A $[\mathcal{K}, \mathcal{KL}]$ Sector-Based Hands-Off Control With Quantization Parameter Mismatch

Ankit Sachan, Sharat Chandra Mahto, Vijay Kumar Singh, Shyam Kamal, *Member, IEEE*, and Thach Ngoc Dinh

***Abstract*—A Hands-off control law is investigated to asymptotically stabilize a nonlinear system with input-quantization. The quantization parameters generated at the coder and decoder sides of the communication channel are mismatched and a time-varying ratio is modeled to show the non-synchronous adjustment of the quantization parameters. By the means of $[\mathcal{K}, \mathcal{KL}]$ sector, the state-space is roughly divided into an attractive and non-attractive regions and a controller based on On-Off logic is utilized to remove the model uncertainties caused by the mismatch of the quantization parameters with limited control effort. Finally, the illustration of the proposed scheme is verified for an electronic circuit.**

***Index Terms*—$[\mathcal{K}, \mathcal{KL}]$ sector, hands-off control, input-quantization, asymptotic stability.**

I. Introduction

HANDS-OFF control [1] is a paradigm of variable structure control which shows a switching action between an open-loop structure and a closed-loop structure of the dynamical system. This equips with a minimum assistance to govern the state towards a stable region. However, the switching criterion of event-triggered control [2] is much likely similar to that of Hands-off control but the main difference is that the state vectors are continuously monitored and the control signal is only updated when its value crosses a certain threshold value obtained from its previous sampled value. More likely, the execution of event-triggered control is done in aperiodic manner and some work referred in [3]–[5]. By the means of Matrosov's proof [6], the state-space is roughly divided into an attractive and non-attractive regions depending upon the sign-definiteness of the derivative of Lyapunov-candidate function. Institutively, some difficulty appears in the construction of an attractive region for nonlinear system because there is no general procedure to find an appropriate Lyapunov function and one should decide by trial and error method. This construction led significant amount of work to recall Zubov method [7] where the Lyapunov function approaches the value 1 on the boundary of attractive region. The computational advantage for this procedure is that the information about the solution is not required. An approach for computing control-Lyapunov function was first introduced by Sontag in [8] which results a universal formula for feedback stabilization. This procedure avoids the dependability of the system to solve for an algebraic/differential Riccati equation [9] for the considered system to decide the boundary of a sector.

In order to design an attractive region for nonlinear system, a $[\mathcal{K}, \mathcal{KL}]$ sector is proposed in [10], [11] and its boundary to separate the regions is decided by a control-Lyapunov function [12]. Here, the Hands-off control seems to perform a switching action to ensure the system stability. Moreover, some additional developments are made in the direction of robustness analysis for nonlinear sector design as in [13], [14].

In modern engineering systems, signal quantization is an ongoing research topic due to wide applications of communication channels with information processing units, see [15]–[17] and references therein. Among these research articles, the important aspect is the quantization of state signal is done before the design of feedback control design. For example, for nonlinear complex system in the busty fading channels, the event-triggering based control law is investigated in [18]. However, the communication saving for complex system is done by integrating the quantization approach with event-based scheme [20]. In addition, in [19], quantized-feedback control for nonlinear system with nonlinear sector design was investigated. In general, the quantization parameters cultivated at both the ends of the communication channel are assumed to be identical. But the discrepancy of quantization parameter occurs because of hardware imperfections and firstly investigated in [21]. Similar discussion to provide the stability conditions for the nonlinear system with encoder/decoder mismatch are attained in [22], [23]. While analyzing the inconsistency in quantization parameters of coder and decoder, its ratio is needed to remain unchanged by adjusting the synchronization of quantization parameters of coder and decoder at every time-instant.

The above discussion motivate us to study mismatch relation between quantization parameter of coder and decoder sides for nonlinear system dealing with $[\mathcal{K}, \mathcal{KL}]$ sector.

In this brief, a Hands-off control is designed for the mismatched relation between the quantization parameter of coder and decoder ends of communication channel. The main contribution includes threefolds. First, a $[\mathcal{K}, \mathcal{KL}]$ sector is designed for the nonlinear system within the $n^{th}-$dimensional space. Second, a time-varying ratio is modeled for showing the mismatched relation of the quantization parameters at the coder and decoder sides. Third, a controller based on On-Off logic is utilized to remove the effect of model uncertainties appeared

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while the input-quantization. Thus, the closed-loop system acquires asymptotic stability.

II. System Description

Consideration of nonlinear-affine system with quantized input in the form of state-space equation as follows:

$$\dot{\zeta} = F(\zeta) + G(\zeta)Q(u(t)) \tag{1}$$

where the functions $F(\zeta)$ and $G(\zeta)$ are $\mathcal{C}^1$ continuously differentiable and maps $F, G : \mathcal{Z} \to \mathbb{R}^n$ where a domain $\mathcal{Z}$ containing an equilibrium point at origin $\zeta = 0$. Here, $\zeta \in \mathbb{R}^n$ is state vector, $u \in \mathbb{R}$ is control vector, and $Q(u(t))$ is the quantized-input. Moreover, the pair$(F(\zeta), G(\zeta))$ is assumed to be stabilizable.

