulieu (2004), Park et al. (2010))	Required
Acoustic emission	Real time monitoring , Applied to structures with limited access, Covers long distances.	Emissions can be very weak, Sometimes hard to detect due to background noise.	(Worden and Dulieu (2004), Farrar and Worden (2007))	Not required
Acoustic-ultrasonics	Easy to use, Sensitive to small defects.	Complex signal, High damping ratio of the wave.	(Chang et al. (2003), Ou and Li (2010))	Not required
Eddy current methods	Simple to implement, Do not require expensive equipment, Detection of damage and corrosion.	Require large amount of power, Complicated data to interpret, Extensive calibration.	(Worden and Dulieu (2004), Sun et al. (1992), Stalenhoef and Raad (2000))	Required
Radiography	Capable of internal damage detection , Permanent record of results. Simple procedure.	Expensive equipment , Expensive to implement , Time consuming.	(Worden and Dulieu (2004), Sazonov et al. (2003)), Ong et al. (1994)	Required
Electrical conductivity	Simple to implement , Low cost.	Require a lot of electrodes.	Sohn et al. (2003)	Not required
Structural vibrations and acoustics	Simple and low cost, Effective for detecting large damage.	Insensitive to small damage, Difficult to excite high frequencies.	(Staszewski et al. (2009), Diamanti and Soutis (2010))	Not required
Magnetic-particle testing	Quick and relatively uncomplicated, Sensitive to surface defects.	Restricted to ferromagnetic materials, Spurious, or non-relevant indications.	(Hamshaw (1996), Fritzen and Kraemer (2009)), Plotnikov (2002)	Required

There are two approaches for SHM techniques. In the first technique, signal received in the active or passive sensor is analyzed to notice some undesirable frequency or other changes whereas in the second technique the signal is compared to historical records to identify the structural condition based on the change of measurements over times. SHM system can monitor the structure in active or passive techniques. An active method uses sensors to interrogate the structure to detect damage. The structure is excited by actuators and response of the structure is measured by sensors. However, the passive method actuators are not required and the state of the structure is received using passive sensors (loading, environmental conditions, acoustic emission from cracks, etc.) that are supervised at the different time and fed back into a structural system. Passive methods only “listen” to the structure but do not interact with it.

Normally, the diagnosis of the damage state involves the following four aspects: (1) the detection of damage, (2) damage localization, (3) damage extent and severity analysis, and (4) the remaining service life of the structure. In order to satisfy the objective of the SHM system, the process can be broken down into four parts (Farrar and Worden (2007)):

(1) Operational evaluation -this step considers the economic and life-safety case, operating environment, limitation on data acquisition, and expected damage modes.

(2) Data acquisition and cleansing - the acquired signal data needs to be gathered from the sensors hardware using a data acquisition system and any necessary data cleansing and normalization must be considered in this step.

(3) Feature extraction and data compression - certain features may have to be extracted

from the signals in the signal processing package for distinguishing damaged and undamaged data.

(4) Statistical model development - a predictive model can be used to provide estimates of the possible structural defects with quantifiable confidence. This step also considers enhancement of the design and optimization of the SHM hardware, e.g. location of the actuators and sensors array. The main components of the SHM system are shown in Figure 1.2.

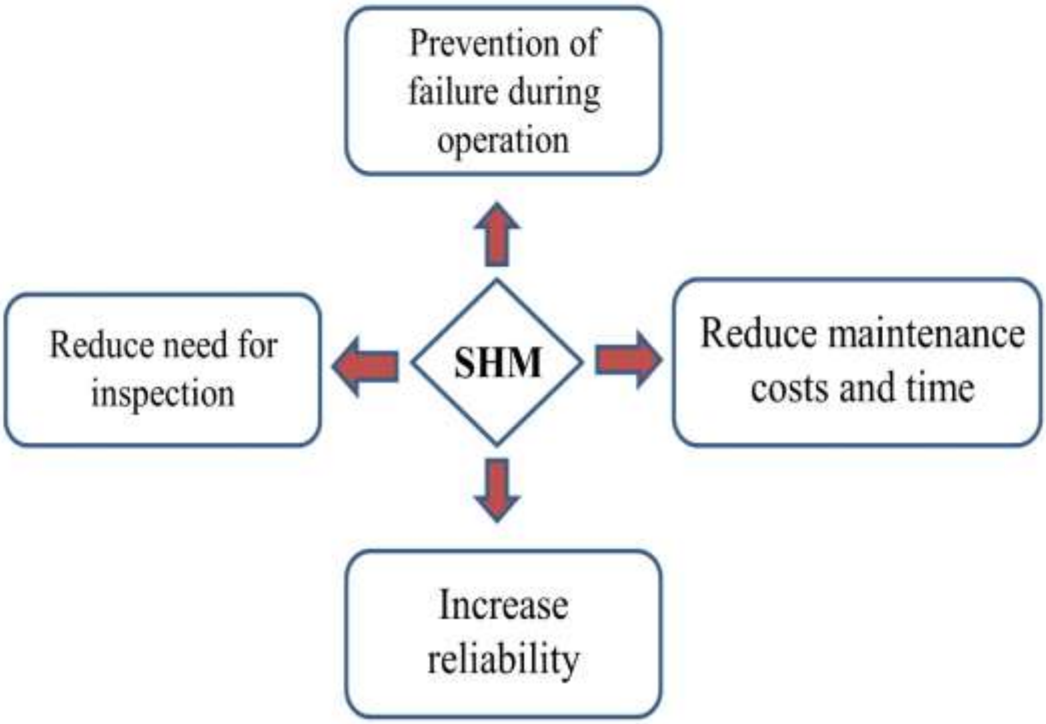


Figure 1.1. The perceptible advantages of SHM over NDT&E

Guided-wave testing has been shown as a very prominent option among active interrogation schemes of SHM. It provides an efficient way to scan relatively wide portions of a structure, estimate the location, severity and variety of damage types, and also it is a well-established practice in the NDT&E industry. The main advantage of guided waves is their ability to travel long distances without significant amplitude decay, which offers large area coverage of the specimen to be investigated. The capability of guided waves to penetrate through the thickness of the plate structure provides good sensitivity to various imperfections such as flaws, cracks, impact damage and delamination. Various key aspects play important role in the successful implementation of guided wave based SHM systems. Each of them by itself is an independent area of research. Figure 1.3 depicts the main aspects in guided wave based online damage detection system.

