CHAPTER 6

OPTIMIZATION OF CE PROCESS PARAMETERS

6.1 Introduction

To utilize the resources available in an effective manner thereby minimizing the wastage of resources available optimization of the process parameters is needed very much now a days so as. In Continuous Extrusion industry one need to know the wheel velocity and extrusion ratio at which the minimum amount of power is consumed so as to minimize the power consumption and better surface quality of product is obtained.

In the present chapter Response Surface Methodology (RSM), Artificial Neural Network (ANN) and Genetic Algorithm (GA) has been used as optimization tools. The RSM technology is better suited for small number of factors i.e. for less number of data sets available. RSM has strong ability to screen the input process parameters by Plackette-Burman Design (PBD) and recognize those parameters which have sufficient impact on output process parameters. Therefore significant and insignificant factors as well as interaction factors can easily be recognized using RSM. On the other side RSM requires well defined range of input process parameters to ensure that the output process parameters under consideration changes in a regular manner within this change. As a matter of fact ANN performs better than RSM in case of nonlinear modeling of process or process parameters.

Efficient modeling of the process parameters has been done through ANN in case of small number of data sets but the accuracy is poor. Accuracy of ANN is sufficiently high in case of large number of data sets available. ANN modeling requires a large number of iterations to obtain an optimal result whereas for RSM it is single step calculation to get an optimal result. Depending on the nonlinearity of the problem and the number of parameters, an ANN model may require a high computational cost to create. Although computationally much more costly than a response model, ANN model provides comparatively accurate value of load required predictions for extrusion of feedstock material.

The optimization of Continuous Extrusion process parameters such as total load, torque, effective stresses and Damage value etc. during extrusion of feedstock material for given values of extrusion parameters like wheel velocities, product diameters, frictional conditions, feedstock temperatures, die temperatures has been done in this chapter using statistical tool Minitab(version 15.1.0.0,USA), Artificial Neural Network and Genetic Algorithms. The Continuous Extrusion process variables such as extrusion wheel velocities, product diameters, frictional conditions, feedstock temperatures, die temperatures having impact on process parameters or response variables of Continuous Extrusion process such as total load, torque, effective stress, Damage value, Product Temperatures, Hardness, Ultimate Tensile Strength, Yield Strength etc. has been screened using Plackett-Burman Design (PBD) and optimum level of the screened components has been determined using Central Composite Design (CCD) method. The details about PBD have been added in Appendix 2.

Optimization of Continuous Extrusion response variables has been done using Response Surface Methodology (RSM) for Aluminum and Copper feedstock materials in section 6.2 and section 6.3 respectively. Numerical modeling of the Continuous Extrusion process parameters has been done using Response Surface Methodology (RSM) for Aluminum feedstock materials in section 6.4. Optimization of Continuous Extrusion process responses through Artificial Neural Network is done in section 6.5. In section 6.6 optimization of the Continuous Extrusion process response has been done through Genetic Algorithms. A brief comparison of results of optimization obtained through Response Surface Methodology, Artificial Neural Network and Genetic Algorithms has been made in section 6.7.

Case I

6.2 Optimization of Extrusion process by Response Surface Methodology (Aluminum feedstock material)

A mathematical model through Response Surface Methodology has been developed to analyze the influence of wheel velocity and extrusion ratio on mechanical properties such as Ultimate Tensile Strength, Yield Strength, %Elongation and Hardness. An optimum value of the extrusion wheel velocity and extrusion ratio has been determined to predict the best mechanical properties of the Continuous Extrusion forming products at different wheel velocities and extrusion ratio. The adequacy of the model has also been tested by the Analysis of Variance test (ANOVA).

The Plackette-Burman Design (PBD) has been applied to find out the significant input process variables of the Continuous Extrusion process having considerable impact on the output process parameters of the Continuous Extrusion process. A two factor three level factorial design has been considered in which each factor is investigated at three widely spaced levels, a high (+1), 0 and a low (-1) level (Plackette and Burman, 1946).A total of two variables has been considered for an experimental design in which Pure Aluminum rod (Al 1100) available commercially of 9.5 mm diameter feedstock has been used as the raw material. The designed developed and fabricated as well as commercially available Continuous Extrusion machine set up has been used to carry out experimentation at different wheel velocity and extrusion ratio.