A. Review of [$\mathcal{K}, \mathcal{KL}$] Sector

Some related notions for [$\mathcal{K}, \mathcal{KL}$] sector are presented below

Definition 1 [10]: A [$\mathcal{K}, \mathcal{KL}$] sector Ψ for the nonlinear system (1) is defined as

$$\Psi = \left\{ \zeta \in \mathbb{R}^n \,:\, \varrho^2(\zeta) \leq \xi^2(\zeta) \right\} \tag{2}$$

inside which a Lyapunov-candidate function $W : \mathcal{Z} \to \mathbb{R}^n$ is bounded between Lipschitz class $\mathcal{K}_\infty$ functions

$$\varphi_1(|\zeta|) \leq W(\zeta) \leq \varphi_2(|\zeta|), \qquad \zeta \in \mathcal{Z} \tag{3}$$

$\varphi_1, \varphi_2 \in \mathcal{K}_\infty$, decreases monotonically with zero-input and its derivative along with the trajectories of the nonlinear system (1) inside the sector satisfies

$$\frac{d}{dt}W(\zeta) = \frac{\partial W}{\partial \zeta}F(\zeta) \leq -\hat{\varphi}_3(|\zeta|), \qquad \forall \zeta \in \Psi \tag{4}$$

where $\hat{\varphi}_3$ is a Lipschitz class $\mathcal{K}$ function. Here [$\mathcal{K}, \mathcal{KL}$] sector parameters, i.e., $\varrho(\zeta)$ and $\xi(\zeta)$ are determined as

$$\varrho(\zeta) = \Xi = \frac{1}{2}\frac{\partial W}{\partial \zeta}G(\zeta), \tag{5}$$

$$\xi(\zeta) = \sqrt{\left|\varphi_3(|\zeta|) - \hat{\varphi}_3(|\zeta|)\right|} = (\phi\varphi_3(|\zeta|))^{1/2} \tag{6}$$

where φ_3 is a Lipschitz class $\mathcal{K}_\infty$ function and positive scalar ϕ $(0 < \phi < 1)$ follows the relation $\xi^2(\zeta) = \phi\varphi_3(|\zeta|)$ and $\hat{\varphi}(|\zeta|) = (1 - \phi)\varphi_3(|\zeta|)$.

The state evolves inside the [$\mathcal{K}, \mathcal{KL}$] sector gradually decreases with zero-input. This sector is not empty and at least hold a zero-state. It is proven in [10] that for any nonlinear system there exist a [$\mathcal{K}, \mathcal{KL}$] sector inside which state converges automatically towards equilibrium point and requires a control input whenever it appears outside the [$\mathcal{K}, \mathcal{KL}$] sector to satisfy the results of Lyapunov proof.

B. Input Quantization

Mathematically, the operator $Q(\cdot)$ is defined by the function $round(\cdot)$ that rounds towards the nearest integer, i.e.,

$$Q(\cdot) = \tau_d(t)round\left(\frac{\cdot}{\tau_c(t)}\right) \approxeq \tau_d(t)q\left(\frac{\cdot}{\tau_c(t)}\right) \tag{7}$$

where $\tau_d(t)$ and $\tau_c(t)$ are the quantization parameters for the decoder and the coder, respectively. In this operation, the input signal $u(t)$ used to stabilize the dynamical signal is assumed to be quantized. Here, the signal $u(t)$ is encoded by the coder and generates a quantized signal, i.e., $\{round(\frac{u(t)}{\tau_c(t)}) \triangleq q(\frac{u(t)}{\tau_c(t)})\}$ then transmitted to the decoder side via communication channel. The decoder receives the signal $q(\frac{u(t)}{\tau_c(t)}) + n(t)$ which is not necessarily identical to the signal transmitted by the coder and the term $n(t)$ consists of overall noise appeared in the channel such as packet loss, time-delay, bit error, etc. According to this, the information of quantization measurement on coder and decoder sides are respectively shown as $q_{\tau_c(t)} = \tau_c(t)q(\frac{u(t)}{\tau_c(t)}),\ \tau_c(t) > 0$ and $q_{\tau_d(t)} = \tau_d(t)(q(\frac{u(t)}{\tau_c(t)}) + n(t)),\ \tau_d(t) > 0$. In case the communication channel is ideal, i.e., $n(t) = 0$, the decoder receives the signal similar to that of coder side and the quantization parameters at the coder side $\tau_c(t)$ and the decoder side $\tau_d(t)$ are the same, i.e., $\tau_c(t) = \tau_d(t)$ for all time instants $t > 0$.