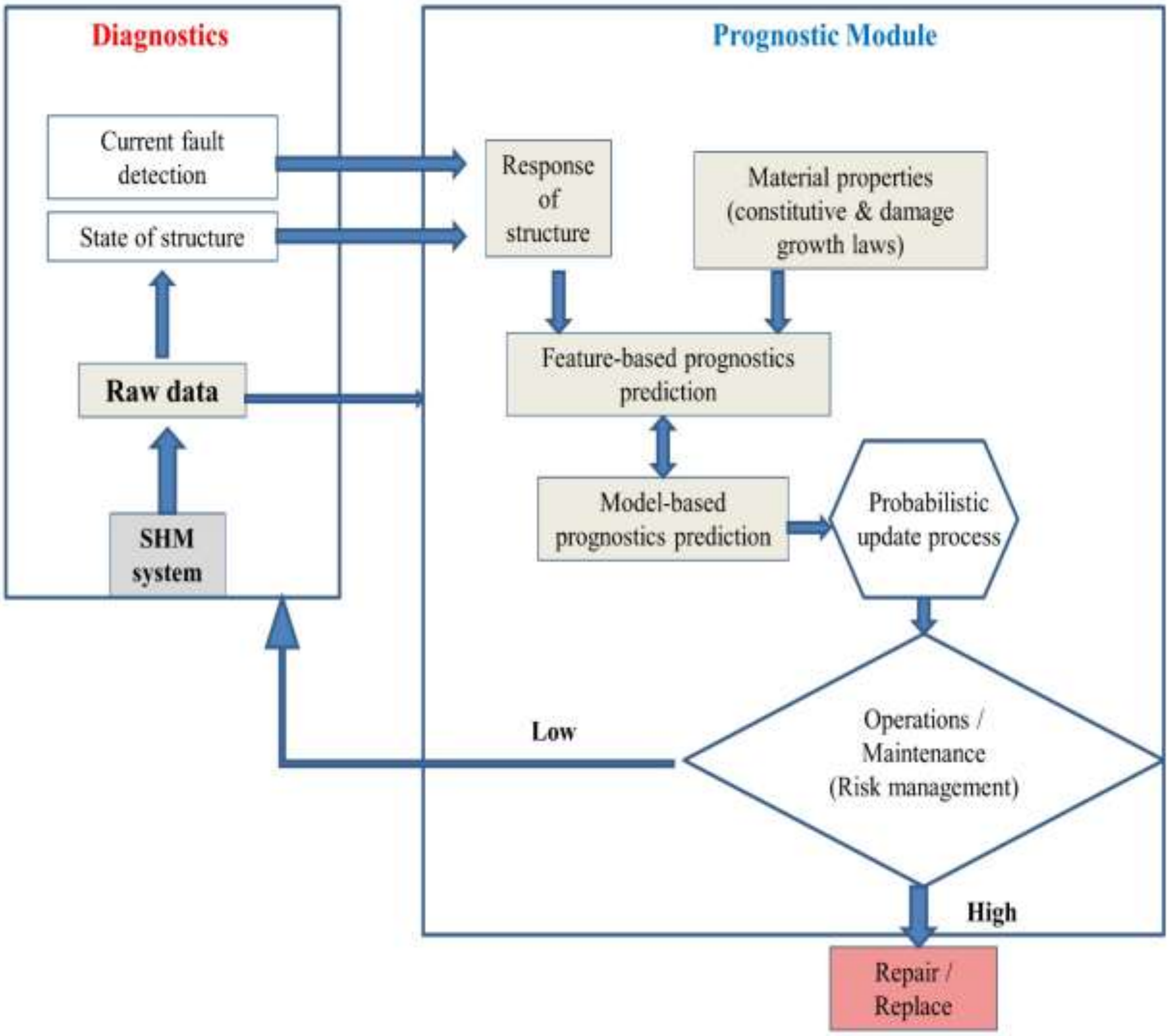


Figure 1.2. Outline of SHM system

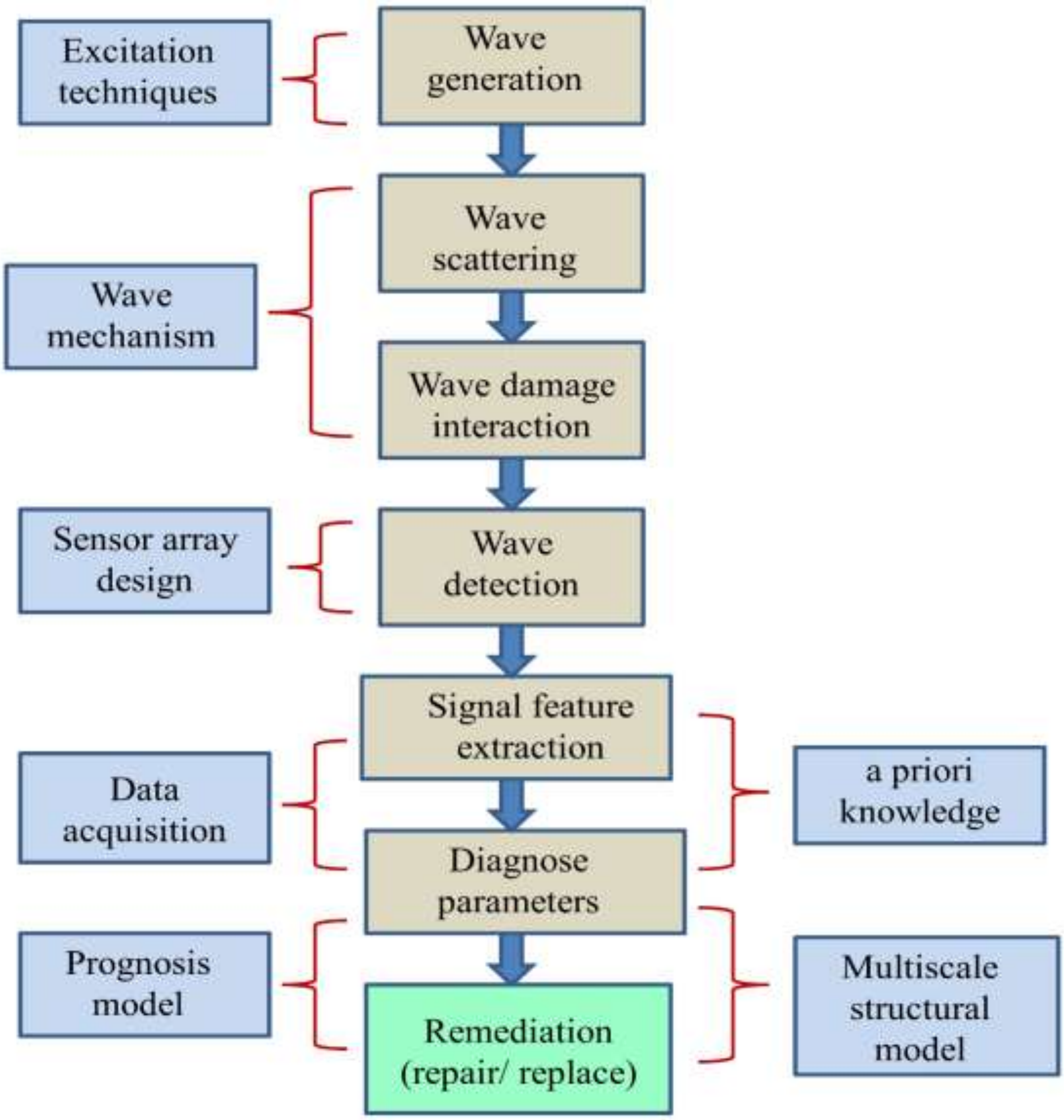


Figure 1.3. Guided wave based SHM system

Lamb waves can be defined as a guided stress waves that propagate along the structures such as plates, pipes rods, shells, and beams. In SHM system, Lamb waves are excited using transducers which produce some high frequency modulated sinusoidal tone burst pulse signal with the limited number of cycles which are received by the same or different transducer in the structure. Lamb waves can be actuated in many possible wave modes having different velocity and shape. Therefore, the selection of the appropriate mode is crucial for enhancing damage detection capabilities of the SHM system.