The responses from 13 individual experiments have been utilized for generating regression coefficient values. The Plackette-Burman Design is based on the first-order polynomial model given as

$$Y = \beta_0 + \sum \beta_i X_i \qquad (i = 1.....,k) \qquad (6.1)$$

Where, Y denotes the response of Continuous Extrusion process, β_0 is model intercept and β_i is the factor estimates. X_i is the level of the i^{th} independent variable. From regression analysis, the variables showing P-values below 5% level ($P<0.05$) is considered to have greater impact on the Continuous Extrusion process response and used further for Central Composite Design (CCD).

For the Continuous Extrusion forming, Pure Aluminum rod (AA 1100) available commercially of 9.5 mm diameter feedstock has been used as the raw material. The Design Developed and fabricated Continuous Extrusion set up have been used to carry out experimentation at different wheel velocity and extrusion ratio. The Aluminum rod is subjected to Continuous Extrusion under the extrusion wheel velocities of 4, 6, 8 and 10 RPM. The Aluminum rod is also subjected to Continuous Extrusion under different extrusion ratios .The Continuous Extrusion products is not subjected to any artificial aging and treatments. The tensile testing of the samples has been performed on INSTRON machine. The Hardness test of the extruded products samples has been performed on Vickers Hardness testing machine and the Hardness values has been obtained.

For finding out the relationship between the process parameters and the mechanical properties, second order polynomial response surface mathematical models can be considered as [Seeman et al., (2009)],[Vettivel et al., (2013)], [Balasubramanian et al., (2006)], [Mahmoud et al., (2011)], [Niranjan et al., (2013)], [Sathyabalan et al., (2009)], [Yigenzu et al., (2013)].

$$Y_u = b_0 + \sum b_i x_{iu} + \sum b_{ii} x^2_{iu} + \sum b_{ij} x_{iu} x_{ju} \qquad (6.2)$$

Where Y_u is the corresponding response; x_{iu} is the coded values of the i-th Continuous Extrusion parameters for the u-th experiments; and b_i, b_{ii}, b_{ij} are the second order regression coefficients. The second term under the summation sign of this polynomial equation corresponds to linear effect, while the third term denotes to the higher order effect. The fourth term of the equation includes the interactive effects of the process parameters.

The investigation made in this section deals with the effects of factors such as extrusion wheel velocity and extrusion ratio. The response parameters are Ultimate Tensile Strength (UTS), Yield Strength (YS), % Elongation and Hardness of the extruded Aluminum feedstock during Continuous Extrusion process. A 2^k factorial with Central Composite second order design has been used (in this case k=2). Table 6.1 shows the experimental parameters used and their levels.

Table 6.1: Experimental parameter and levels

Factors	**Level**		
	-1	0	1
Wheel Velocity	4	6	8
Extrusion Ratio	2.5	1.84	1.41

Modeling and Optimization (Ultimate Tensile Strength)

Table 6.2 shows the experimental plan and result for Ultimate Tensile Strength based on Central Composite second order rotatable design for Aluminum feedstock material. The test of significance of UTS has been carried out using the quadratic model. The results of the quadratic model for UTS are given in Table 6.3.The value of R^2 and adjusted R^2 are 95.96 %and 93.07%.This means that the regression model provides a complete relationship between the dependent and independent variables. The associated p value for the model is <0.05(i.e., α=0.05 or 95% confidence), indicating that the model is considered to be statistically significant. Table 6.4 shows the result of analysis of variance (ANOVA) for Ultimate tensile strength (UTS).