However, for practical engineering applications, it is quite difficult to generate similar quantization parameters at both ends of the communication channel. Thus, we need to show the relation between the coder/decoder schemes. For this, the adjustment for the coder and the decoder quantization parameters are to be synchronized at every time instant such that the ratio $\frac{\tau_d(t)}{\tau_c(t)}$ remain unchanged throughout the process. Here the bandwidth of input quantization is inevitable so the zoom variables are required for the adjustment of system evolution information. Roughly, zoom variables are composed of two stages, i.e., zoom-out stage and zoom-in stage. At zoom-out stage, the value of quantization parameter increased until the states can be appropriately captured. At zoom-in stage, decrease in the value of quantization parameter in such a way to force the state to the origin. Thus, a time-varying parameter is considered to establish a relation between the quantization parameters as follows:

$$\nu(t) \in (\nu_{\min}, \nu_{\max}) = \frac{\tau_d(t)}{\tau_c(t)} \tag{8}$$

where $\nu_{\min}$ and $\nu_{\max}$ are two positive scalars, satisfying the condition $\nu_{\max} \geq \nu_{\min} > 1$.

Combining (1), (7) and (8), the nonlinear system can be rearranged as

$$\begin{aligned}\dot{\zeta}(t) &= F(\zeta) + G(\zeta)\tau_d(t)q\left(\frac{u(t)}{\tau_c(t)}\right)\\ &= F(\zeta) + G(\zeta)\frac{\tau_d(t)}{\tau_c(t)}\tau_c(t)q\left(\frac{u(t)}{\tau_c(t)}\right)\\ &= F(\zeta) + G(\zeta)\nu(t)q_{\tau_c(t)}(u(t))\end{aligned} \tag{9}$$

Let us define the quantization error as follows:

$$e_{\tau_c(t)} := q_{\tau_c(t)}(u(t)) - u(t) \tag{10}$$

Combining Eqs. (9) and (10), one can easily obtain that

$$\begin{aligned}\dot{\zeta}(t) &= F(\zeta) + G(\zeta)\nu(t)\big(u(t) + e_{\tau_c(t)}\big)\\ &= F(\zeta) + G(\zeta)u(t) + G(\zeta)(\nu(t) - 1)u(t) + G(\zeta)\nu(t)e_{\tau_c(t)}\end{aligned} \tag{11}$$

Thus, a robust control technique is designed to deal with nonlinear system subjected to quantization parameter mismatch for achieving asymptotic stability. Before main results, the following lemma is discussed.

*Lemma 1 (**Hölder's Inequality [24]**):* For $\alpha,\ \beta \in \mathbb{R}^n$, $p \geq 1$ and $q \geq 1$, the following inequality holds:

$$|\alpha^\top \beta| \leq |\alpha|_p|\beta|_q, \qquad p^{-1} + q^{-1} = 1. \tag{12}$$

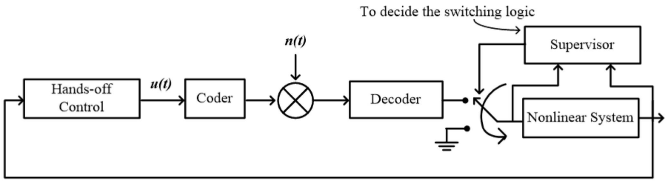

Fig. 1. Schematic diagram for Hands-off control with input quantization.

III. Main Result

In this section, a Hands-Off control based on $[\mathcal{K}, \mathcal{KL}]$ sector is studied and the quantization parameter mismatch for control-input is considered to achieve asymptotic stability for nonlinear system.

Theorem 1: Corresponding to the nonlinear-affine system (1), the Hands-off control is designed for $[\mathcal{K}, \mathcal{KL}]$ sector with quantization parameter mismatch as follows:

$$u(t) \triangleq \chi(\varrho(\zeta), \xi(\zeta))(u_1(t) + u_2(t)) \tag{13}$$

where

$$u_1(t) \triangleq -\left(\frac{\partial \Xi}{\partial \zeta} G(\zeta)\right)^{-1} \left(\frac{\partial \Xi}{\partial \zeta} F(\zeta) + K\varrho(\zeta)\right) \tag{14}$$

$$u_2(t) \triangleq -\frac{1}{1-\lambda}\left(\frac{\partial \Xi}{\partial \zeta} G(\zeta)\right)^{-1} [\Pi_1(\zeta) + \Pi_2]\mathrm{sat}\left(\frac{\varrho(\zeta)}{\epsilon}\right) \tag{15}$$