1.2 Elastic Wave Propagation

The behavior of particle oscillation during wave propagation is called as wave mode. On the basis of particle motion in the material structure, four fundamental wave modes are classified as (1) longitudinal waves or pressure waves, (2) shear waves or transverse waves, (3) surface waves or Rayleigh waves, and (4) plate waves or Lamb waves. Longitudinal waves are also known as the primary waves or P-waves. The direction of particle oscillation is parallel to the direction of wave propagation. Shear waves are also called as transverse or secondary waves and these waves occur when the displacement of structure's particles motion is perpendicular to the direction of propagation and there are two polarizations. Both waves, namely longitudinal waves and transverse waves are known as bulk waves and these waves are non-dispersive. The speed of the bulk wave depends on the material properties of the structure.

1.2.1 Rayleigh waves

Rayleigh waves also known as the surface waves and propagate near to the surface of the solid body. Rayleigh waves travel with longitudinal and transverse particle oscillations where amplitude decreases rapidly with surface depth. The effective length of penetration in the solid structure is less than a wavelength and polarization of wave lie in a plane perpendicular to the body surface. The elliptical path of a particle in surface is due to simultaneous longitudinal and transverse displacement. At the extremity of transverse displacements, longitudinal displacement is zero and at the mean position of transverse displacement, the longitudinal displacement is maximum. This results an elliptical path of particle displacement. Figure 1.4 shows Rayleigh wave propagation in the x-direction and normal to surface along z-direction. Rayleigh wave speed is slightly less than that of shear waves. The wave speed C_{RW} is commonly defined as:

$C_{RW} = C_{SW}\left(\frac{0.862+1.14\nu}{1+\nu}\right)$, where C_{SW} is shear wave speed and ν is the Poisson ratio of the medium.

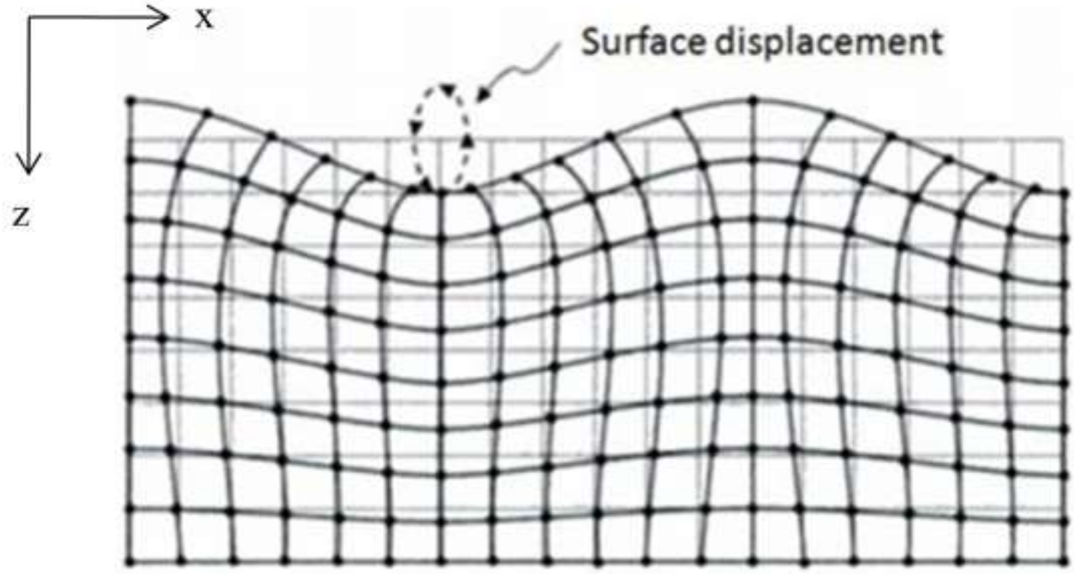


Figure 1.4. Surface displacement pattern in Rayleigh wave (Cheeke (2002))

In 1924, Stoneley discovered Rayleigh wave at the interface between two solid materials which is called now as Stoneley wave. In 1926, Love observed Shear Horizontal (SH) waves in a thin layer, and it is consequently called Love waves.

1.2.2 Lamb waves

Lamb waves are the type of guided waves of plane strain that can propagate in a free plate. Lamb waves are commonly used in the guided wave based SHM because they can travel a long distance in structures and are well understood in mechanics and mathematics. Figure 1.5 shows displacement u in the direction of wave propagation (x-direction) and v through the thickness (y-direction). Figure 1.6 represents particle motion in the y-direction as transverse waves (red color arrows) and particle motion in the x-direction as longitudinal waves (black color arrows), and the blue color arrows show the direction of wave propagation. Guided Lamb waves take place when the plate thickness is of the same order as the wavelength.

