Preface

Thesis contains seven chapters in which Chapter 2 is split into two separate sections. Throughout the study one-, two-dimensional standard, as well as fractional order solute transport models, are considered.

Chapter 1 is an introductory chapter where some basic definitions and literature review related to the field have been discussed. Also, mathematical modeling and numerical approach used in the thesis are explained. Historical background of Hydrogeology and the theory of porous media is added here. The brief introduction of the fractional calculus and its application is also discussed. The definitions of special functions and orthogonal polynomials, which are used in subsequent chapters are included.

In Chapter 2, numerical solution of the solute transport system is derived using spectral collocation method with finite difference method. In Section 2.1 of Chapter 2, Chebyshev collocation method is used to reduce the one-dimensional advection-dispersion equation having source/sink term with given boundary conditions and initial condition into a system of ordinary differential equations which are solved using finite difference method. The solution profile of solute concentration factor for both conservative and non-conservative systems are calculated numerically which are presented through graphs for different particular cases. The salient feature of this chapter is the comparison of the trend of numerical solution with the existing analytical solution thereby validating our considered numerical technique.

In Section 2.2, a drive is taken to compute the solution of spatial fractional order advection-dispersion equation having a source/sink term with given initial and boundary conditions. During numerical solution of the considered problem, the same method is applied which is already discussed in Section 2.1 where fractional derivatives are considered is Caputo sense. The striking feature of the chapter is the fast transportation of solute concentration as and when the system approaches fractional order from the standard order for specified values of the parameters of the system.

In Chapter 3, the numerical solution of the two-dimensional solute transport system in a homogeneous porous medium of finite-length is obtained. The considered transport system has the terms accounting for advection, dispersion and first-order decay with first-type boundary conditions. Initially, the aquifer is considered solute free, and a constant input-concentration is considered at inlet boundary. The solution is describing the solute concentration in rectangular inflow-region of the homogeneous porous media. The numerical solution is derived using a powerful method viz., spectral collocation method. The numerical computation and graphical presentations exhibit that the method is effective and reliable during the solution of the physical model with complicated boundary conditions even in the presence of reaction term.

Chapter 4 deals with the numerical solutions of a class of NPDEs subject to initial and boundary conditions. In the proposed approach shifted Chebyshev polynomials are considered to approximate the solutions together with shifted Chebyshev operational matrix and spectral collocation method. The benefit of this method is that it converts such problems in the systems of algebraic equations which can be solved easily. To show the efficiency, high accuracy and reliability of proposed approach, a comparison between the numerical results of some illustrative examples and their existing analytical results from the literature are reported. There is high consistency between the approximate solutions and their exact solutions to a higher order of accuracy. The error analysis for each case exhibited through graphs and tables confirms the exponential convergence rate of the proposed method.

In Chapter 5, the method discussed in chapter 4 is extended to get the numerical solution of a class of fractional differential equations namely space fractional order reaction-convection-diffusion equations subject to initial and boundary conditions. In the proposed approach shifted Jacobi polynomials are used to approximate the solutions together with shifted Jacobi operational matrix of fractional order and spectral collocation method. The proposed approach is efficient to solve the linear as well as non-linear fractional differential equations. To show the reliability, validity and high accuracy of the proposed approach, the numerical results of some illustrative examples are reported, which are compared with the existing analytical results already reported in the literature. The error analysis for each case exhibited through graphs and tables confirms the exponential convergence rate of the proposed method.

Chapter 6 deals with the solution of non-linear partial differential equations arise in porous media using the approach already discussed in chapter 5. The salient feature of the chapter is the exhibition of sub-diffusion nature of solution profile for different particular cases in the presence or absence of the source/sink term.

In Chapter 7, a new algorithm is proposed to solve the standard as well as fractional order linear/non-linear two-dimensional partial differential equations subject to initial and boundary conditions. Here approximation technique is used to solve the considered problem with the triple shifted Legendre polynomials and spectral collocation method. During differentiation, standard, as well as fractional operational

matrices of the shifted Legendre polynomials, are used. The proposed algorithm is based on the fact that the terms of the considered problems are approximated through a series expansion of triple shifted Legendre polynomials and then collocated these on Legendre Gauss-Lobatto points which convert the considered problems into the system of linear/non-linear algebraic equations. The systems having unknown coefficients can be solved using the standard numerical technique. Graphical presentations and tabular representations of some considered examples are illustrated to make a comparison with the existing analytical solutions which demonstrate the applicability, efficiency, and reliability of the proposed algorithm.