and the switching logic $\chi(\varrho(\zeta), \xi(\zeta)) = \begin{cases} 0, & \text{if } \zeta \in \Psi \\ 1, & \text{if } \zeta \notin \Psi. \end{cases}$ Here, a saturation function of high-slope, i.e., $1/\epsilon$ is considered in place of signum function to eliminate chattering such that $sat(\varrho(\zeta)) = \begin{cases} \varrho(\zeta), & \text{if } |\varrho(\zeta)| \le 1 \\ sign(\varrho(\zeta)), & \text{if } |\varrho(\zeta)| > 1 \end{cases}$. For good approximation, the limit of $\epsilon \to 0$ and the saturation function $sat(\varrho(\zeta)/\epsilon)$ approaches to signum function $sign(\varrho(\zeta))$. The $[\mathcal{K}, \mathcal{KL}]$ sector with $\xi^2(\zeta) = \phi\varphi_3(|\zeta|)$ and $\hat{\varphi}_3(|\zeta|)$ for positive constant ϕ $(0 < \phi < 1)$ provides a stable region in the state-space such that the state converges to the origin for closed-loop system to attain asymptotic stability. The design parameters are chosen as $\Pi_1(\zeta) \ge \lambda[|\frac{\partial \Xi}{\partial \zeta} F(\zeta)| + K|\varrho(\zeta)|]$, $\Pi_2 \ge \nu_{\max}|\frac{\partial \Xi}{\partial \zeta} G(\zeta)|\Delta\tau_c$ and $\lambda \ge \max\{|\nu_{\min} - 1|, |\nu_{\max} - 1|\}$ where Δ is scalar term to satisfy the relation $|e_{\tau_c}(t)| \le \Delta\tau_c(t)$ and the positive constant K is large enough to satisfy the condition $K > \max\{(\frac{\partial \Xi}{\partial \zeta} G(\zeta))/2, K_0\}$ and hold the inequality relation

$$2K_0\phi\varphi_3(|\zeta|) + \varrho^\top(\zeta)\frac{\partial \Xi}{\partial \zeta} F(\zeta) + \left(\frac{\partial \Xi}{\partial \zeta} F(\zeta)\right)^\top \varrho(\zeta) > 0. \tag{16}$$

Proof: The initial value of the state is assumed to be outside of the $[\mathcal{K}, \mathcal{KL}]$ sector ζ such that the control law determined in (13)-(15) is active for every $\zeta \notin \Psi$ and swtiching logic $\chi(\varrho(\zeta), \xi(\zeta)) = 1$.

With this, the time-derivative of $\varrho^2(\zeta)$ for quantization parameter mismatch with condition $|\varrho(\zeta)| > \xi(\zeta)$ is as follows:

$$\begin{aligned} \frac{d}{dt}\varrho^2(\zeta) &= 2\varrho(\zeta)\dot{\varrho}(\zeta) = 2\varrho(\zeta)\frac{\partial \Xi}{\partial \zeta}\frac{d\zeta}{dt} \\ &= 2\varrho(\zeta)\Big[\frac{\partial \Xi}{\partial \zeta} F(\zeta) + \frac{\partial \Xi}{\partial \zeta} G(\zeta)u(t) + (\nu(t)-1)\frac{\partial \Xi}{\partial \zeta} G(\zeta)u(t) \\ &\quad + \nu(t)\frac{\partial \Xi}{\partial \zeta} G(\zeta)e_{\tau_c}(t)\Big] \end{aligned}$$

By (13)-(15) and $|\nu(t) - 1| \le \max\{|\nu_{\min} - 1|, |\nu_{\max} - 1|\} \le \lambda$, we have

$$\begin{aligned} \varrho(\zeta)\dot{\varrho}(\zeta) &= -K\varrho^2(\zeta) + \varrho(\zeta)\lambda\frac{\partial \Xi}{\partial \zeta} G(\zeta)u_2(t) + \varrho(\zeta)(\nu(t)-1) \\ &\times \frac{\partial \Xi}{\partial \zeta} G(\zeta)u_2(t) + \varrho(\zeta)(1-\lambda)\frac{\partial \Xi}{\partial \zeta} G(\zeta)u_2(t) + \varrho(\zeta) \\ &\times \left[(1-\nu(t))\left\{\frac{\partial \Xi}{\partial \zeta} F(\zeta) + K\varrho(\zeta)\right\} + \nu(t)\frac{\partial \Xi}{\partial \zeta} G(\zeta)e_{\tau_c(t)}\right] \end{aligned} \tag{17}$$

By the definition of ∞-norm for relation $|xy| \le |x||y|$ and noting that the quantization error $|e_{\tau_c}(t)| \le \Delta\tau_c(t)$ for a portion of (17), we get

$$\begin{aligned} &(1-\nu(t))\left\{\frac{\partial \Xi}{\partial \zeta} F(\zeta) + K\varrho(\zeta)\right\} + \nu(t)\frac{\partial \Xi}{\partial \zeta} G(\zeta)e_{\tau_c(t)} \\ &\le |1-\nu(t)|\left\{\left|\frac{\partial \Xi}{\partial \zeta} F(\zeta) + K\varrho(\zeta)\right|\right\} + |\nu(t)|\left|\frac{\partial \Xi}{\partial \zeta} G(\zeta)\right||e_{\tau_c}(t)| \\ &\le \lambda\left\{\left|\frac{\partial \Xi}{\partial \zeta} F(\zeta)\right| + K|\varrho(\zeta)|\right\} + \nu_{\max}\left|\frac{\partial \Xi}{\partial \zeta} G(\zeta)\right|\Delta\tau_c(t) \\ &\le \Pi_1(\zeta) + \Pi_2 \end{aligned} \tag{18}$$