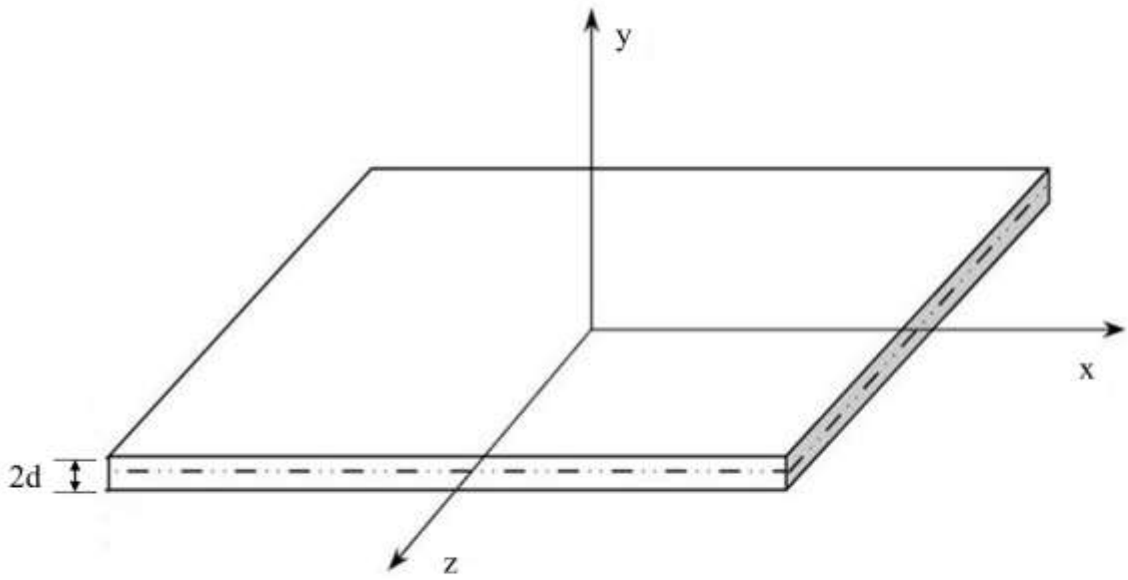


Figure 1.5. A thin plate of 2*d* thickness

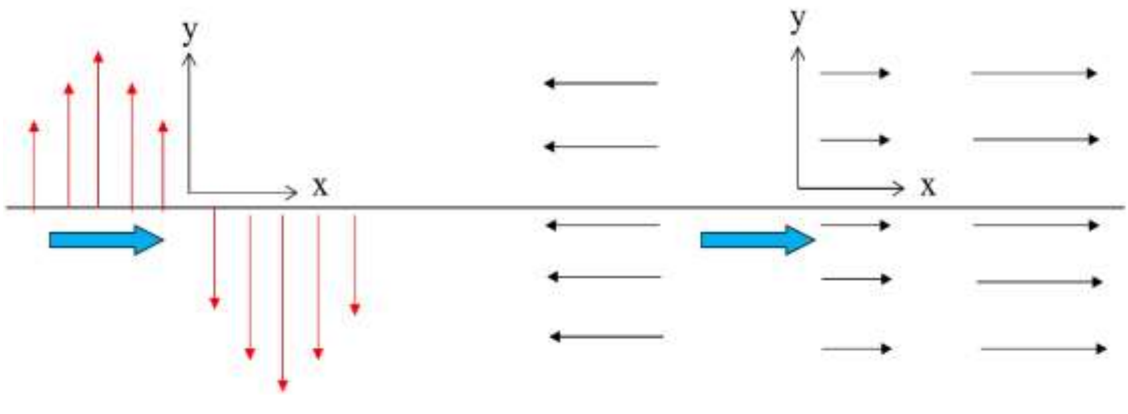


Figure 1.6. Particle motion of Lamb waves in x and y-direction

The governing equations of motion for Lamb wave are

$$\rho \frac{\partial^2 u}{\partial t^2} = \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + f_x \tag{1.1a}$$

$$\rho \frac{\partial^2 v}{\partial t^2} = \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + f_y \tag{1.1b}$$

Where f_x and f_y are the body forces, and ρ is density. The normal stresses σ_{xx} and σ_{yy} , and the shear stress τ_{xy} can be expressed as:

$$\sigma_{xx} = (\lambda + 2\mu)\frac{\partial u}{\partial x} + \lambda \frac{\partial v}{\partial y} \tag{1.2a}$$

$$\sigma_{yy} = \lambda \frac{\partial u}{\partial x} + (\lambda + 2\mu)\frac{\partial v}{\partial y} \tag{1.2b}$$

$$\tau_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \tag{1.2c}$$

Where $\lambda = \dfrac{E\nu}{(1+\nu)(1-2\nu)}$ and $\mu = \dfrac{E}{2(1+\nu)}$ are Lamé constants. E and ν are Young's modulus and Poisson ratio, respectively. Substituting relation 1.2(a)-(c) in equations (1.1a) and (1.1b), we get

$$(\lambda + 2\mu)\frac{\partial^2 u}{\partial x^2} + (\lambda + \mu)\frac{\partial^2 v}{\partial x \partial y} + \mu \frac{\partial^2 u}{\partial y^2} + f_x = \rho \frac{\partial^2 u}{\partial t^2} \tag{1.3}$$

$$(\lambda + 2\mu)\frac{\partial^2 v}{\partial y^2} + (\lambda + \mu)\frac{\partial^2 u}{\partial x \partial y} + \mu \frac{\partial^2 v}{\partial x^2} + f_y = \rho \frac{\partial^2 v}{\partial t^2} \tag{1.4}$$

The equations for Lamb wave can be expressed by using longitudinal velocity

$C_L^2 = \frac{\lambda + 2\mu}{\rho}$ and shear velocity $C_T^2 = \frac{\mu}{\rho}$ as:

$$C_L^2 \frac{\partial^2 u}{\partial x^2} + (C_L^2 - C_T^2)\frac{\partial^2 v}{\partial x \partial y} + C_T^2 \frac{\partial^2 u}{\partial y^2} + f_x = \frac{\partial^2 u}{\partial t^2} \quad (1.5)$$

$$C_L^2 \frac{\partial^2 v}{\partial y^2} + (C_L^2 - C_T^2)\frac{\partial^2 u}{\partial x \partial y} + C_T^2 \frac{\partial^2 v}{\partial x^2} + f_y = \frac{\partial^2 v}{\partial t^2} \quad (1.6)$$

Now using the Helmholtz decomposition method for the above wave equation, the displacement field is separated into two uncoupled wave equations under the plain strain condition as:

governing longitudinal wave, $$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\omega^2}{C_L^2}\phi = 0 \quad (1.7)$$

governing transverse wave, $$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\omega^2}{C_T^2}\psi = 0 \quad (1.8)$$

Where ϕ and Ψ are two potential functions and ω is circular frequency of the external pulse excitation employed to the surface of the plate structure. Since the time dependence is assumed harmonic in the form $e^{-i\omega t}$, the general solution to governing wave equation can be expressed as:

$$\phi = \Phi(y)\exp\left(i(kx - \omega t)\right) = \left(A_1 \sin(py) + A_2 \cos(py)\right) * \exp\left(i(kx - \omega t)\right) \quad (1.9a)$$

$$\psi = \Psi(y)\exp\left(i(kx - \omega t)\right) = \left(B_1 \sin(py) + B_2 \cos(py)\right)\exp\left(i(kx - \omega t)\right) \quad (1.9b)$$

$$p^2 = \frac{\omega^2}{C_L^2} - k^2,\ q^2 = \frac{\omega^2}{C_T^2} - k^2,\ k = \frac{2\pi}{\lambda_{wave}} \tag{1.9c}$$

Subscripts 1 and 2 of longitudinal wave constant A and transverse wave constant B indicate outward and inward directions of propagation. The four constants depend on the boundary conditions. k and λ_{wave} are circular wave number and wavelength of the wave. The field variables involve sines and cosines with the argument (y), which are odd for sines and even for cosines. Considering that the trigonometric function tangent is defined with sine and cosine which have symmetric and anti-symmetric properties, respectively, the solution is divided into two sets of wave modes which are symmetric and anti-symmetric modes (Rose (1999)). As a result of plane strain, the displacements in the wave propagation direction x and normal direction y can be described as:

$$u = \frac{\partial \phi}{\partial x} + \frac{\partial \psi}{\partial y},\ v = \frac{\partial \phi}{\partial y} - \frac{\partial \psi}{\partial x}, \text{and } w = 0 \tag{1.10}$$

Therefore,

$$\sigma_{xx} = \lambda\left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2}\right) + 2\mu\left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \psi}{\partial x \partial y}\right) \tag{1.11a}$$

$$\sigma_{yx} = \mu\left(\frac{\partial^2 \phi}{\partial x \partial y} - \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}\right) \tag{1.11b}$$

$$\sigma_{yy} = \lambda\left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2}\right) + 2\mu\left(\frac{\partial^2 \phi}{\partial y^2} - \frac{\partial^2 \psi}{\partial x \partial y}\right) \tag{1.11c}$$

By applying traction free boundary conditions at upper and lower surfaces of plate as:

$$u(x,t) = u_0(x,t) \text{ (displacement)} \tag{1.12a}$$

$$t_i = \sigma_{ji}\eta_j \text{ (traction)} \quad (1.12b)$$

$$\sigma_{yx} = \sigma_{yy} = 0 \text{ at } y = \pm d/2 \quad (1.12c)$$

where d is the plate thickness and η is unit normal. The resulting displacement, stress and strain fields depend upon the type of wave modes. We can obtain the general description of Lamb waves (symmetric and antisymmetric modes) in an isotropic and homogeneous plate:

$$\frac{\left(k^2 - q^2\right)\sin\left(\frac{qd}{2}\right)}{2ikp\left(\sin\left(\frac{pd}{2}\right)\right)} = \frac{-2\mu ikq\left(\cos\left(\frac{qd}{2}\right)\right)}{\left(\lambda k^2 + \lambda p^2 + 2\mu p^2\right)\cos\left(\frac{pd}{2}\right)} \quad (1.13)$$

$$\frac{\left(k^2 - q^2\right)\sin\left(\frac{pd}{2}\right)}{2ikp\left(\sin\left(\frac{qd}{2}\right)\right)} = \frac{-2\mu ikq\left(\cos\left(\frac{pd}{2}\right)\right)}{\left(\lambda k^2 + \lambda p^2 + 2\mu p^2\right)\cos\left(\frac{qd}{2}\right)} \quad (1.14)$$

