

## **EFFECT OF DISPERSIVE MATERIALS ON A SQUARE PHOTONIC CRYSTAL FIBER**

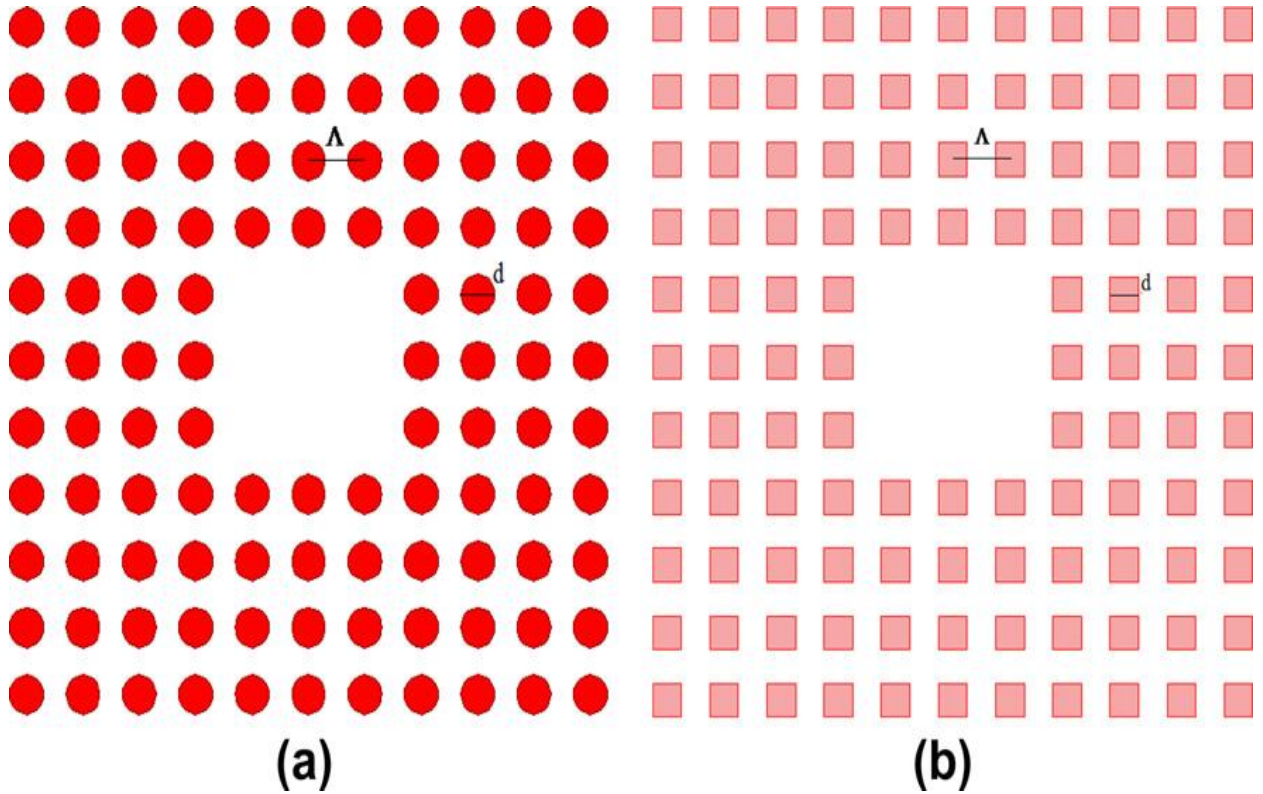
In this chapter, a solid core photonic crystal fiber (PCF) with square lattice of air holes is proposed and numerically investigated. The air holes of the PCF are filled with metals which follow Drude-Lorentz model of dispersion. Chromatic dispersion and normalized frequency ( $V_{\text{eff}}$ ) parameters are investigated using frequency domain algorithm based on semi-vectorial finite difference time domain (FDTD) method. The frequency range is chosen as 0.4 to 1.0  $\mu\text{m}$  for numerical computation of the square PCF to calculate the dispersion and  $V_{\text{eff}}$ . This frequency range is chosen since it almost includes visible light communication wavelength span (0.375- 0.78  $\mu\text{m}$ ) and also the metals that are applied in our PCF follow Drude-Lorentz condition in this range. High negative dispersion is observed when metals are applied in the cladding air holes of the PCF as compared to that of the PCF with cladding containing only air holes. In case of Aluminum as the metal we have got highest dispersion in the negative direction which is -44000 ps/nm-km. The  $V_{\text{eff}}$  parameters obtained for PCF are also highest ( $\sim 160$ ) in case of Aluminium applied to the cladding holes.

### **3.1 GENERAL**

Significant research is being carried on photonic crystal fibers (PCF) from a long time since PCF was first proposed by P. Russell in the year 1991. Holey or micro-structured optical fibers have numerous design possibilities and these fibers could appear as one of the