where the design parameters $\Pi_1(\zeta)$ and Π_2 follows the relation

$$\begin{cases} \Pi_1(\zeta) \ge \lambda\left\{\left|\frac{\partial \Xi}{\partial \zeta} F(\zeta)\right| + K|\varrho(\zeta)|\right\} \\ \Pi_2 \ge \nu_{\max}\left|\frac{\partial \Xi}{\partial \zeta} G(\zeta)\right|\Delta\tau_c(t) \end{cases} \tag{19}$$

By Hölder inequality [24], and by utilizing the design $u_2(t)$ is mentioned in (15), a part of (17) is simplified, we get

$$\begin{aligned} &\varrho(\zeta)(1-\lambda)\frac{\partial \Xi}{\partial \zeta} G(\zeta)u_2(t) + \varrho(\zeta)[(1-\nu(t)) \\ &\quad \times\left\{\frac{\partial \Xi}{\partial \zeta} F(\zeta) + K\varrho(\zeta)\right\} + \nu(t)\frac{\partial \Xi}{\partial \zeta} G(\zeta)e_{\tau_c(t)}\Big] \\ &\le \varrho(\zeta)(1-\lambda)\frac{\partial \Xi}{\partial \zeta} G(\zeta)u_2(t) + |\varrho(\zeta)|_1\big[\Pi_1(\zeta) + \Pi_2\big] \\ &= \varrho(\zeta)(1-\lambda)\frac{\partial \Xi}{\partial \zeta} G(\zeta)\left\{-\frac{1}{1-\lambda}\left(\frac{\partial \Xi}{\partial \zeta} G(\zeta)\right)^{-1}\right. \\ &\quad \left.\times\big[\Pi_1(\zeta) + \Pi_2\big]\varrho(\zeta)\mathrm{sat}\left(\frac{\varrho(\zeta)}{\epsilon}\right)\right\} + |\varrho(\zeta)|_1\big[\Pi_1(\zeta) + \Pi_2\big] \\ &= -\big[\Pi_1(\zeta) + \Pi_2\big]\varrho(\zeta)\mathrm{sat}\left(\frac{\varrho(\zeta)}{\epsilon}\right) \\ &\quad + |\varrho(\zeta)|_1\big[\Pi_1(\zeta) + \Pi_2\big] = 0 \end{aligned} \tag{20}$$

On the other hand, by inserting the control signal $u_2(t)$ (15) and the inequality relation (19) in some other part of (17), we have

$$\begin{aligned} &\varrho(\zeta)\lambda\frac{\partial \Xi}{\partial \zeta} G(\zeta)u_2(t) + \varrho(\zeta)(\nu(t)-1)\frac{\partial \Xi}{\partial \zeta} G(\zeta)u_2(t) \\ &\le \varrho(\zeta)\lambda\frac{\partial \Xi}{\partial \zeta} G(\zeta)\left\{-\frac{1}{1-\lambda}\left(\frac{\partial \Xi}{\partial \zeta} G(\zeta)\right)^{-1}[\Pi_1(\zeta) + \Pi_2]\right. \\ &\quad \left.\times\ \mathrm{sat}\left(\frac{\varrho(\zeta)}{\epsilon}\right)\right\} + |\varrho(\zeta)|\lambda\left|\frac{\partial \Xi}{\partial \zeta} G(\zeta)\left\{-\frac{1}{1-\lambda}\right.\right. \\ &\quad \left.\left.\times\left(\frac{\partial \Xi}{\partial \zeta} G(\zeta)\right)^{-1}[\Pi_1(\zeta) + \Pi_2]\right\}\right| \\ &\le -\frac{\lambda}{1-\lambda}[\Pi_1(\zeta) + \Pi_2]\varrho(\zeta)\mathrm{sat}\left(\frac{\varrho(\zeta)}{\epsilon}\right) \end{aligned}$$

$$
\begin{aligned}
&+ |\varrho(\zeta)|\lambda\Big|\frac{1}{1-\lambda}\Big|\big|[\Pi_1(\zeta)+\Pi_2]\big|\Big|\mathrm{sat}\Big(\frac{\varrho(\zeta)}{\epsilon}\Big)\Big| \\
&\le -\frac{\lambda}{1-\lambda}[\Pi_1(\zeta)+\Pi_2]\varrho(\zeta)\mathrm{sat}\Big(\frac{\varrho(\zeta)}{\epsilon}\Big) \\
&+ \Big|\frac{\lambda}{1-\lambda}\Big|\big|[\Pi_1(\zeta)+\Pi_2]\big| = 0 \qquad (21)
\end{aligned}
$$

By substituting the values of (20) and (21) in (17), we can see that

$$\frac{d}{dt}\varrho^2(\zeta) \le -2K\varrho^2(\zeta) \qquad (22)$$

This claims that the state moves towards the $[\mathcal{K}, \mathcal{KL}]$ sector in some finite interval of time with sufficiently large controller gain K.