(i) Symmetric modes

After rearranging symmetric modes equation (1.13), we get:

$$\frac{\tan\left(\frac{qd}{2}\right)}{\tan\left(\frac{pd}{2}\right)} = \frac{4k^2\mu qp}{\left(\lambda k^2 + \lambda p^2 + 2\mu p^2\right)\left(k^2 - q^2\right)} \quad (1.15)$$

The denominator on the RHS of equation (1.15) can be defined by the wave velocities and parameters p and q.

Longitudinal velocity expression

$$\lambda = C_L^2\rho - 2\mu \tag{1.16}$$

Using equation (1.16) in denominator of RHS of equation (1.15) as

$$\lambda k^2 + \lambda p^2 + 2\mu p^2 = \lambda\left(k^2 + p^2\right) + 2\mu p^2$$

$$= \left(C_L^2\rho - 2\mu\right)\left(k^2 + p^2\right) + 2\mu p^2$$

$$= C_L^2\rho\left(k^2 + p^2\right) - 2\mu k^2 \tag{1.17}$$

Using equations (1.9c) we get

$$\lambda k^2 + \lambda p^2 + 2\mu p^2 = \rho\omega^2 - 2C_T^2\rho k^2$$

$$= C_T^2\rho\left[\left(\frac{\omega}{C_T}\right)^2 - 2k^2\right]$$

$$= C_T^2\rho\left(q^2 - k^2\right)$$

$$= \mu\left(q^2 - k^2\right) \tag{1.18}$$

After substituting equation (1.18) in equation (1.15), we get dispersion equation for symmetric modes as:

$$\frac{\tan\left(qd/2\right)}{\tan\left(pd/2\right)} = \frac{4k^2qp}{\left(-q^2 + k^2\right)^2} \tag{1.19}$$

(ii) Antisymmetric modes

Similarly, dispersion equation for antisymmetric modes as:

$$\frac{\tan\left(qd/2\right)}{\tan\left(pd/2\right)} = \frac{\left(-q^2+k^2\right)^2}{4k^2qp} \tag{1.20}$$

The equations (1.19) and (1.20) are known as Rayleigh-Lamb frequency equations and can be utilized to calculate the velocity at which a wave of a particular frequency will propagate in structure. For solution, the dispersion equations (1.19) and (1.20) can be rearranged as:

$$\frac{\tan\left(qd/2\right)}{q} + \frac{4k^2 p\tan\left(pd/2\right)}{\left(k^2-q^2\right)^2} = 0 \tag{1.21}$$

$$q\tan\left(qd/2\right) + \frac{\left(k^2-q^2\right)^2\tan\left(pd/2\right)}{4k^2qp} = 0 \tag{1.22}$$

The dispersion equations can be satisfied by an infinite number of real roots of wavenumbers at a particular frequency. The graphic representation of solutions of above dispersion equations is called the dispersion curves (Rose (1999)). Dispersion curves are used to express and calculate the relationship among frequency, thickness, and phase/group velocity. Figure 1.7 shows the displacement direction of particles motion in the symmetric and antisymmetric Lamb wave modes. Lamb waves in a plate consist of symmetric and antisymmetric modes (as shown in Figure 1.8) are denoted as S_i and A_i respectively. The subscript i shows order $(i = 0,1,2,3...)$. S_0 and A_0 are the lowest order modes and known as fundamental symmetric and antisymmetric Lamb modes. The symmetric modes are showing compressional nature (thickness bulging as well as contracting) and have the radial in-plane displacement of particles, while antisymmetric modes are showing flexural behavior (constant thickness flexing) and have out of plane displacement. Mostly, the magnitude of S_i modes is smaller than that of A_i modes under the same excitation condition.

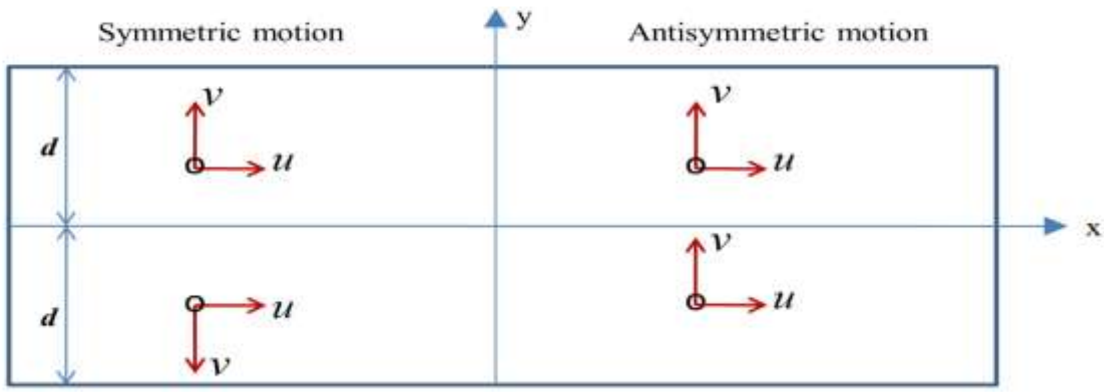


Figure 1.7. Sketch diagram of particles motion in the symmetric and antisymmetric wave modes

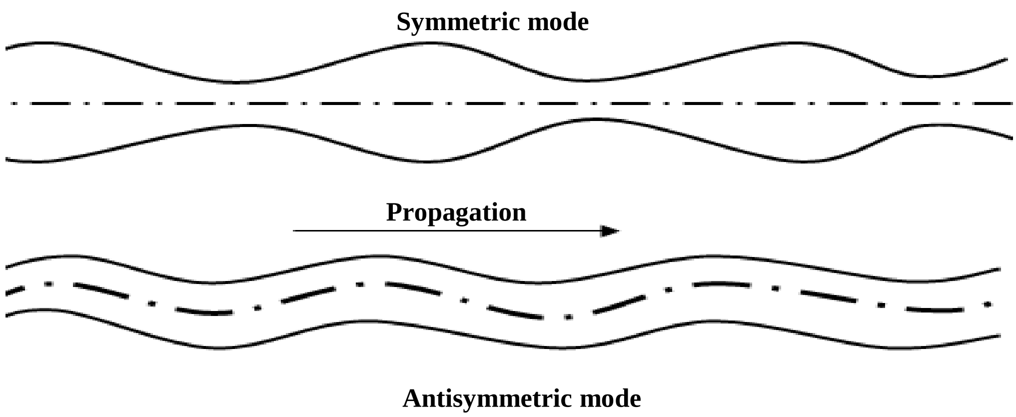


Figure 1.8. Sketch diagram of Lamb wave propagating modes in isotropic plate

1.2.2.1 Phase Velocity (c_p) *and Group Velocity* (c_g)

The Lamb waves are dispersive for each solutions of equation (1.21) and (1.22) and characterized by the phase (c_p) and group (c_g) velocities. The individual waves move together and the resulting wave speed known as group velocity which may be different from the phase velocities. The phase velocity is the velocity of propagating wave of a single frequency and defined as:

$$c_p = \frac{\omega}{2\pi}\lambda_{wave} \tag{1.23}$$

Using equations (1.9c) in above equation, we get

$$c_p = \frac{\omega}{k} \tag{1.24}$$

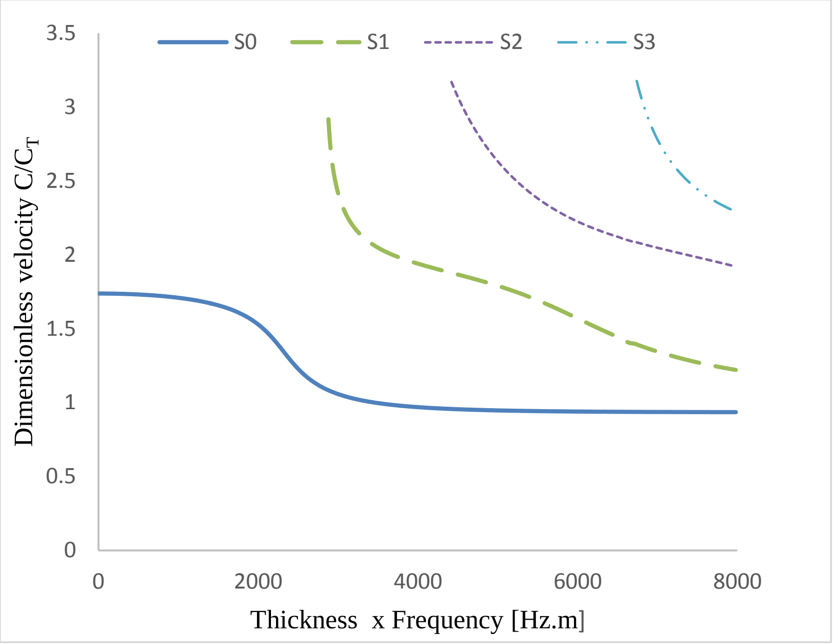
The phase velocity is dependent on frequency in dispersive mediums. The group velocity is referred to as the propagation velocity of a wave packet (overall shape of the amplitudes of the wave). The group velocity is dependent on frequency and plate thickness, defined as (Rose (1999)):

$$c_g = c_p^2\left[\left(c_p - (fd)\frac{dc_p}{d(fd)}\right)\right]^{-1} \tag{1.25}$$

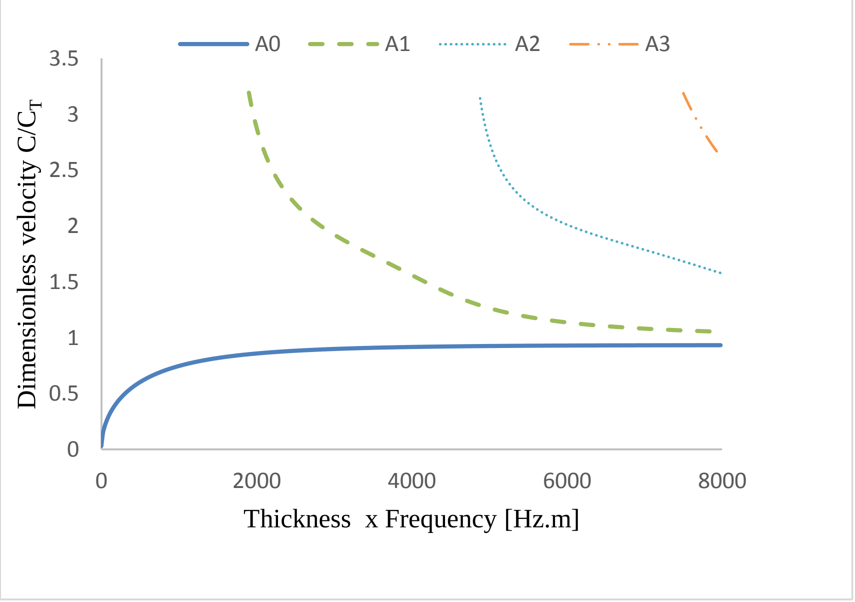
The phase velocity is identical to the group velocity when the phase velocity is independent of frequency.

1.2.2.2 Dispersion curves

The dispersive behavior of Lamb waves means that the different frequency components of waves propagate at different velocities and that the shape of the wave envelope (packet) changes as it propagates by solid media. The propagation characteristic in a given material is described with their dispersion curves, which are plots of phase/group velocity verses excitation frequency-thickness product. Dispersion curves are obtained numerically by solving the Lamb wave equations. The independent variable is iteratively solved for the product of frequency-thickness. These values of phase velocity and product of frequency-thickness are plotted to get the dispersion curve.