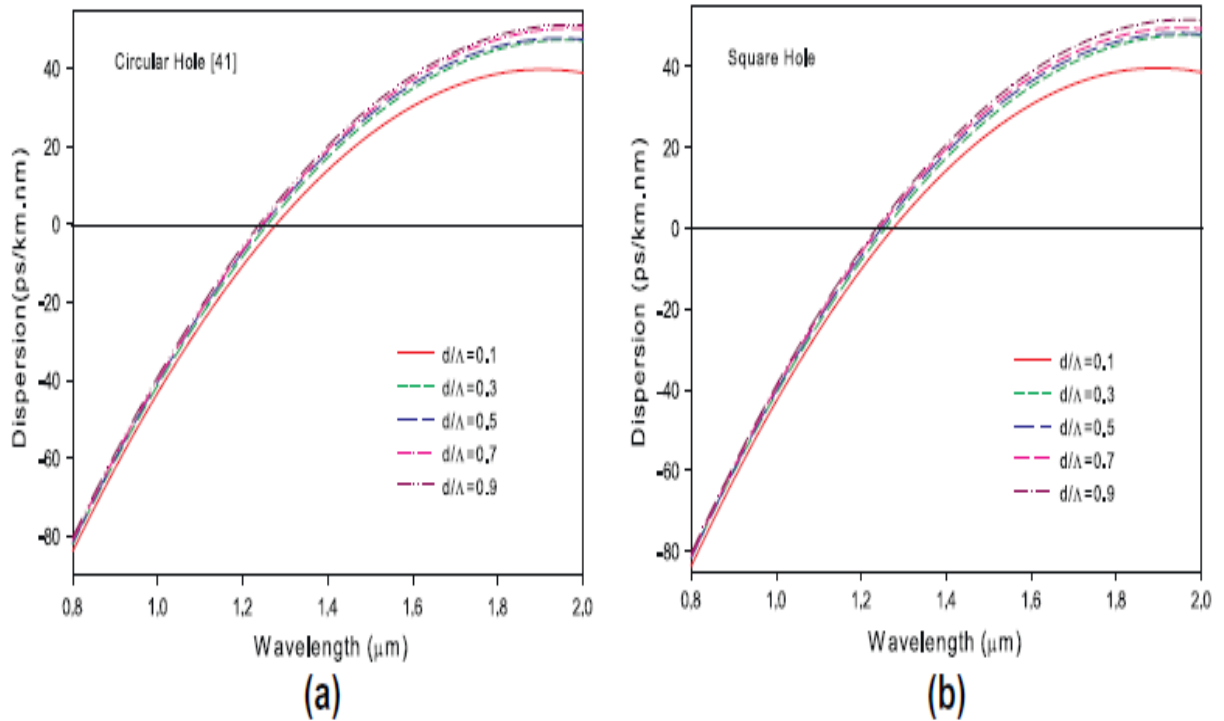
biggest candidates for next generation submarine and terrestrial transmission sources. PCF contains an array of air holes running parallel to its length as part its cladding, the design which makes it different from the conventional single or multi-mode optical fibers [116]. PCFs show extraordinary optical properties for several potential applications which are not achievable in conventional optical fibers. Due to the unique physical structure of PCF containing air-holes in the cladding, they can be used as dispersion compensating fibers (DCF). Apart from application as DCF, photonic crystal fibers can show other special characteristics also such as high birefringence [117], polarization maintenance [85], controllable nonlinearity [113], large mode-area [118] and so on. The aim of this work is to exploit the design of PCF to show novel dispersion characteristics by filling the air holes of PCF with metals which follow Drude-Lorentz model. Based on the air holes arrangement in the cladding, PCFs can be classified into several categories such as hexagonal [119], octagonal [56], elliptical [64] and rectangular PCF [70] etc. There are few works that have been carried out on PCF containing square lattice of air holes as their cladding [120], we have also taken the same structure for study in our work since square PCF is relatively newer design as compared to the conventional circular or hexagonal lattice. H. D. Inci et al. [120] in 2012 theoretically studied large solid-core square-lattice silica photonic crystal fibers with square air-holes. They considered solid-core PCFs with square-lattice structures of circular or square air-holes, as shown in Figure 3.1, where  $d$  is the diameter (or width) of the air-holes, and  $\Lambda$  is the pitch of the cladding. The pitch length is chosen as  $\Lambda = 4.2 \mu\text{m}$  for both of the structures. Refractive index of silica which forms the photonic crystal background and solid-core is  $n_{co} = 1.45$ . The analysis includes relative air-hole sizes  $d/\Lambda$  from 0.1 to 0.9 and the wavelength range from 0.8 to 2.0  $\mu\text{m}$ . A commercial software

BandSOLVE based on plane wave expansion method has been used to find out the effective index of the PCF structures.



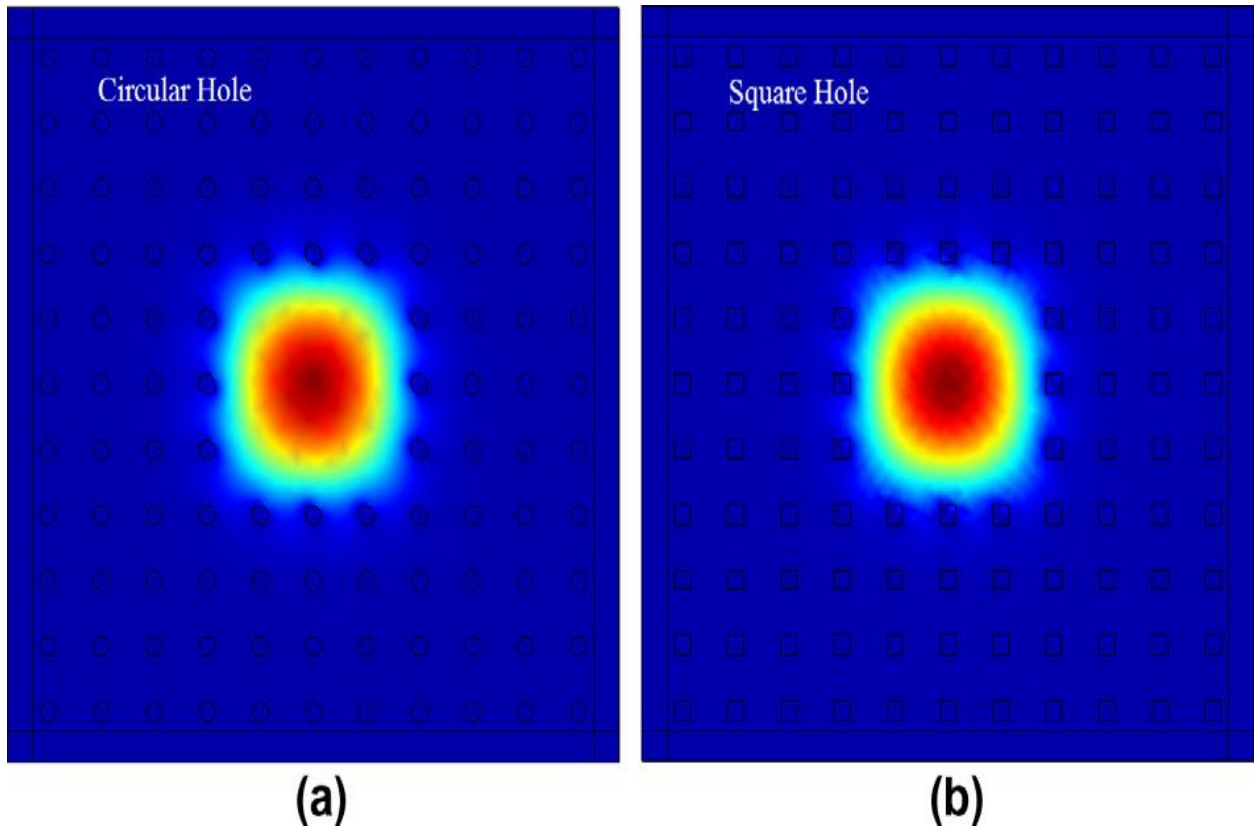
**Figure 3.1** Cross-sections of the solid-core PCFs with: (a) circular holes and (b) square holes.