At the same time, the Lyapunov stability of the closed-loop system (1) by the use of Hands-off control (13)-(15) follows:

$$
\begin{aligned}
\dot{W}(\zeta) &= \frac{\partial W}{\partial\zeta}F(\zeta) + \frac{\partial W}{\partial\zeta}G(\zeta)u(t) + \frac{\partial W}{\partial\zeta}G(\zeta)(\nu(t)-1)u(t) \\
&\quad + \frac{\partial W}{\partial\zeta}G(\zeta)\nu(t)e_{\tau_c}(t) \\
&= \varrho^2(\zeta) - \xi^2(\zeta) - \hat{\varphi}_3(|\zeta|) + 2\varrho(\zeta)u(t) \\
&\quad + 2\varrho(\zeta)(\nu(t)-1)u(t) + 2\varrho(\zeta)\nu(t)e_{\tau_c}(t) \\
&= \varrho^2(\zeta) - \xi^2(\zeta) - \hat{\varphi}_3(|\zeta|) \\
&\quad + 2\varrho(\zeta)\left\{-\Big(\frac{\partial W}{\partial\zeta}G(\zeta)\Big)^{-1}\Big(\frac{\partial W}{\partial\zeta}F(\zeta) + K\varrho(\zeta)\Big)\right\} \\
&\quad + 2\varrho(\zeta)u_2(t) + 2\varrho(\zeta)(\nu(t)-1)u_2(t) + 2\varrho(\zeta)\nu(t)e_{\tau_c}(t) \\
&\quad + 2\varrho(\zeta)(1-\nu(t))\Big(\frac{\partial W}{\partial\zeta}G(\zeta)\Big)^{-1}\Big(\frac{\partial W}{\partial\zeta}F(\zeta) + K\varrho(\zeta)\Big)
\end{aligned}
$$

In term of $\varrho^2(\zeta) > \xi^2(\zeta)$, we have

$$
\begin{aligned}
\dot{W}(\zeta) &\le -2\Big(\frac{\partial W}{\partial\zeta}G(\zeta)\Big)^{-1}K\xi^2(\zeta) - \hat{\varphi}_3(|\zeta|) + 2\varrho(\zeta)\lambda u_2(t) \\
&\quad + 2\varrho(\zeta)(\nu(t)-1)u_2(t) - 2\varrho(\zeta)\Big(\frac{\partial W}{\partial\zeta}G(\zeta)\Big)^{-1}\frac{\partial W}{\partial\zeta}F(\zeta) \\
&\quad + 2\varrho(\zeta)(1-\nu(t))\Big(\frac{\partial W}{\partial\zeta}G(\zeta)\Big)^{-1}\Big(\frac{\partial W}{\partial\zeta}F(\zeta) + K\varrho(\zeta)\Big) \\
&\quad + 2\varrho(\zeta)(1-\lambda)u_2(t) + 2\varrho(\zeta)\nu(t)e_{\tau_c}(t) \qquad (23)
\end{aligned}
$$

Following the equation (18), one can solve

$$
\begin{aligned}
&(1-\nu(t))\Big(\frac{\partial W}{\partial\zeta}G(\zeta)\Big)^{-1}\Big(\frac{\partial W}{\partial\zeta}F(\zeta) + K\varrho(\zeta)\Big) + \nu(t)e_{\tau_c}(t) \\
&\le \Big(\frac{\partial W}{\partial\zeta}G(\zeta)\Big)^{-1}[\Pi_1(\zeta)+\Pi_2] \qquad (24)
\end{aligned}
$$

By inserting the control signal $u_2(t)$ (15) in some part of (23), we have

$$
\begin{aligned}
&\varrho(\zeta)(1-\lambda)u_2(t) + \varrho(\zeta)\Bigg[(1-\nu(t))\Big(\frac{\partial W}{\partial\zeta}G(\zeta)\Big)^{-1} \\
&\quad \times\Big(\frac{\partial W}{\partial\zeta}F(\zeta) + K\varrho(\zeta)\Big) + \nu(t)e_{\tau_c}(t)\Bigg] \\
&= \varrho(\zeta)(1-\lambda)\left\{-\frac{1}{1-\lambda}\Big(\frac{\partial \varXi}{\partial\zeta}G(\zeta)\Big)^{-1}[\Pi_1(\zeta)+\Pi_2]\right. \\
&\quad \left.\times\,\mathrm{sat}\Big(\frac{\varrho(\zeta)}{\epsilon}\Big)\right\} + |\varrho(\zeta)|\Big(\frac{\partial \varXi}{\partial\zeta}G(\zeta)\Big)^{-1}[\Pi_1(\zeta)+\Pi_2] \\
&= -\Big(\frac{\partial \varXi}{\partial\zeta}G(\zeta)\Big)^{-1}[\Pi_1(\zeta)+\Pi_2]\varrho(\zeta)\mathrm{sat}\Big(\frac{\varrho(\zeta)}{\epsilon}\Big) \\
&\quad + |\varrho(\zeta)|\Big(\frac{\partial \varXi}{\partial\zeta}G(\zeta)\Big)^{-1}[\Pi_1(\zeta)+\Pi_2] = 0 \qquad (25)
\end{aligned}
$$