It can be seen from Table 6.4 that the standard F value for 95% confidence limit is 5.05. Hence the model is found to be adequate and statistically significant. It is also seen from Table 6.3 that the p values, the main effect X_2 and second order effect of X_1 and X_2 is much more significant. The other model terms can be regarded as insignificant due to their probabilities values being more than 0.05. Figure 6.1 depicts the normal probability of residuals for UTS. It is observed from the plot that residuals values are distributed normally and in a straight line and hence the model is adequate.

Using the results presented in Table 6.3, the derived model is shown as:

$$\sigma=106.207-0.208X_1-0.708X_2-1.768X_1^2-0.581X_2^2+0.375X_1X_2 \tag{6.3}$$

Table 6.2: Experimental plan and result for Ultimate Tensile Strength based on Central Composite second order rotatable design

Experiment No.	**Wheel Velocity (RPM)**		**Product Diameter (mm)**		**UTS (MPa)**
	Coded	**Actual**	**Coded**	**Actual**	
1	-2	1	0	7	101.000
2	1	10	1	8	103.500
3	-1	4	1	8	103.000
4	0	7	0	7	106.500
5	0	7	-2	5	106.500
6	0	7	0	7	106.500
7	2	13	0	7	100.000
8	-1	4	-1	6	105.500
9	0	7	0	7	106.500
10	1	10	-1	6	104.500
11	0	7	2	9	104.000
12	0	7	0	7	106.500
13	0	7	0	7	106.500

Table 6.3: Test for significance of UTS

Terms	**Coefficient**	***t* – value**	***P* – value**
Constant	106.207	374.757	0.000
Wheel Velocity	-0.208	-1.048	0.330
Product diameter	-0.708	-3.562	0.009
Wheel Velocity *Wheel Velocity	-1.768	-12.286	0.000
Product diameter* Product diameter	-0.581	-4.036	0.005
Wheel Velocity*Product diameter	0.375	1.089	0.312

Table 6.4: Test result of ANOVA for UTS

Source	**DF**	**Sum of Squares**	**Mean Sum of squares**	***F*-value**	***P*-value**
Regression	5	78.9082	15.7816	33.25	0.000
Linear	2	6.5417	3.2708	6.89	0.022
Square	2	71.8040	35.9020	75.64	0.0000
Interaction	1	0.5625	0.5625	1.19	0.312
Residual error	7	3.3226	0.4747		
Lack of fit	3	3.3226	1.1075		
Pure error	4	0.0000	0.0000		
Total	12	82.2308			

