

LIST OF CONTENTS

	Page No.
List of Figures	x
List of Tables	xv
List of Abbreviations	xvi
List of Symbols	xviii
Preface	xx
 Chapter 1: Introduction and Theoretical Background	
1.1 Structural Health Monitoring	1
1.2 Elastic Wave Propagation	8
1.2.1 Rayleigh waves.....	8
1.2.2 Lamb waves.....	9
1.2.2.1. Phase velocity (c_p) and Group velocity (c_g)	17
1.2.2.2. Dispersion curves	18
1.2.2.3. Excitation and generation of Lamb waves	21
1.3 Motivation and Problem Statement	23
1.4 Contributions of the Thesis Work	26
1.5 Organization of Thesis	29
 Chapter 2: Literature Review	
2.1 Early Developments and Fundamentals	32
2.1.1 Parameters of Lamb waves for damage detection	33
2.1.1.1. Amplitudes of the waves	33
2.1.1.2. Phase velocity and group velocity	34
2.2 Analysis and Simulation of Lamb Wave	35
2.2.1 Analytical analysis	36
2.2.2 Numerical analysis	38
2.3 Processing of Lamb Wave Signal	43
2.3.1 Time domain analysis	44
2.3.2 Frequency domain	45
2.3.3 Integrated time-frequency domain analysis	45
2.4 Error Estimation	47

2.5 Observations Based on the Literature Review	50
2.6 Research Context	52

Chapter 3: Wavelet Transform and Multiscale Analysis

3.1 Literature Review of Wavelets	53
3.1.1 History of wavelets	53
3.1.2 Wavelets for solution of PDEs	55
3.1.3 Wavelet based multiscale methods	57
3.1.4 Wavelet based finite element method for wave propagation	58
3.2 Wavelet Overview	59
3.2.1 Continuous wavelet transform (CWT)	60
3.2.2 Discrete wavelet transform (DWT)	61
3.2.3 Multiscaling using wavelets	62
3.2.4 Multi-scale decomposition of finite element matrix using wavelets	68
3.3 Numerical Results	72
3.3.1 Transient scalar wave	73
3.3.2 Time harmonic scalar wave	74
3.3.3 Lamb waves	79

Chapter 4: Multiscale Simulation Using Nonstandard Wavelet Transform

4.1 Introduction	90
4.2 Mathematical Formulation	92
4.2.1 Nonstandard wavelet transform	92
4.2.2 Multiscaling using wavelets	95
4.2.3 Nonstandard multi-scale decomposition of finite element matrix	96
4.3 Results and Discussion	103

Chapter 5: Wavelets for *A Posteriori* Error Estimation

5.1 Introduction	116
5.2 Wavelet Based Error Estimation	118
5.3 Numerical experiments	125

Chapter 6: Lamb Wave Signal Processing

6.1 Diagnosis of Structural Cracks in the Plate-Like Structures	134
6.1.1 Introduction	134

6.1.2 Mathematical formulation	137
6.1.2.1. Wavelet transform for denoising of signal	137
6.1.2.2. Matched filter and wavelet matched filter methods	140
6.1.2.2. Criteria for selecting the optimal wavelet using Shannon's entropy	142
6.1.3 Results and discussion	143
6.1.3.1. Single wave mode excitation	144
6.1.3.2. Multi wave mode excitation	161
6.2 Modeling of Lamb Wave for Cylindrical Structures	173
 Chapter 7 Conclusions and Future Work	
2.1 Conclusions	187
2.2 Future Research Directions	189
 References	 191
List of Publications	222