Similarly, by inserting the control signal $u_2(t)$ (15) in some other part of (23) and noting $|\nu(t)-1| \le \max\{|\nu_{\min}-1|, |\nu_{\max}-1|\} \le \lambda$, $0<\lambda<1$, we have

$$
\begin{aligned}
&\varrho(\zeta)\lambda u_2(t) + \varrho(\zeta)(\nu(t)-1)u_2(t) \\
&\le \varrho(\zeta)\lambda\left\{-\frac{1}{1-\lambda}\Big(\frac{\partial \varXi}{\partial\zeta}G(\zeta)\Big)^{-1}[\Pi_1(\zeta)+\Pi_2]\mathrm{sat}\Big(\frac{\varrho(\zeta)}{\epsilon}\Big)\right\} \\
&\quad + |\varrho(\zeta)|\left|\left\{-\frac{\lambda}{1-\lambda}\Big(\frac{\partial \varXi}{\partial\zeta}G(\zeta)\Big)^{-1}[\Pi_1(\zeta)+\Pi_2]\mathrm{sat}\Big(\frac{\varrho(\zeta)}{\epsilon}\Big)\right\}\right| \\
&\le -\frac{1}{1-\lambda}\Big(\frac{\partial \varXi}{\partial\zeta}G(\zeta)\Big)^{-1}[\Pi_1(\zeta)+\Pi_2]\varrho(\zeta)\mathrm{sat}\Big(\frac{\varrho(\zeta)}{\epsilon}\Big) \\
&\quad + \frac{\lambda}{1-\lambda}|\varrho(\zeta)|\Big(\frac{\partial \varXi}{\partial\zeta}G(\zeta)\Big)^{-1}\big|[\Pi_1(\zeta)+\Pi_2]\big| = 0. \qquad (26)
\end{aligned}
$$

By inserting the value of (25) and (26) in (23) and noting $\xi^2(\zeta) = \phi\alpha_3(|\zeta|)$, we can see that

$$
\begin{aligned}
\dot{W}(\zeta) &\le -\Big(\frac{\partial \varXi}{\partial\zeta}G(\zeta)\Big)^{-1}\Bigg[2K\phi\varphi_3(|\zeta|) + \varrho^\top(\zeta)\frac{\partial \varXi}{\partial\zeta}G(\zeta) \\
&\quad + \Big(\frac{\partial \varXi}{\partial\zeta}G(\zeta)\Big)^\top\varrho(\zeta)\Bigg] - \hat{\varphi}_3(|\zeta|) \\
&\le -\hat{\varphi}_3(|\zeta|),\quad \forall\zeta\notin\Psi \text{ and } \chi(\varrho(\zeta),\xi(\zeta)) = 1. \qquad (27)
\end{aligned}
$$

After the state moves to the $[\mathcal{K}, \mathcal{KL}]$ sector, the Hands-off control is set to zero and the switching logic $\chi(\varrho(\zeta), \xi(\zeta)) = 0$. For this, the Lyapunov stability of the open-loop system for constraint $\varrho(\zeta) \le \xi(\zeta)$ follows:

$$
\begin{aligned}
\dot{W}(\zeta) &\le \varrho^2(\zeta) - \xi^2(\zeta) - \hat{\varphi}_3(|\zeta|) \\
&\le -\hat{\varphi}_3(|\zeta|),\quad \forall\zeta\in\Psi \text{ and } \chi(\varrho(\zeta),\xi(\zeta)) = 0. \qquad (28)
\end{aligned}
$$

This claims that the Lyapunov function monotonically decreases with the Hands-off control (13)-(15) to ensure asymptotic stability. ■