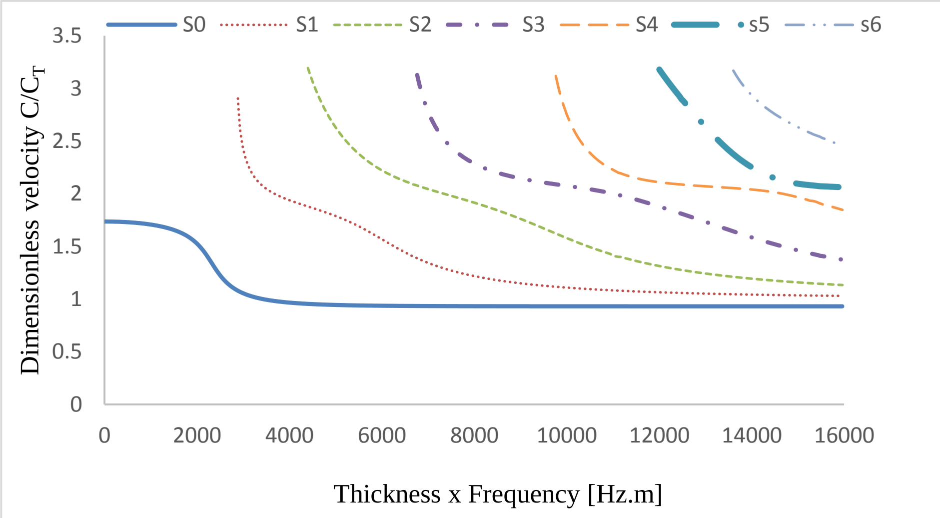


(a) symmetric modes

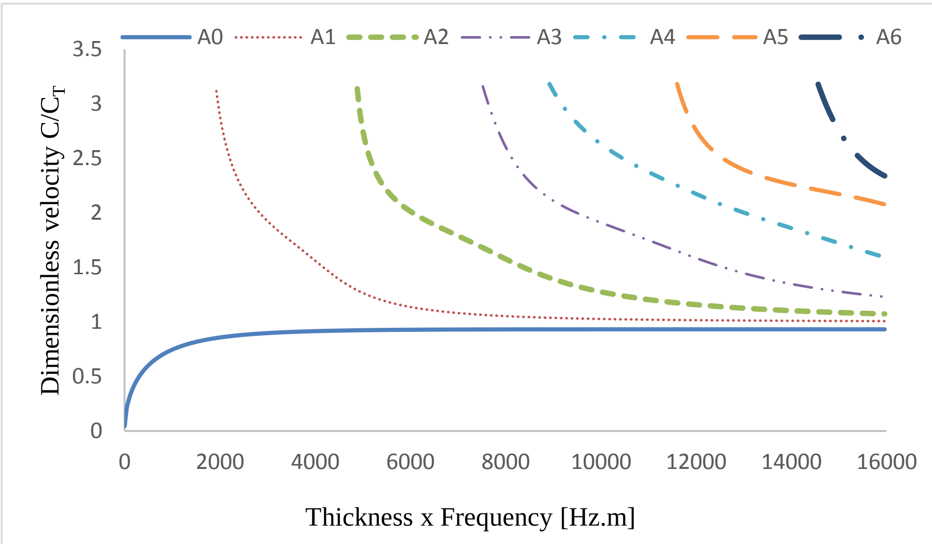


(b) anti-symmetric modes

Figure 1.9. Dispersion curves for Lamb modes in Al plate with 1 mm thickness



(a) symmetric modes



(b) anti-symmetric modes

Figure 1.10. Dispersion curves of Lamb modes in Al plate with 2 mm thickness

To obtain an in-depth understanding of the physics of Lamb wave propagation in thin plate structure, the dispersion curves are numerically generated. For this purpose, the isotropic Aluminum (Al) plate is considered. The material properties of Al assumed are Poisson's ratio 0.33; density 2700 kg/m^3; Young's modulus 69 GPa. Transverse velocity C_T and longitudinal velocity C_L are 3130 m/s and 6320 m/s, respectively. Figure 1.9

shows the dispersion curves in Al plate of 1mm thickness. The dispersion curves in Figures 1.9 (a) and 1.9 (b) tend to converge at the dimensionless velocity value 1, which is true for all dispersion curves of Lamb waves. Figure 1.10 depicts the dispersion curves of symmetric and anti-symmetric wave modes in Al plate with 2 mm thickness.

1.2.2.3 Excitation and generation of Lamb waves

In last two decades, several devices are developed to actively excite and sense guided Lamb waves in various SHM and NDT&E applications. Many types of transducers such as angle beam transducers, comb actuators, magnetostrictive tapes are broadly employed to test the integrity of welds, pipes and plate structures. However, these transducers can endure from the large variation in mechanical impedance between air or fluid and specimen under inspection, resulting in small precision. Above-mentioned transducers produce transverse or longitudinal waves into the specimen and the preference of proper incidence angles allows for the generation of guided elastic waves. These transducers/devices are too large and bulky to be permanently attached or embedded in the specimen structure and also they require relatively high voltages to drive so inspection process is not cost effective. Therefore, in recent years various new transducers such as Piezoelectric lead Zirconate Titanate (PZT) elements, Polyvinylidene Fluoride (PVDF) and piezoelectric polymer film are developed for the guided wave excitation and reception. Guided waves can be actuated and sensed by a variety of means, which are roughly summarized in Table 1.2.

Ultrasonic probes have been widely used to excite and receive Lamb wave with excellent precision and controllability. Ultrasonic probes are coupled with adjustable angle wedges to avoid the complexity of multimode wave propagation and to actuate a pure Lamb wave mode. This technique is not suitable for detecting near-surface damage where reflections from a crack/flaw are inadequate within the wavelength of the transmitted ultrasonic pulse. Limited access to the complex geometry of structure also decreases the practical applications of the ultrasonic probe. Non-contact actuation of guided waves through laser based ultrasonic is a prominent technique for high precision damage detection (Staszewski *et al.* (2004)).

Table 1. 2. Comparison of Lamb wave transducers with other NDE transducers (Boffa (2016))

Sensor	**Applications/features**	**Available style**
Ultrasonic probe	Flaw and thickness detection, Efficient.	Contact, air/fluid-coupled
Laser interferometer	High precision, Expensive.	Contactless
Piezoceramics	Active sensor, Cheap, High-frequency response.	Attaching, embedding
Piezoelectric point and PVDF	Damage detection, Cheap, Application for flat and non-flat shapes.	Attaching, embedding
EMAT	Avoidance of physical contact, Narrow band.	Contact, attaching
Accelerometer	Acceleration detection, High-frequency response.	Attaching
Shape memory alloy	Deformation detection, Low frequency response.	Attaching, embedding
Magnetic sensor	Large deformation with magnetic leakage, Magnetic field required.	Contact, attaching
Optical fibre	Deformation and temperature detection,	Embedding
AE sensor	Passive sensor, Changes in physical property only.	Attaching, embedding
Eddy-current transducer	Electromagnetic impedance detection, Good for composites.	Attaching
Strain gauge	Low frequency response, Cheap, Deformation measurement.	Attaching

Depending on the practical application, a laser-based ultrasonic provides broadband or narrowband excitation. This technique allows effective inspection of complicated geometries and curved surfaces where access is not viable and can offer extensive ranges from nanoscale defects to a small crack. However, the cost of the whole setup can restrict broad application.

In recent years PZT elements are extensively used for excitation plus acquisition of guided wave and show excellent performance in damage detection. PZT elements are suitable for attachment to a host structure as an actuator as well as a sensor because of their negligible mass, thin, and small size. Besides, PZT delivers the wide range of frequency responses, less power consumption. However, small driving force, low fatigue life, and brittleness are demerits of the PZT elements. Furthermore, a PZT element shows certain nonlinear behavior and hysteresis under high-temperature or large stain/voltage. PVDF is novel interdigital transducers which are employed in online crack detection techniques for the excitation of wave signal. The advantages of PVDF transducer are superior flexibility, higher dimensional stability, constant piezoelectric coefficients throughout time, more versatile applications with reduced cost. However, PVDF is mainly used as a sensor because of its weak driving force, and as actuators perform only under a very low-frequency range up to 500 Hz (Wilcox *et al.* (2001), Giurgiutiu *et al.* (2003)).

1.3 Motivation and Problem Statement

Among the well-established SHM methodologies discussed in above section, the elastic guided wave based SHM has gained attention in the past few years because of its advantages: capable to inspect a large area of structure in a short time, sensitive to small damage and discontinuities, able to identify both surface and internal damage, low energy consumption, no need of transducers movement, quick and repeatable, and being cost-effective (Su and Ye (2009)). This dissertation work is based on using Lamb waves, which is a part of an *active* interrogation technique.

Unlike conventional ultrasonic techniques, which depend on point by point inspections, Lamb waves inspect large structure through thickness even in harsh environmental conditions. Hence using this technique, the significant part of a specimen can be monitored by measuring the wave characteristic such as wave modes, amplitude, and velocity. Elastic waves have been extensively used for damage detection applications, especially for smart structures with permanently embedded or integrated sensors. Although Lamb waves are effective tools for evaluation of structural integrity yet many complexities exist in the proper implementation of guided Lamb wave. Therefore, the understanding of the fundamental mechanics and physics behind the Lamb waves and their multi-mode propagation and interaction characteristics within the host structure is crucial for the selective actuation and receiving of wave modes and excitation frequency. Other major complexities are interpretation of the received signals obtained through sensors. The complexity mainly comes from the multi-layer structure, edge reflections, a large number of transducers, anisotropy, and possible non-homogeneity of structures.