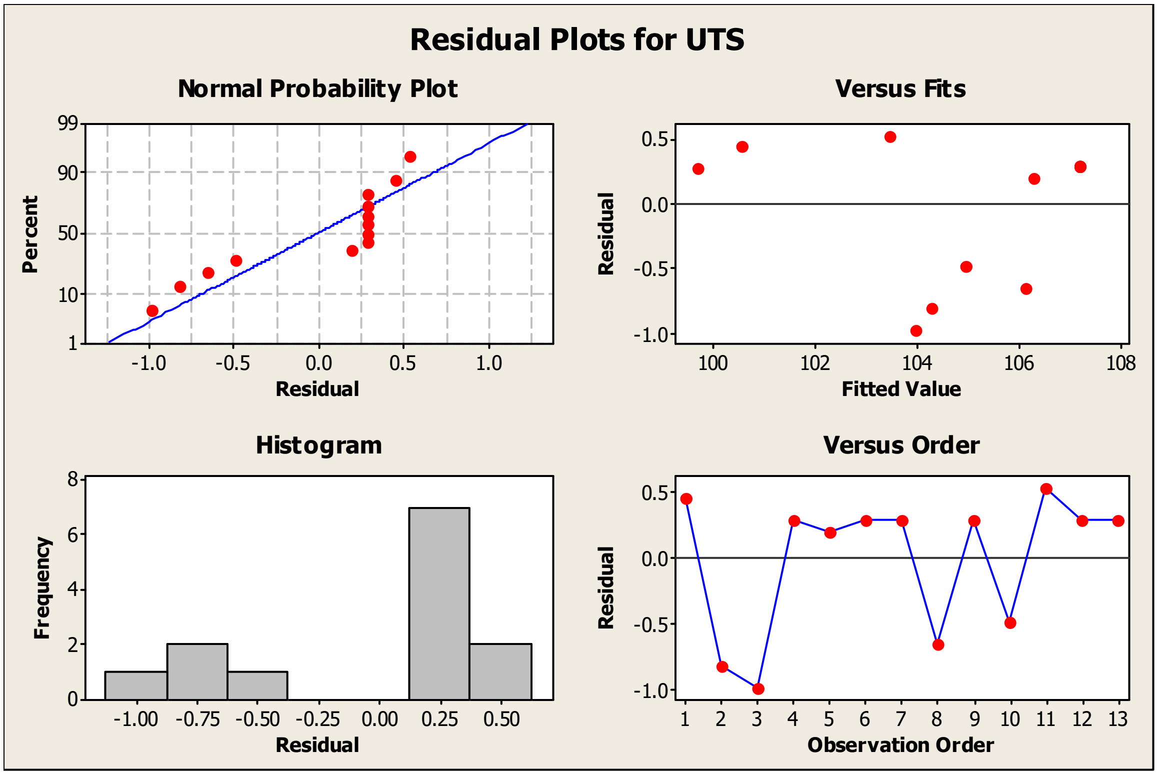

Figure 6.1: Residual plots for UTS

It is observed from the Figure 6.1 that residuals values are distributed normally and in a straight line and hence the model is adequate. Figure 6.2, 6.3 and 6.4 shows the Contour plot, surface plot and optimization plot respectively of Ultimate Tensile Strength (UTS) with variation of wheel velocity and product diameter.

It can be inferred from these plots that as the extrusion wheel velocity and product diameter or extrusion ratio increases, the Ultimate Tensile Strength of the of the extruded feedstock through Continuous Extrusion process increases and reaches to a maximum value and then further increase in the value of extrusion wheel velocity and product diameter or extrusion ratio results in decrease in the value of Ultimate Tensile Strength. The optimization plots reveals that the maximum Ultimate Tensile Strength (UTS) of 107.45 MPa can be achieved for Aluminum feedstock material corresponding to optimum wheel velocity of 6.57 rpm and product diameter of 6.33 mm.