IV. Simulation Example

The performance of the proposed Hands-off control (13)-(15) with quantization parameter mismatch is demonstrated through an application of nonlinear electronic circuit [25]:

$$\ddot{\vartheta} + \epsilon(1-\vartheta)\dot{\vartheta} + \vartheta = u \qquad (29)$$

where v is the voltage output, $\epsilon = L/C$ represents the system constant with inductor value $L = 0.1$mH and capacitor value $C = 400\mu$F, u is a varying current source considered as a control signal to obtain a forced equation. The nonlinearity of the circuit is caused by an resistive element. Let us rewrite the above equation in state-space form:

$$\dot{\zeta}_1 = \zeta_2,\quad \dot{\zeta}_2 = -\zeta_1 - \frac{1}{2}(1-\zeta_1)\zeta_2 + u \qquad (30)$$

where $\zeta_1 := \vartheta$, $\zeta_2 := \dot{\vartheta}$. Let the Lyapunov function for the state-space model is $W = \frac{7}{2}\zeta_1^2 + 2\zeta_1\zeta_2 + \frac{2}{3}\zeta_2^2$. Thus, the $[\mathcal{K}, \mathcal{KL}]$ sector parameter is determined as $\varrho(\zeta) = \varXi = \zeta_1 + \frac{2}{3}\zeta_2$.

For simulation, let the initial state $\zeta(0) := [5\ 5]^\top$ and the controller gain $K = 0.8$. Now consider the evolution of

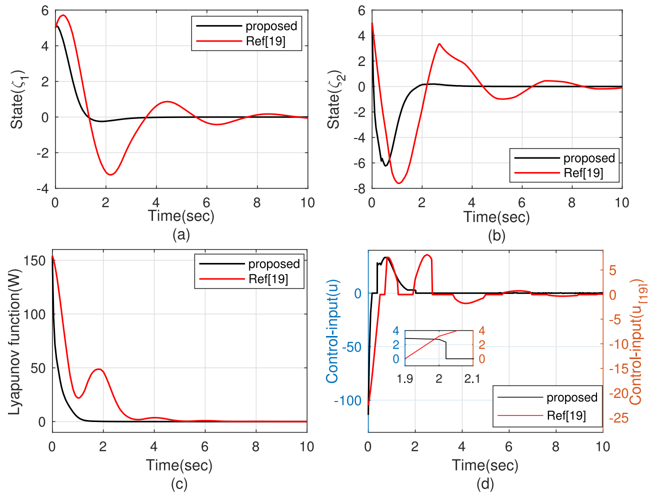

Fig. 2. Response curves for electronic circuit system (a) State ζ_1, (b) State ζ_2, (c) Lyapunov-candidate function W, and (d) Control-input $u(t)$.

$\tau_c(t)$ and $\tau_d(t)$ to illustrate the effect of quantization parameter mismatch is as follows:

$$\tau_c(t) = \begin{cases} 1, & \text{if } t \leq 0.5 \\ 0.6, & \text{if } 0.5 < t \leq 2 \\ 0.3, & \text{if } 2 < t \leq 5 \\ 0.2, & \text{if } t \geq 5 \end{cases}, \quad \tau_d(t) = \begin{cases} 1.2, & \text{if } t \leq 0.4 \\ 0.2, & \text{if } 0.4 < t \leq 2.5 \\ 0.1, & \text{if } 2.5 < t \leq 6 \\ 0.04, & \text{if } t \geq 6 \end{cases} \tag{31}$$

With this selection, we can see that there is a mismatch in the quantization parameters of coder and decoder. Thus, the ratio $\nu(t) = \frac{\tau_d(t)}{\tau_c(t)}$ is time-varying. However, to meet the requirement of the ratio $\nu(t) = \frac{\tau_d(t)}{\tau_c(t)}$ to remain unchanged, we restrict the condition to switch both the quantization parameters at the same time instant. Here, the minimum and the maximum of the ratio is obtained as $\nu_{\min} = 0.2$ and $\nu_{\max} = 1.2$ and furthermore, we calculate $\lambda \geq \max\{|\nu_{\min} - 1|, |\nu_{\max} - 1|\} = 0.8$. Applying the Hands-off control law proposed in Theorem 1, the response curves for the states ζ_1, ζ_2, the Lyapunov-candidate function W and the control-input $u(t)$ are depicted in Fig. 2.

It can be seen that the states of electronic circuit converges to $[\mathcal{K}, \mathcal{KL}]$ sector with Hands-off control and afterwards move to the origin automatically as shown in Figs. 2(a), (b). Similarly, the response curve of Lyapunov-candidate function in Fig. 2(c) is strictly decrease towards the origin. Finally, the Hands-off control in Fig. 2(d) is entailed by the switching mechanism to decide its operationality, i.e., control signal appears whenever inequality condition does not satisfy the relation given in (2). Thus, the controller switch to zero whenever the state lies inside the $[\mathcal{K}, \mathcal{KL}]$ sector.

V. Concluding Remarks

In this brief, the Hands-off control design for nonlinear system dealing with quantization parameter mismatch was investigated. First, a $[\mathcal{K}, \mathcal{KL}]$ sector is designed within the space by the results of comparison function. Second, a time-varying relation for the quantization parameters at both the ends of communication channel is established. At last, the stabilization of the dynamical system is achieved by the proposed control law. An electronic circuit is illustrated to verify the effectiveness of the proposed design.

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