LIST OF FIGURES

Figure 1.1. The perceptible advantages of SHM over NDT&E.....	5
Figure 1.2. Outline of SHM system.	6
Figure 1.3. Guided wave based SHM system.	7
Figure 1.4. Surface displacement pattern in Rayleigh wave (Cheeke (2002)).....	9
Figure 1.5. A thin plate of $2t$ thickness.	10
Figure 1.6. Particle motion of Lamb waves in x and y-direction.....	10
Figure 1.7. Sketch diagram of particles motion in the symmetric and anti-symmetric wave modes.	17
Figure 1.8. Sketch diagram of Lamb wave propagating modes in isotropic plate.	17
Figure 1.9. Dispersion curves for Lamb modes in Al plate with 1 mm thickness.	19
Figure 1.10. Dispersion curves of Lamb modes in Al plate with 2 mm thickness.	20
Figure 3.1. Haar scaling and wavelet function.....	65
Figure 3.2. D4 scaling function and D4 wavelets.	65
Figure 3.3. B-spline scaling function and B-spline wavelets.....	66
Figure 3.4. Multi-scale decomposition.....	71
Figure 3.5. Snapshots of displacements at $t = 0.95$ s with various mesh number.....	75
Figure 3.6. Snapshots of displacements at different time instants with 100×100 mesh number.	76
Figure 3.7. Displacement variations along the x-axis at $t = 0.95$ s.	76
Figure 3.8. Displacement variations along the x-axis.	77
Figure 3.9. Displacement response u for time harmonic scalar wave.....	78
Figure 3.10. Contour plots of the displacement in the x-direction at four different time frame for isotropic plate by wavelet-based multiscale method.....	80
Figure 3.11. Excitation signal	81

Figure 3.12. Wave propagation for 400 kHz center frequency in plate at different nodes.	82
Figure 3.13. Response of plate with no. of elements per wavelength.....	83
Figure 3.14. Comparison of plate response at wavelet transform level 1.....	85
Figure 3.15. Comparison of plate response at wavelet transform level 2.....	85
Figure 3.16. Comparison of plate response at wavelet transform level 3.....	86
Figure 3.17. Comparison of plate response at wavelet transform level 4.....	86
Figure 4.1. Organization of a nonstandard form of the operator T	95
Figure 4.2. Multi-scale decomposition.....	99
Figure 4.3. Excitation signal for Lamb wave with higher harmonics.	104
Figure 4.4. Plate geometry, boundary conditions, forcing function and micro crack/ material softening.....	104
Figure 4.5. Convergence of Lamb wave response at healthy plate.....	105
Figure 4.6. Comparison of response of plate for 40, 80 and 120 elements per wavelength for higher harmonics.	106
Figure 4.7. Comparison of higher harmonic Lamb waves plates response at wavelet transform level.	107
Figure 4.8. Comparison of non-dimensional L_2 norm value for the higher harmonic signal with various wavelet.	108
Figure 4.9. Nodal displacement (mm) vs time (microsecond) at sensing location for a healthy plate.	110
Figure 4.10. Nodal displacement for a plate with crack.	110
Figure 4.11. Comparison of damage plate response at wavelet transform level 3.....	111
Figure 4.12. Comparison of non-dimensional L_2 norm for various wavelet in damage plate.....	111

Figure 4.13. The waveform of higher harmonics Lamb waves at the center frequency of 400 kHz.	112
Figure 4.14. The spectrum of higher harmonics Lamb waves at the center frequency of 400 kHz.	112
Figure 5.1. Orthogonality.	121
Figure 5.2. The solution at $t = 1.6$ with mesh size 40×40	127
Figure 5.3. Comparison of exact, two scale, and proposed error estimators.	128
Figure 5.4. The solution at $t = 1.1$ s with mesh size 40×40	129
Figure 5.5. Comparison of two scale, and proposed error estimators.	130
Figure 5.6. Discretized one quarter of the rectangular plate with a hole.	131
Figure 5.7. Strain energy using two scale difference.	132
Figure 5.8. Strain energy using wavelet coefficient.	132
Figure 6.1. Two level filter bank for DWT analysis.	139
Figure 6.2. Proposed WMFM for damage identification.	142
Figure 6.3. Schematic of FE model.	145
Figure 6.4. Sensor output for S_0 mode excitation for the healthy and damage plate. ...	146
Figure 6.5. FE simulation of Lamb wave.	146
Figure 6.6. Shannon's entropy curves of the DWT from Lamb wave signal.	149
Figure 6.7. Shannon's entropy curves of the CWT from Lamb wave signal.	150
Figure 6.8. RMSE in various SNR level computed from wavelet transform.	152
Figure 6.9. A contaminated signal with SNR 0 dB for damage plate.	153
Figure 6.10. Wavelet transform for the contaminated signal with SNR (0 dB) for damaged plate.	153
Figure 6.11. Contaminated signal with SNR (0 dB) convoluted with healthy plate signal.	154

