Preface

Moving boundary problems have a wide range of applications in science, engineering and industry. These problems occur in many important physical processes, such as melting and solidification of the material, recrystallization of materials, evaporation of droplets, sedimentation process, oxygen diffusion problem, particle dissolution in solid media, cryosurgery, tumour growth, etc. These problems are interesting because of the diversity of their applications. Many important physical processes involving moving boundary/ boundaries have not been effectively studied and are not understood till now. To have a better and adequate understanding of these problems, correct mathematical models of the physical phenomenon and terrific computations are required.

Moving boundary problems are typical problems from point of view of mathematics because of its nonlinear nature and presence of moving interface / interfaces. Exact solutions of these problems are difficult. Only a few exact solutions for restricted cases are available in literature. Hence, various numerical methods and approximate methods have been used to solve these problems.

In this thesis, the author presents a study of some mathematical models of moving boundary problems and its solution by approximate techniques, viz. Homotopy perturbation method, Adomian decomposition method and optimal homotopy asymptotic method. The techniques adopted here are recent, accurate and minimize the size of the calculations.

This thesis consists of six chapters. Chapter I is introductory in nature which includes moving boundary problems, the literature review, Homotopy perturbation method, Adomian decomposition method, optimal homotopy asymptotic method and some definitions on fractional calculus.

In chapter II, The work presents a mathematical model describing the time fractional anomalous-diffusion process of a generalized Stefan problem which is a limit case of a shoreline problem. In this model, the governing equations include a fractional time derivative of order $0 < \alpha \leq 1$ and variable latent heat. The approximate solution of the problem is obtained by homotopy perturbation method. The results thus obtained are compared graphically with an exact solution of integer order. A brief sensitivity study is also performed.

In chapter III, A mathematical model of the movement of the shoreline in a sedimentary ocean basin is discussed. The model includes space-time fractional derivative in Caputo sense and variable latent heat term. An approximate solution of the problem is obtained by Adomian decomposition method and the results thus obtained are compared graphically with an exact solution of integer order $(\beta = 1, \alpha = 1)$. Three particular cases, the standard diffusion, the time-fractional and the space-fractional diffusions are also discussed. The model and solution are generalization of previous works.

In chapter IV, Adomian decomposition method is successfully applied to find an approximate analytical solution of a Stefan problem subject to periodic boundary condition. By using initial and boundary conditions, the explicit solutions of the temperature distribution and the position of moving interface are evaluated and numerical results are depicted graphically. The method performs extremely well in terms of efficiency, simplicity and accuracy.

In chapter V, The work presents a fractional mathematical model of a one phase Stefan problem with latent heat a power function of position. The model includes Caputo space-time fractional derivatives and time dependent surface heat flux. An approximate solution of this model is obtained by optimal homotopy asymptotic method to find temperature distribution in the domain

$0 \le x \le s(t)$ and tracking or location of interface. The results thus obtained are compared graphically with existing exact solution.

In the last chapter, Adomian decomposition method and optimal homotopy asymptotic method are successfully applied to find an approximate solution of a two moving boundaries problem governed with fractional time derivative in Caputo sense of order $\alpha \in (0,1]$. This problem occurs in controlled drug release from a polymeric matrix. The results thus obtained are compared with the existing analytical solutions for the particular cases which are in good agreement.