The dispersion curves for the two structures have been plotted in Figure 3.2. As seen from these figures, the dispersion shows almost the same behavior for the structures with circular holes and square air-holes. Also, the zero dispersion points remain the same for the two structures. It can be observed that the dispersion property and the zero dispersion of the PCF studied do not depend much on  $d/\Lambda$  values, especially for the smaller wavelengths.



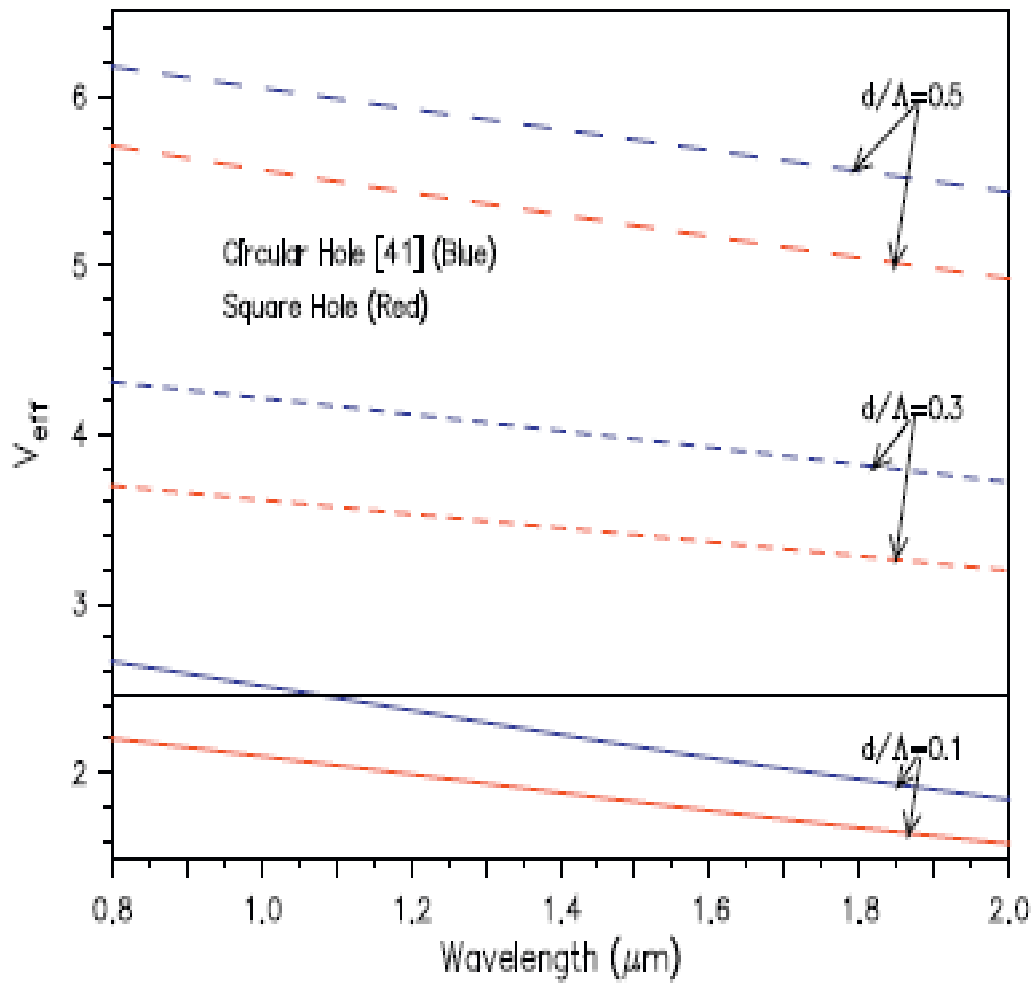
**Figure 3.2** Dispersion curves based on  $d/\Lambda$  values for (a) circular holes structure and (b) square holes structure.

To have a look on the intensity based profile and appearance of the output light in two directions X and Y, the 2D intensity distributions of the PCFs considered are calculated and provided in Figure 3.3. These distributions show a square-like shape of the field in lateral direction.



**Figure 3.3** The intensity profile and shape of the output light in two dimensions of the structures with  $d/\Lambda = 0.3$  at  $\lambda = 1.55 \mu\text{m}$  (a) the circular air-holes and (b) square air-holes.

The V-parameter values are plotted in Figure 3.4 for  $d/\Lambda$  ranging from 0.1 to 0.5.  $V_{\text{eff}}$  of square holes based PCF is found to be lower than that of circular holes based PCF for all wavelengths.



**Figure 3.4** V-parameter versus wavelength for  $\Lambda = 4.2 \mu\text{m}$ , for the structures with circular air-holes (blue) and square air-holes (red).

Normalized frequency ( $V_{\text{eff}}$ ) or V-parameter is also an important quantity regarding PCF, it shows the number of modes that can be propagated simultaneously inside an optical fiber. M. D. Nielsen and N. A. Mortensen have studied photonic crystal fiber design based on the V-parameter in their work [47]. They have varied the air hole size of PCF, computed  $V_{\text{eff}}$  and plotted it against the ratio of diameter of air hole to the pitch.

In this paper we have applied metals in the air holes of the PCF and computed dispersion parameters and  $V_{\text{eff}}$ . The dielectric function (permittivity) of those metals follow Drude-Lorentz dispersive model and their refractive index varies with the incident field. The study of transmission and intensity characteristics of gold by Drude and Drude-Lorentz model was carried out by A. Vial et al. [121]. In addition to gold we have also studied and compared the dispersion characteristics of silver, copper, aluminium, chromium and nickel applied to PCF. The refractive index of the metals used in our work is calculated, at different wavelengths directly through the values of their permittivity at those wavelengths. The equation for calculating permittivity of metals at various wavelengths is taken from the work of [122], while the parameters required of each metal to be put in the equation are obtained from [123]. All the metals taken into consideration in our work, respond differently to the input electromagnetic wave. The dielectric function of each metal depends upon the magnitude of the input wavelength. Thus refractive index of the metal which depends on the dielectric function also varies with the incident wavelength. When these metals are applied to PCF, then the effective index of the PCF could be controlled by the variation of the refractive index of metals, which in turn is related to the input wavelength. Using the effective index of PCF, dispersion parameter (D) and normalized frequency values ( $V_{\text{eff}}$ ) are computed at different wavelengths. In our work, the effective refractive index of PCF without and with inclusion of metals is calculated for the wavelength range 0.4-1.0  $\mu\text{m}$ . This wavelength span is chosen since there is a significant change in the permittivity of the metals used, with the input Gaussian pulse. Also, the visible region of wavelength stretches nearly from 0.4-0.7  $\mu\text{m}$  and testing of light emitting diodes (LEDs) emitting light in this region is under progress to make them act as prominent optical sources [124, 125]. Thus a PCF working as a dispersion compensating fiber in and around the

visible wavelength range is a relatively newer application of PCF and will be highly appreciated in the near future.

