Chapter 8

Overall Conclusions and Scope for Future Work

8.1 Overall Conclusion

Groundwater contamination is one of the serious issues for life on earth since the groundwater mainly fulfills the basic needs of fresh water. The area below the earth surface is known as an aquifer which is a porous type structure. So in throughout work I have considered the solute transport models which greatly describe the movement of solute in a porous type structure. To study and analyze the considered models a drive has been made to solve these models numerically subject to initial and boundary conditions. To solve the considered problems, the existing spectral approach is considered and the extension of the approach is made according to the considered problems.

The general solute transport model is the reaction-advection-dispersion equation (RADE) since it has the combined effects of advection, dispersion and reaction processes due to which solutes are transported down with the stream along the flow also get dispersed and sometimes react with the medium through which it moves. Mathematically it is represented as

$$\frac{\partial c}{\partial t} = \nabla.(d\,\nabla c) - \nabla.(vc) + R,$$

where c describes the species concentration for mass transfer problem and temperature for heat transfer problem, d is the mass diffusivity for particle motion and thermal diffusivity for heat transfer, v is the average velocity and for flows in porous media v is the superficial velocity, R is the reaction term for the species c.

To analyze the movement of solute in a homogeneous aquifer firstly, I have considered the one-dimensional solute transport model which is simply an advection-dispersion equation having source/sink term with given first type source boundary conditions and initial condition. To describe the movement of solute concerning the column length, the considered problem is solved numerically using a spectral collocation method namely Chebyshev collocation method where finite difference scheme is used to overcome with the temporal derivative. Here the second-kind shifted Chebyshev polynomials are used to approximate the solution. And the solution profile has been drawn for both conservative and non-conservative system for a different time. The salient feature of this chapter is the comparison of the trend of numerical solution with the existing analytical solution thereby validating our considered numerical technique.

Secondly, I have considered the fractional order form the previous model and get the numerical solution of the considered model using the same approach discussed above where the fractional derivative have been considered in Caputo sense. The striking feature of the chapter is the fast transportation of solute concentration as and when the system approaches fractional order from the standard order for specified values of the parameters of the system which greatly described the physical phenomena where the rate of transportation is much faster than the usual one.

To understand the physical problems in the more accurate way I have considered the two-dimensional solute transport models with first-order decay and first-type boundary conditions. Initially, the aquifer is considered solute free, and a constant input-concentration is considered at inlet boundary. To draw the solution profile, I have solved the considered model using Chebyshev collocation method where first-type Chebyshev polynomials are used to approximate the solution and an unconditionally stable and up-to second-order accurate temporal scheme is used to overcome with the temporal derivative. Here the solution profiles the movement of solute in rectangular inflow-region of the homogeneous porous media. The numerical computation and graphical presentations exhibit that the method is effective and reliable during the solution of the physical model with complicated boundary conditions even in the presence of reaction term.

In most of the situation to describe the physical phenomena in the more accurate way, we get the nonlinear standard, as well as fractional-order models. So here I have also discussed a numerical scheme based on the spectral and operational matrix approach which is useful to solve these types of models as discussed in Chapter 4, 5 & 6. I have also extended this scheme to solve the standard as well as fractional order linear/non-linear two-dimensional models which can be seen in Chapter 7.

So here we can say that the overall work can be categorized into three section. First one is the study and analysis of the standard order solute transport models. The second one is the study and analysis of the fractional order solute transport models. And the last one is the extension of the existing numerical approach to solve the considered problem.

8.2 Scope for Future Work

The two-dimensional fractional order linear problems do not have a precise analytic solution. Especially it is hard to get for nonlinear equations in fractional order systems. Approximate analytical methods and numerical methods are very useful for solving these types of equations. Achieving computationally efficient solutions of these evolutions for different particular cases are very challenging jobs. The mathematical software viz., Mathematica, Matlab, Mathcad are needed during numerical computations. Due to physical relevance and important applications, there is still plenty of scopes for researchers to explore nonlinear fractional order RADE subject to different types of initial and boundary conditions viz., Dirichlet boundary conditions, Neumann boundary conditions and Robin-type boundary condition, mainly in two-dimensional, which have motivated myself to propose a number of mathematical models of physical interests and then predict the physical nature. Also, both are very much interested in showing the effect of nonlinearity appeared in reaction term as well as the dispersion term on the solution profiles.

Recently, many researchers are involved in the field of aerosol transportation due to its direct link to climate change. Many models on aerosol transport have been developed viz., aerosol model for atomic spectrometry, a model in one-dimension for simulation of aerosol transport and deposition in human lung, the aerosol in human lung in two and three dimension, motion of inertial spheroidal particles in a shear flow near a solid wall with special application to aerosol transport in microgravity, aerosol transport in sequentially bifurcating airways ,the effect of electrohydrodynamic flows and turbulence on aerosol transport, an aerosol transport model for coastal area and calculated effect of surf produced aerosols on processes in the marine atmospheric boundary layer, a model for Micro-particle transport and deposition in a human oral airway, a model of aerosol microscopic module for large scale, a model to validate the size-resolved particle dry depositions scheme for application in aerosol transport model etc. the Shifted

Legendre collocation method is effective, accurate and easy to implement for solving the considered two-dimensional time fractional order reaction-advection–dispersion equation in the finite domain and even can be useful to solve space-time fractional order two-dimension problem with arbitrary initial and boundary conditions. Due to the tremendous application of Aerosol problems in biology and engineering sciences the extension of the numerical technique to solve the three-dimensional space-time fractional order Aerosol transport equation could be a research work in the future endeavor.