Figure 6.12. The response of WMFM for the signal with SNR (0 dB).	154
Figure 6.13. A contaminated signal with SNR (10 dB) and coherent noise.	156
Figure 6.14. Wavelet transform for the contaminated signal with SNR (10 dB) and coherent noise.	157
Figure 6.15. MFT response for the contaminated signal with SNR (10 dB) and coherent noise.	157
Figure 6.16. WMFM response for the contaminated signal with SNR (10 dB) and coherent noise.	158
Figure 6.17. FE model.....	162
Figure 6.18. Sensor output for S_0 mode excitation for the healthy and damaged plate.	162
Figure 6.19. Shannon's entropy curves of the DWT from Lamb wave signal.	163
Figure 6.20. Shannon's entropy curves of the CWT from Lamb wave signal.	164
Figure 6.21. RMSE in various SNR level computed from wavelet transform.	166
Figure 6.22. A contaminated signal with SNR level 0 dB for damaged plate.	167
Figure 6.23. Response of wavelet transform for the noisy damaged plate signal with SNR (0 dB).	167
Figure 6.24. MFT response for the noisy signal with SNR (0 dB) convoluted with healthy signal.	168
Figure 6.25. The response of WMFM for the noisy damaged plate signal with SNR (0 dB).....	168
Figure 6.26. A contaminated damaged plate signal with SNR (5 dB) and coherent noise.	169
Figure 6.27. Wavelet Transform for the contaminated signal with SNR (5 dB) and coherent noise.	169

Figure 6.28. MFT response for the contaminated signal with SNR (5 dB) and coherent noise.	170
Figure 6.29. The response of WMFM for the contaminated signal with SNR 5 dB. ..	170
Figure 6.30. The configuration of a hollow cylinder with circumferential notch.....	174
Figure 6.31. Snapshots of FE Simulation of Lamb wave propagation in brass pipe and scattering.	175
Figure 6.32. Sensor output for the L (0, 2) mode excitation for healthy and damaged cylinder.....	175
Figure 6.33. Shannon's entropy curves of the DWT from Lamb wave signal.	176
Figure 6.34. Shannon's entropy curves of the CWT from Lamb wave signal.	177
Figure 6.35. RMSE in various SNR level computed from wavelet transform.	179
Figure 6.36. A contaminated signal with SNR (0 dB) for damaged cylinder.....	180
Figure 6.37. Response of wavelet transform for the noisy damaged cylinder signal with SNR (0 dB).....	180
Figure 6.38. MFT response for the noisy signal with SNR (0 dB) convoluted with healthy signal.	181
Figure 6.39. The response of WMFM for the noisy damaged cylinder signal with SNR (0 dB).....	181
Figure 6.40. A contaminated damaged cylinder signal with SNR (5 dB) and coherent noise.	182
Figure 6.41. Wavelet Transform for the contaminated signal with SNR (5 dB) and coherent noise.	182
Figure 6.42. MFT response for the contaminated signal with SNR (5 dB) and coherent noise.	183
Figure 6.43. The response of WMFM for the contaminated signal with SNR 5 dB. ..	183