### 3.2 THEORETICAL METHOD

A number of techniques have been utilized to solve a photonic crystal fiber to find out its various parameters such as effective index, confinement loss, effective area etc. These techniques include plane wave expansion (PWE), finite element method (FEM), multi pole method (MPM), finite difference time domain (FDTD) and beam propagation method (BPM). To implement our model of photonic crystal fiber we have utilized finite difference method to solve a semi-vectorial wave equation [126], given as below:

$$\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial}{\partial x} \left( \frac{1}{\epsilon_r} \frac{\partial \epsilon_r}{\partial x} E_x \right) + \frac{\partial^2 E_x}{\partial y^2} + (k_0^2 \epsilon_r - \beta^2) E_x = 0 \quad (3.1)$$

In the above equation,  $k_0$  and  $\beta$  stand for wave number and propagation constant respectively of the input wave whereas  $\epsilon_r$  is the dielectric function of the specific medium of the PCF considered. In the same equation, the terms of interaction between x and y-directed polarized light are neglected and the equation is expressed for quasi TE-mode. The cross sectional area of the PCF is converted into a hypothetical mesh and each node of that mesh is numbered. Finite-difference of each term of TE-mode expression is computed for every individual node, which further gives an Eigen-value equation of the form mentioned below,

$$[M_{i,j}] \{\Phi\} = \beta^2 \{\Phi\} \quad (3.2)$$



In the precedent equation,  $M_{i,j}$  is a Hermitian operator and gives the real values of  $\beta$ , using which we can calculate the effective refractive index of the cladding region of PCF. Here, we use analytical boundary condition in which it is assumed that x-polarized electric field decays exponentially at the edges of node boundaries.

The dielectric function for the holes of the PCF containing dispersive materials can be calculated by using Drude-Lorentz model, as shown below:

$$\epsilon_{DL}(\omega) = \epsilon_{x,y,\infty} - \frac{\omega_D^2}{\omega(i\gamma_D + \omega)} - \frac{\Delta\epsilon \cdot \Omega_L^2}{(\omega^2 - \Omega_L^2) + i\Gamma_L \omega} \quad (3.3)$$

Here,  $\epsilon_{x,y,\infty}$ ,  $\Delta\epsilon$ ,  $\omega_D$  denote relative permittivity at infinite frequency, weighing factor and plasma frequency respectively. Whereas  $\gamma_D$ ,  $\Omega_L$ ,  $\Gamma_L$  are the damping coefficient, oscillator strength and spectral width of Lorentz Oscillator respectively. The above dielectric function is then transferred to the eigen-value Equation (3.2) to calculate the values of  $\beta$  which are further deployed to find the effective refractive index of PCF. With the help of the values of effective index, dispersion parameters for the PCF can be calculated by using Equation 3.4 below:

$$D = - \frac{\lambda}{c} \frac{\partial^2 (\text{Re}[n_{\text{eff}}])}{\partial \lambda^2} \quad (3.4)$$

Here  $D$  is known as chromatic dispersion which is the summation of material as well as waveguide dispersion. The parameter  $c$  is the speed of light in vacuum whereas  $\lambda$  is the wavelength of the field travelling in the fiber and  $\text{Re}(n_{\text{eff}})$  shows the real component of  $n_{\text{eff}}$ . In this paper the computed values of  $n_{\text{eff}}$  are equivalent to their real counterparts. The waveguide dispersion can be calculated from the same equation as that is used to calculate chromatic dispersion but the material dispersion is evaluated using Sellmeier's formula.

The equation to determine normalized frequency ( $V_{\text{eff}}$ ) of any PCF is given as :

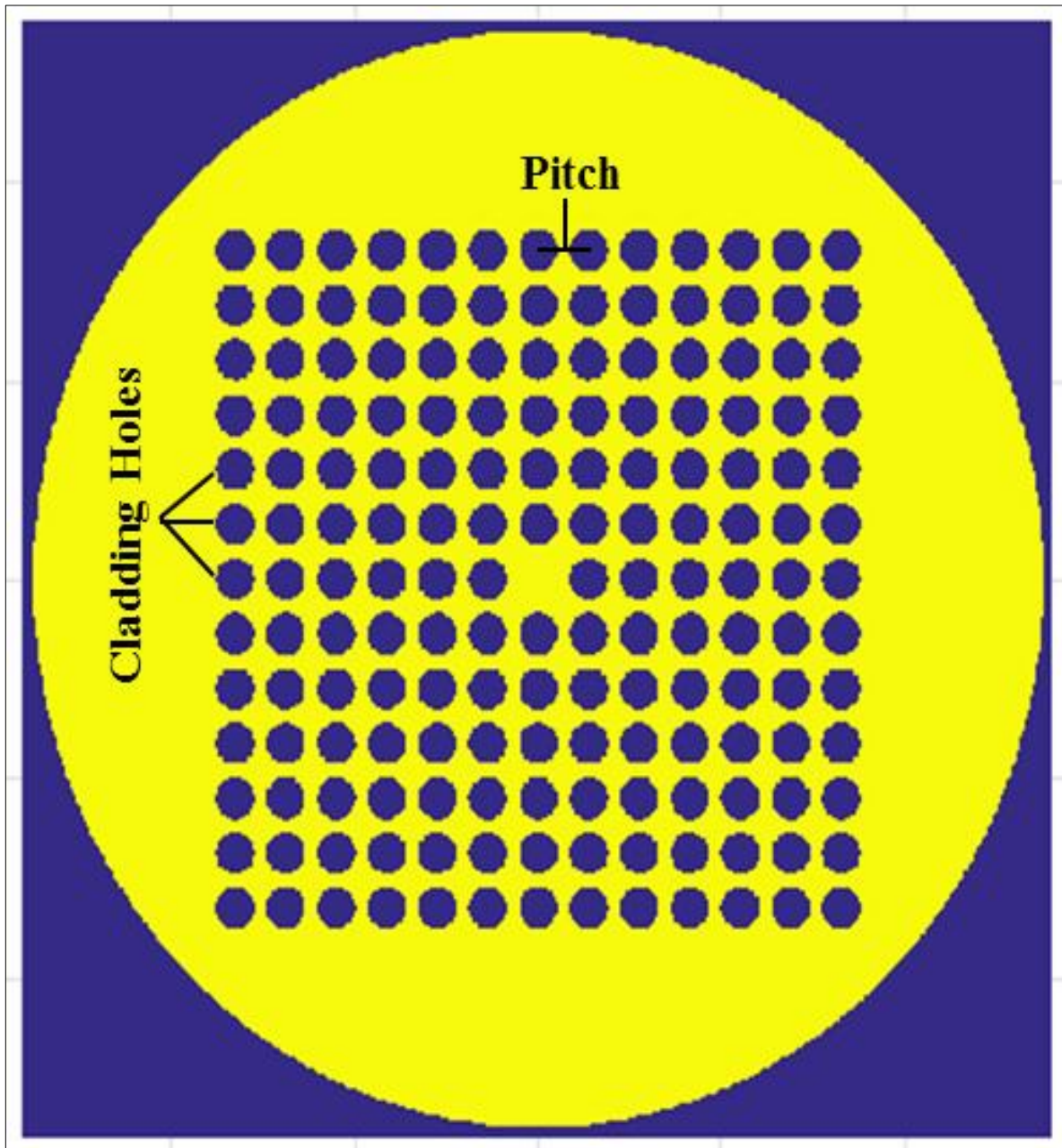
$$V_{\text{eff}} = \frac{2\pi}{\lambda} \Lambda (n_{\text{co}}^2 - n_{\text{eff}}^2)^{1/2} \quad (3.5)$$

