Fractional calculus has become very popular among the researchers in the last few decades, because of its wide applicability in many fields. Fractional differential equations are obtained by replacing the integer order derivative by fractional order. These differential equations can model even those physical situations to an extent of accuracy which integer order calculus cannot. The major advantage of fractional calculus over integer order calculus is its nonlocal effect which is also called its memory effect. Fractional order model gives the greater degree of flexibility.

This thesis consists of six chapters beginning with introduction as **Chapter 1**, which contains some of the basics of fractional calculus, and literature review related with this field.

In the **Chapter 2**, Homotopy analysis method is used to obtain approximate analytic solution of the time-fractional diffusion-wave equation with given initial conditions. A special drive has been given to show the effect of reaction term with long term correlation to the diffusion-wave solutions for various values of anomalous exponent to constitute a good mathematical model useful for various engineering and scientific systems. Effects of parameters on the solution profile are calculated numerically and presented through graphs for different particular cases. Sub-diffusion and Superdiffusion phenomena for birth and death processes are also shown through figures.

In the **Chapter 3**, investigative study is done for the solutions of differential equations with fractional order derivatives applying a modified fractional order variational iteration method. The Lagrangian multiplier in the proposed method is expanded in the form of fractional order Taylor series, which helps to achieve the solutions in approximate analytical forms.

In the **Chapter 4**, an endeavor is made to solve the variable order fractional diffusion equations using a powerful method viz., Homotopy Analysis method. It is demonstrated how the method can be used while solving approximately two types of variable order fractional diffusion equations having physical importance. Numerical simulation results show that the method is reliable and effective for solving fractional order diffusion equations even when the order of the derivative is varying with respect to space or time or both or it is dependent upon some other parameters.

In the **Chapter 5**, Homotopy analysis method is successfully used to find the approximate solution of fractional order Van der Pol equation. The fractional derivative is described in the Caputo sense. The numerical computations of convergence control parameters for the acceleration of convergence of approximate series solution through the analysis of

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minimization of error for different particular cases are obtained and the results are depicted through graphs. The salient feature of the chapter is the graphical presentation of achieving limit cycles for different parameters.

In the **Chapter 6**, an effort has been given to solve fractional order vibration equation with the help of operational matrix with Bernstein polynomial basis. Numerical calculations are carried out for integer order as well as fractional order derivatives for different particular cases, which are depicted through graphs.