The development of an efficient SHM system involves multidisciplinary research challenges, as is discussed in the previous sections. The associated multidisciplinary research field requires a lot of attention for better performance of guided wave. While the advances in Lamb wave technologies have established the feasibility of guided Lamb wave based SHM, there remain several challenges for applications, especially for complex geometries structures with multiple damages. The followings are the basic challenges and problems in wave-based SHM systems, and these are the open area for the research at present

(1) *Accurate, efficient, and versatile numerical method for elastic wave propagation and interaction mechanism-*

Numerical simulation of wave propagation is essential to understand the physical phenomenon of the wide variety of practical problems. However, the requirement of minimum grid point density per wavelength limits the computational stability and accuracy as well as simulation of engineering application by numerical method. The purpose of this thesis is to provide an improved framework for simulation of linear and nonlinear elastic wave propagation and damage identification techniques feasible in the context of online SHM. The simulation of multiple harmonics requires a very high mesh resolution. A wavelet-based adaptive multiscale technique efficiently compresses the resultant large stiffness matrix from finite element discretization for the propagation of such wave. This dynamic adaptive grid selection is based on the facts that very few wavelet coefficients are required to represent a short pulse containing higher harmonics.

(2) *Generalized error estimation techniques for wave propagation and practical engineering problems-*

It is commonly accepted that error estimation and adaptation increase the usefulness and reliability of numerical computations. Estimating the error in a numerical solution and generating appropriate meshes are complicated tasks. The error estimation in computation provides: (i) the choice of error tolerance, (ii) confidence in the computed solution increases, and (iii) freedom from the tedious task of generating a very dense mesh.

(3) ***Signal processing and data interpretation as well as damage diagnosis methodologies-***

The conventional guided wave-based damage detection system commonly comprises of many sensors for accumulating the structural response. Raw signals containing the response of the structure, are obtained through sensors after transmission over a long distance and these are often corrupted by high frequency ambient random noise and coherent noise. The actuation and reception of undesirable modes, edge reflections and a large number of transducers are the main sources of coherent noise (Alleyne and Cawley (1997)). Both the random and coherent noise do not allow easy interpretation of the received signals. Thus, signal processing and denoising techniques are essential to extract signal features from the measured noisy signal before implementing in the defect identification algorithm in order to predict the physical condition of the structure.

These challenges engage many researchers in this multidisciplinary area of research for better implementation and enhancement of wave application. Hence, this thesis addresses the above issues of Lamb wave based SHM. This work focuses on utilization of wavelet-based multiresolution analysis for the numerical method, error estimation, signal processing and data interpretation.

1.4 Contributions of the Thesis Work

The research goal of this Ph.D. work falls under three underlying prospects. Firstly, to develop accurate, and versatile computational method for efficient simulation of guided wave propagation and interaction with damage in two-dimensional structures.

This research covers the wavelet-based multi-scale simulation and numerical experiments. For enhancement of computational performance of the proposed algorithm and handling high mesh density problem, nonstandard wavelet method is used to develop an efficient framework for simulation of wave propagation problem. The main objective of the work is to further improve computational performance using the multiresolution analysis. Our focus is on the recognition of computational bottlenecks and their elimination. This advanced modeling technique is capable to model both linear and nonlinear elastic wave propagation in complex geometry.

The first part of this study comprises the mathematical foundation of compactly-supported wavelets based multiresolution analysis. Coupling of wavelet based multiscale approach with finite element method is used for the dynamic analysis of wave propagation in plate-like structures. The research results demonstrate the feasibility of wavelet in the analysis of linear and nonlinear waves and provide a foundation for future research. In order to formulate the wave propagation problems, a MATLAB program is developed for various structural conditions. This generalized program uses various wavelet functions for the Lamb wave method. Subsequently, a more efficient algorithm based on the nonstandard form of the wavelet transform is tested. The method can detect damage, high gradients and other sharp changes in the solution. In addition, the inverse of the nonstandard form of the operator matrix is used.

The second aspect of this dissertation work is to develop and implement the wavelet-based *a posteriori* error indicator and mesh refinement criteria for wave propagation as well as practical engineering problems. This study presents a method to estimate local and global errors by using wavelet-based error estimation technique. The aim is to decrease the functional error by increasing mesh density in the areas where the functional and solution residual generate the major error in the solution. The finite element solution of engineering problems is transformed into the multilevel decomposition of wavelet space. The error estimations in the present setting do not have any problem due to complex domain of engineering structures. The proposed method is an efficient technique which can be applied in some small region as well as complete domain.

The third aspect of the thesis is to develop an efficient noise filtering technique for the time frequency analysis of multimode guided Lamb wave signal. Filtration of time frequency information of multimode Lamb waves through the noisy signal is investigated in this study using Matched Filtering Technique (MFT), wavelet denoising methods. and Wavelet Matched Filter Method (WMFM). The proposed WMFM technique, a combination of the wavelet transform and MFT, can significantly improve the accuracy of the filtered signal and identify relatively small damage in contaminated signal. As the selection of an appropriate mother wavelet function is very crucial in the efficiency and accuracy of the signal transformation, the present analysis is based on Shannon's entropy of wavelet coefficients for both the Discrete Wavelet Transform (DWT) as well as Continuous Wavelet Transform (CWT) and this selection is further justified by Root Mean Square Error (RMSE) of filtered signal. In addition, the proposed filtering technique is tested for complex structures like the hollow cylindrical shell.

The approaches and techniques which are developed in this research can be applied in many ways as a part of an elastic wave SHM system and other wave-based practical applications. Linear and nonlinear wave propagation and interaction with crack are also considered for the purpose of optimization of discontinuities detection and quantification.

1.5 Organization of Thesis

The present thesis is divided into seven chapters including chapter 1 in which an overview of the present work along with the importance of the problem and the motivation for current work is presented.

Chapter 2 presents a review of the existing literature and the state-of-art for guided elastic waves to provide a deeper background and foundation for the thesis. The literature survey focuses on the background of ultrasonic waves, their prior application to NDT&E and SHM, classical and non-classical approaches for damage detection algorithms. At the end, observations on the present state-of-the art and objective of the present work are presented.

Chapter 3 deals with the theory and mathematical formulation of the wavelet transform that is used in the thesis. This chapter introduces the numerical formulation of wavelet based finite element method by describing the concept of multiscale, adaptive mesh scheme for the simulation of wave propagation, and their implementation. Numerical examples are presented to validate the approach.

The solution methodology based on the nonstandard form of the wavelet transform to solve the linear and nonlinear wave is presented in Chapter 4. This chapter is concerned with the detection and location of the damage using the proposed technique. The accuracy and stability of present solution methodology are established via detailed convergence study.

Chapter 5 focuses on the application of wavelet to error estimation, along with a state-of-art in error estimation. In this chapter, an *a posteriori* error estimation technique to assess the discretization error of standard finite element approximations for the is proposed.

Chapter 6 provides background information on the field of guided wave SHM. This chapter presents filtering of Lamb waves in the process of crack detection in the structures, particularly in extremely noisy conditions. The concepts of single mode and multimode excitation are illustrated. Subsequently, the desirable requirements for robust signal processing of complicated time traces are also described.

Chapter 7 summarizes the current research findings and provides suggestions for future research.