$\Lambda$  (pitch) stands for the distance between the centers of two consecutive air-holes in Equation (3.5) while  $n_{\text{co}}$  indicates the refractive index of core of the PCF. The type of mode propagation in the PCF can be decided on the basis of the values of  $V_{\text{eff}}$  obtained. A PCF has the ability to operate in multimode state in addition to single mode during the propagation of an electromagnetic wave.

### 3.3 NUMERICAL ANALYSIS AND RESULTS

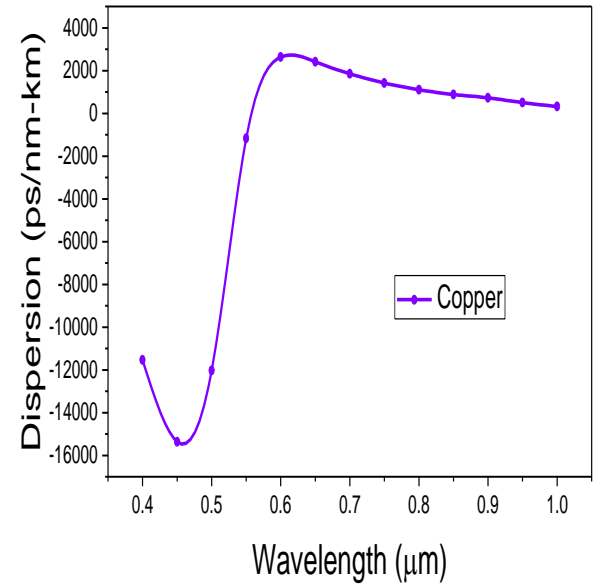
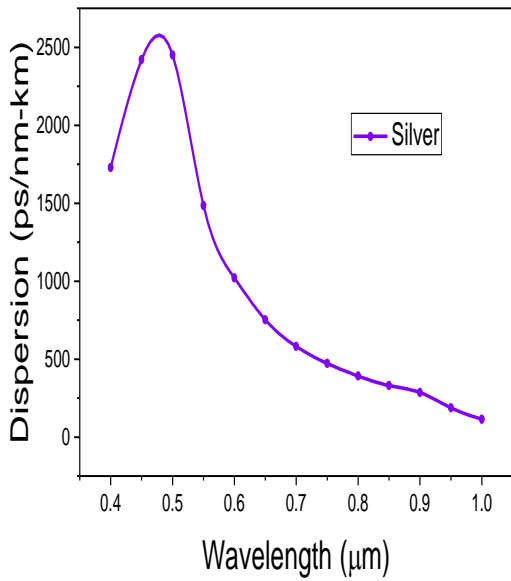
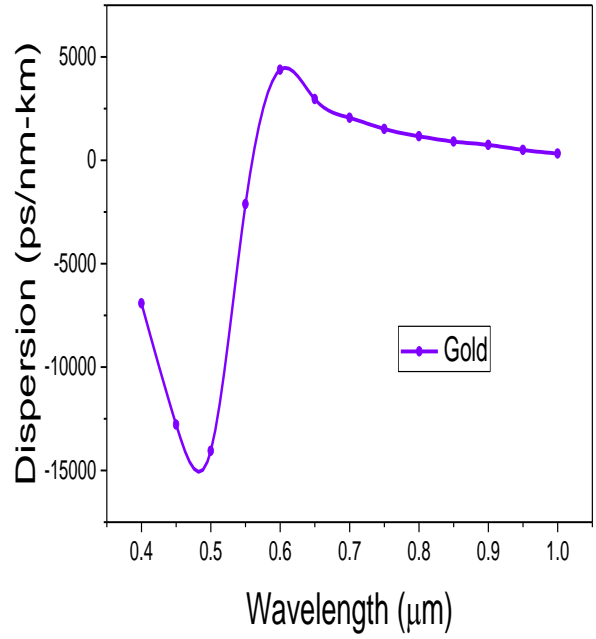
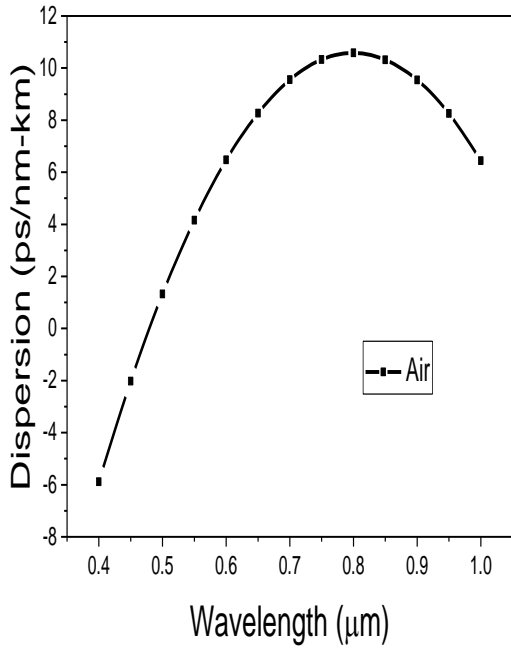
We have deployed a square photonic crystal fiber in our computation whose cross-section is as shown in Figure 3.5. There are 6 square rings of air holes contained in the PCF and the diameter of each air hole is taken to be 0.9  $\mu\text{m}$ . The distance between two consecutive air-holes, also known as pitch ( $\Lambda$ ) is fixed at 5.6  $\mu\text{m}$ . We have taken the background material of PCF as silica with refractive index as 1.46 which is an average of its index in the wavelength range 0.4-1.0  $\mu\text{m}$ . The cladding diameter of the PCF is adjusted to be 100  $\mu\text{m}$  in dimension, although this amount has a negligible result on the computation and hence can be ignored since the electromagnetic modes are restricted only up to the air hole region in our case. Since our PCF has a solid core, therefore light travels in the core through an effective index guiding phenomena. The square lattices of air holes have an influence on the total amount of light reflected back into the core of the PCF.