LIST OF TABLES

Table 1.1. An overview of the most commonly employed NDT&E techniques.	2
Table 1.2. Comparison of Lamb wave transducers with other NDE transducers.	22
Table 4.1. Comparison of computational time for the simulation of Lamb wave.	113
Table 6.1. Comparison of RMSE, correlation coefficient and post improved SNR of the wavelet denoising.....	159
Table 6.2. Comparison of RMSE, correlation coefficient and post improved SNR of the MFT	160
Table 6.3. Comparison of RMSE, correlation coefficient and post improved SNR of the WMFM.....	160
Table 6.4. Comparison of RMSE, correlation coefficient and post improved SNR of the wavelet based denoising using bior4.4 wavelet function.....	171
Table 6.5. Comparison of RMSE, correlation coefficient and post improved SNR of the MFT	172
Table 6.6. Comparison of RMSE, correlation coefficient and post improved SNR of the WMFM using bior4.4 wavelet function	172
Table 6.7. Comparison of RMSE, correlation coefficient and post improved SNR of the wavelet based denoising using bior4.4 wavelet function.....	184
Table 6.8. Comparison of RMSE, correlation coefficient and post improved SNR of the MFT	184
Table 6.9. Comparison of RMSE, correlation coefficient and post improved SNR of the WMFM using bior4.4 wavelet function	184

LIST OF ABBREVIATIONS

Al	Aluminium
AR	Autoregressive
ARIMA	Autoregressive Integrated Moving Average
AWGN	Additive White Gaussian Noise
BEM	Boundary Element Method
CWT	Continuous Wavelet Transform
D4	Daubechies order 4
DOF	Degree of Freedom
DWT	Discrete Wavelet Transform
EFIT	Elastodynamic Finite Integration Technique
FCM	Finite Cell Method
FDM	Finite Difference Method
FEM	Finite Element Method
FFT	Fast Fourier Transform
FT	Fourier Transform
FVM	Finite Volume Method
HHT	Hilbert-Huang transforms
HP	High Pass
HT	Hilbert Transform
IDWT	Inverse Discrete Wavelet Transform
LISA	Local Interaction Simulation Approach
LP	Low Pass
MA	Moving Average
MFT	Matched Filter Technique
MP	Matching Pursuit
NDT&E	Non-Destructive Testing & Evolution
NS	Non-Standard
PDEs	Partial Differential Equations
PVDF	Polyvinylidene Fluoride
PZT	Piezoelectric lead Zirconate Titanate
RHP	Reverse High Pass
RLP	Reverse Low Pass

RMSE	Root Mean Square Error
SBFEM	Scaled Boundary Finite Element Method
SCM	Spectral Cell Method
SEM	Spectral Element Method
SHM	Structural Health Monitoring
SIM	Sharp Interface Model
SNR	Signal-to-Noise Ratio
STFT	Short Time Fourier Transform
ToF	Time of Flight
WMFM	Wavelet Matched Filter Method
WT	Wavelet Transform
WVD	Wigner-Ville Distribution

LIST OF SYMBOLS

A_0	Fundamental anti-symmetric Lamb modes
(c_p)	Phase velocity
(c_g)	Group velocity
$c_{j,k}$	Scaling coefficients
C_L and C_T	Longitudinal and shear velocity
$d_{j,k}$	Wavelet coefficients
E	Young's Modulus
f_x and f_y	Body forces
h	Mesh size
k	Circular wave number
$[K]$	Stiffness matrix
$[M]$	Mass matrix
P_j	Projection onto the space V_j
Q_j	Projection onto the space W_j
r	Correlation coefficient
S_0	Fundamental symmetric Lamb modes
$[T_j]$	Wavelet transformation matrix
u_x	Velocity in X-direction
u_y	Velocity in Y-direction
$[u]$	Unknown coefficient vectors
$[\ddot{u}]$	Unknown coefficient vectors
V_i	Space spanned by a set of scaling function
W_i	Space spanned by a set of wavelet function
\bar{x}	Mean of original signal
Δx	Element size
\bar{y}	Mean of denoised signal

ν	Poisson ratio
σ_{xx} and σ_{yy}	Normal stresses
τ_{xy}	Shear stress
λ and μ	Lamé constants
λ_{wave}	Wavelength
ϕ	Scaling function
ψ	Wavelet function
ω	Circular frequency