Initially the effective refractive index of square PCF at different wavelengths in the range 0.4-1.0  $\mu\text{m}$  has been calculated.

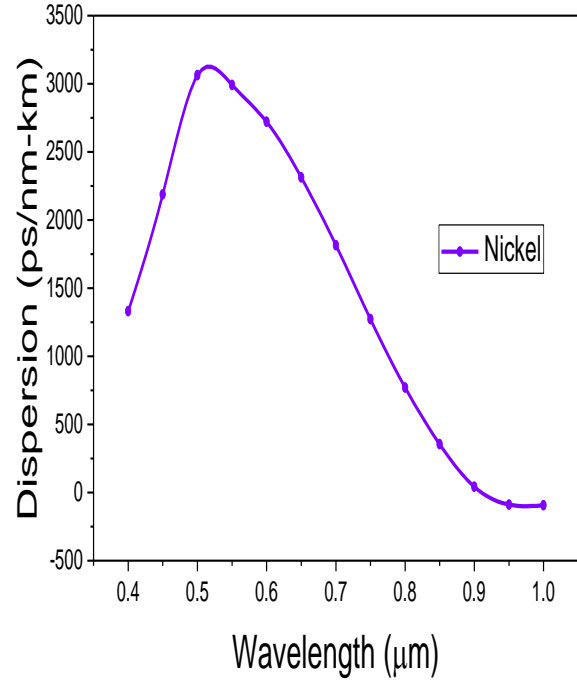
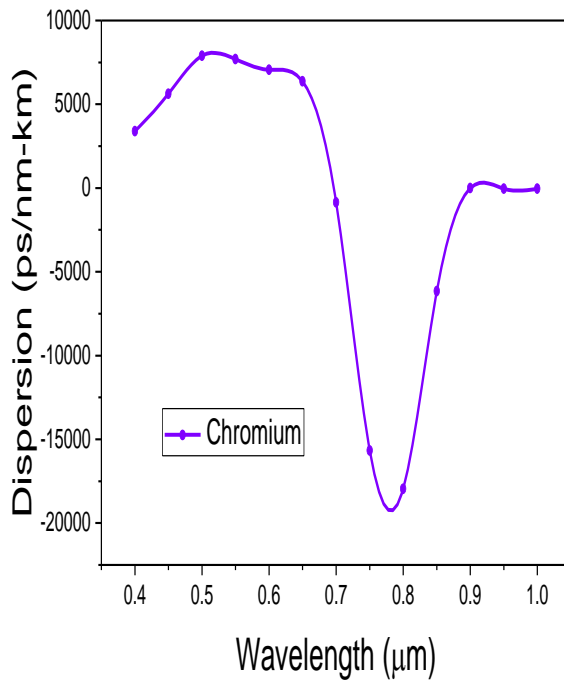
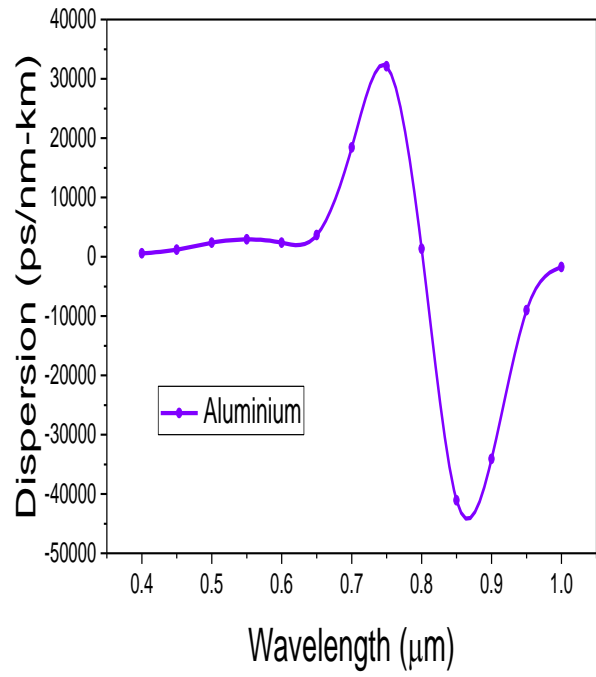
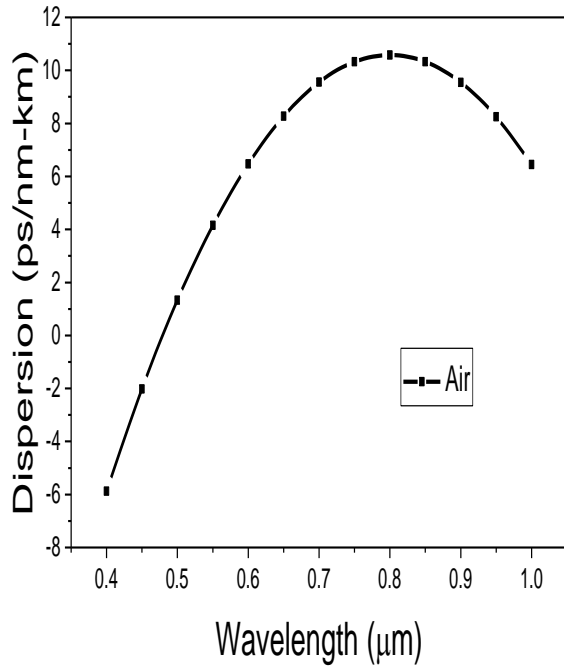


**Figure 3.5** Cross-Section of the square photonic crystal fiber.

Using this effective index, dispersion parameters (D) have been calculated and plotted against wavelength as shown in Figure 3.6 (a). We can see that below 0.48  $\mu\text{m}$  values of D are negative and while moving to the right in increasing direction of wavelength there is observed an increase in the values of D up to 0.8  $\mu\text{m}$ . An approximately flattened dispersion is visible in the range 0.7-0.9  $\mu\text{m}$  with a variation in D as only 1.0 ps/nm-km. After the dispersion calculations with air in the holes of PCF, metals have been applied to the holes one after the other and dispersion parameters have been calculated. The dispersion graphs of PCF containing Gold (Au), Silver (Ag) and Copper (Cu) have been plotted against wavelength using the dispersion parameters as shown in Figure 3.6 (b), (c) and (d) respectively. For the PCF containing gold there is observed negative dispersion up to 0.56  $\mu\text{m}$  and as we move in the increasing direction of wavelength, positive dispersion is seen till 1.0  $\mu\text{m}$ . The highest negative dispersion is visible at 0.48  $\mu\text{m}$  with value of D as -15054 ps/nm-km. In case of silver as the cladding filling material of PCF there can be noticed a positive dispersion in the entire wavelength range considered, with the highest value of dispersion as 2580 ps/nm-km at nearly 0.48  $\mu\text{m}$ . Considering the PCF with copper being filled in the air holes, there is observed both positive as well as negative dispersion. The maximum dispersion in positive direction is 2673 ps/nm-km at 0.6  $\mu\text{m}$  while the highest dispersion in the negative side is -15483 ps/nm-km at about 0.46  $\mu\text{m}$ . Three other metals viz. Aluminium (Al), Chromium (Cr) and Nickel (Ni) which follow Drude-Lorentz dispersive model are also applied in the air holes of the PCF in succession. The dispersion graphs of Al, Cr and Ni are plotted against wavelength in Figure 3.7's (b), (c) and (d) respectively along with that of PCF containing only air in the cladding holes in Figure 3.7 (a) for the basis of comparison.



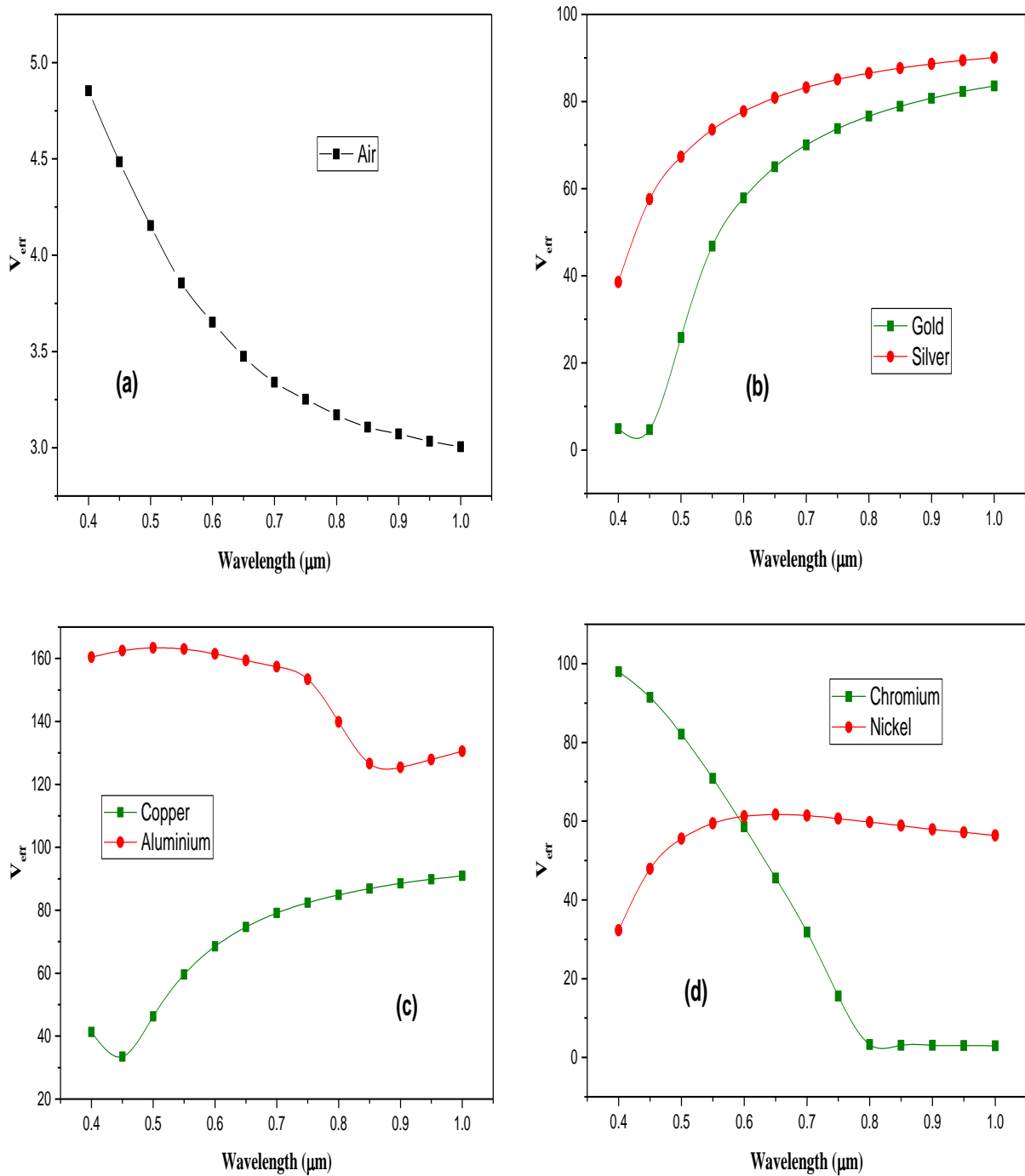
**Figure 3.6** Dispersion graphs of the square PCF containing only (a) Air, (b) Gold, (c) Silver and (d) Copper in the cladding holes.



**Figure 3.7** Dispersion graphs of the square PCF containing only (a) Air, (b) Aluminium, (c) Chromium and (d) Nickel.

The dispersion plot of Al shows both positive and negative variety of dispersion with highest positive value of  $D$  as 32410 ps/nm-km at 0.74  $\mu\text{m}$  while lowest negative  $D$  as -44000 ps/nm-km at nearly 0.86  $\mu\text{m}$  wavelength. Talking about the dispersion graph of Cr, there is obtained a highest positive dispersion equal to 7980 ps/nm-km at 0.50  $\mu\text{m}$  whereas on negative side dispersion at the base point is -19270 ps/nm-km with its wavelength as 0.78  $\mu\text{m}$ . Finally, through the metal Ni's dispersion curve it is observed that in the wavelength range 0.4-0.9  $\mu\text{m}$  there is positive dispersion and between 0.9-1.0  $\mu\text{m}$  wavelength span only the values of dispersion are negative. The extreme dispersion on the positive side is visible as 3133 ps/nm-km at the wavelength 0.51  $\mu\text{m}$ .

Figure 3.8 shows the normalized frequency ( $V_{\text{eff}}$ ) of PCF plotted against wavelength for air as well as other metals filled in the holes of that PCF. In case of air as the medium in the cladding holes of PCF, the maximum value of  $V_{\text{eff}}$  is 4.85 at 0.4  $\mu\text{m}$  and minimum as 3.0 at 1.0  $\mu\text{m}$  wavelength as shown in the Figure 3.8 (a). In the Figure 3.8 (b) the minimum value of  $V_{\text{eff}}$  for gold filled PCF is visible as 4.64 at 0.45  $\mu\text{m}$  while for silver filled PCF as 38.8 at 0.4  $\mu\text{m}$ . The maximum value of  $V_{\text{eff}}$  for gold case is seen as 83 and silver case as 90 both at 1  $\mu\text{m}$  wavelength. From the Figure 3.8 (c) it is observed that the minimum value of  $V_{\text{eff}}$  for copper embedded PCF is 33.24 at 0.45  $\mu\text{m}$  whereas the maximum value goes up to 91 at 1.0  $\mu\text{m}$  wavelength. In the same figure it can be seen that the least value of  $V_{\text{eff}}$  for PCF containing aluminium is 125 at 0.9  $\mu\text{m}$  and higher most 163.5 at 0.5  $\mu\text{m}$  wavelength. At last, the lowest value of  $V_{\text{eff}}$  for chromium based PCF is 3.0 while for nickel based PCF is 32.21 at 1.0  $\mu\text{m}$  and 0.4  $\mu\text{m}$  respectively as displayed in Figure 3.8 (d). From the same figure the higher most  $V_{\text{eff}}$  is observed as 97.94 at 0.4  $\mu\text{m}$ , 62 at 0.65  $\mu\text{m}$  for chromium and nickel filled PCF respectively.



**Figure 3.8** Wavelength dependence of normalized frequency for square lattice PCF with (a) Only Air, (b) Gold and Silver, (c) Copper and Aluminium & (d) Chromium and Nickel.



### 3.4 CONCLUSION

A square photonic crystal fiber has been modeled and investigated for its applications. Metals which follow Drude-Lorentz dispersive model are put into the air holes of the PCF and the calculation of dispersion along with normalized frequency parameters has been done in the wavelength range 0.4-1.0  $\mu\text{m}$ . We see that the dispersion graph of PCF with only air in the cladding holes is confined in the range -5.9 to +10.6 ps/nm-km on the vertical dispersion parameter scale. In the same graph dispersion flattening is observed between 0.7-0.9  $\mu\text{m}$  wavelength with a deviation of only 1.0 ps/nm-km in dispersion. The dispersion graphs of the metals applied in the PCF viz. gold, copper, aluminium and chromium show a large amount of negative dispersion for the wavelength taken into consideration. Silver and nickel doped PCF dispersion curves show mostly positive dispersion with only -100 ps/nm-km negative dispersion in case of nickel from wavelength 0.9 to 1.0  $\mu\text{m}$ . Positive dispersion generated in a conventional optical fiber in the range of thousands of ps/nm-km could be compensated with the help of metal based PCF's. The highest negative dispersion is visible in the case of metal aluminium applied to PCF which is equal to -40000 ps/nm-km whereas the lowest negative dispersion is seen for the metal nickel applied to PCF as -100 ps/nm-km. So the metal aluminum could be the best choice to be employed into PCF for compensating large amounts of positive dispersion. Modern communication techniques employ LED's and lasers as optical sources, working in the visible domain of the wavelength spectra. In our work a square photonic crystal fiber working in the range 0.4-1.0  $\mu\text{m}$  wavelength, which includes the visible spectra is shown to be able to compensate a huge amount of positive dispersion in the order of tens of thousands. This is possible because of the filling of air holes of PCF with metals that obey Drude-Lorentz dispersive model.

Talking about the normalized frequency, the PCF with only air in the cladding holes show a decrease in the values of  $V_{\text{eff}}$  with increase in the wavelength. The difference observed in the initial and final value of  $V_{\text{eff}}$  is 1.85 for the wavelength range 0.4 to 1.0  $\mu\text{m}$ . When metals are included in the PCF there are a variety of normalized frequency curves obtained. Gold, silver and copper filled PCF show an increase in the amount of  $V_{\text{eff}}$  with increase in wavelength. Aluminium embedded PCF shows nearly constant variation of  $V_{\text{eff}}$  in the wavelength ranges 0.4-0.7  $\mu\text{m}$  and 0.85- 1.0  $\mu\text{m}$  with only 2.5 percent divergence in  $V_{\text{eff}}$  parameter. There is almost a linearly decreasing slope of normalized frequency in case of chromium stocked PCF in the wavelength range 0.4-0.8  $\mu\text{m}$  and thereafter a constant value of  $V_{\text{eff}}$ , equal to 3 is maintained. The normalized frequency curve of nickel loaded PCF shows an increase in the wavelength range 0.4-0.65  $\mu\text{m}$  and acquires almost a uniform state up to 1.0  $\mu\text{m}$ . Thus the number of modes that can enter and travel along the length of a photonic crystal fiber for a particular wavelength range can be adjusted according to the metal chosen for filling the cladding holes of the fiber.