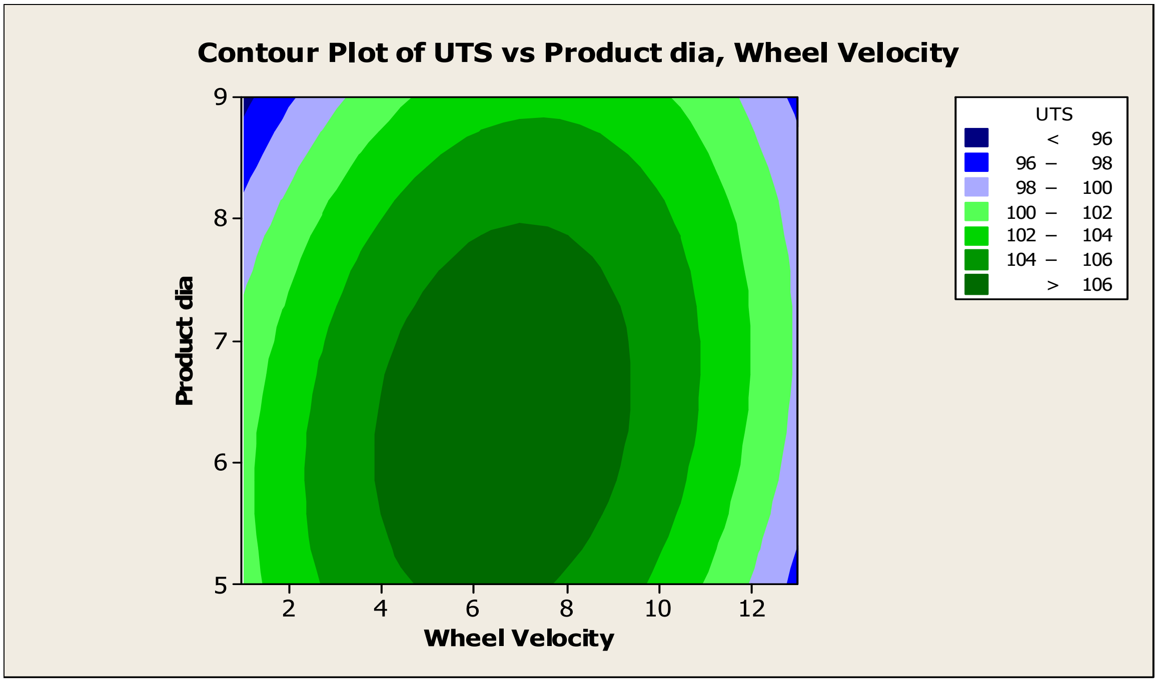

Figure 6.2: Contour plot of UTS with variation of wheel velocity and product diameter

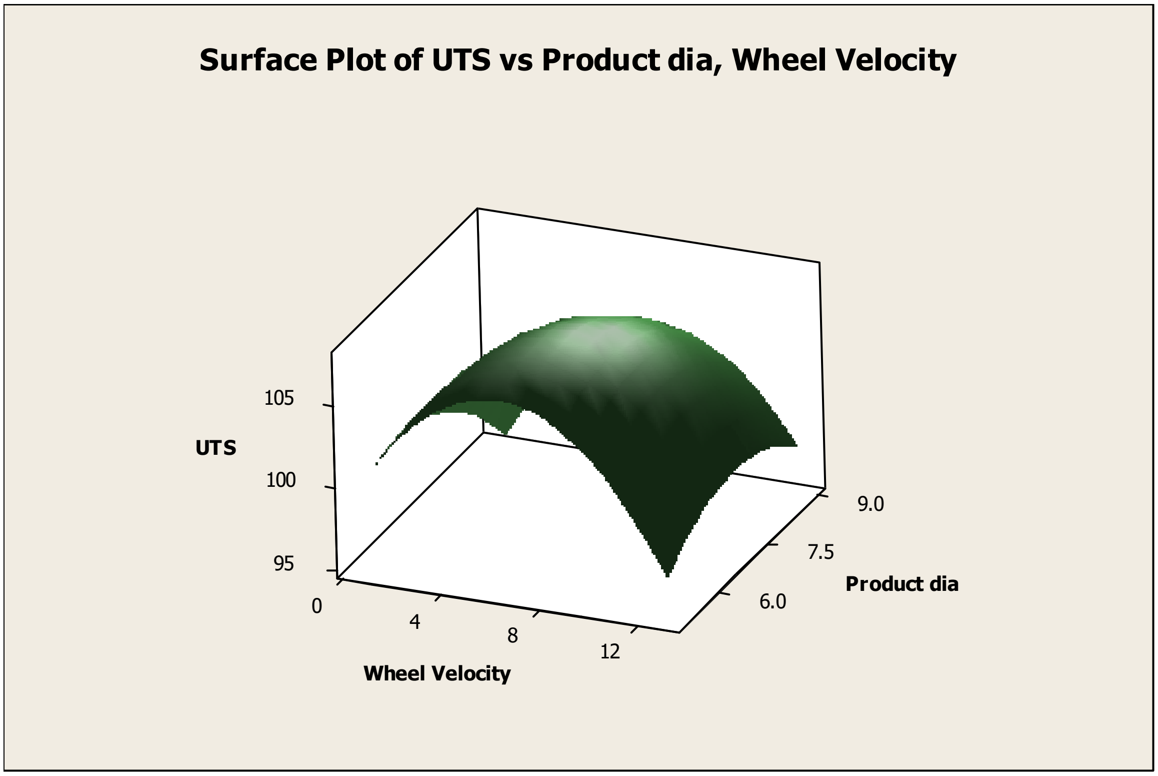

Figure 6.3: Surface plot of UTS with variation of wheel velocity and product diameter

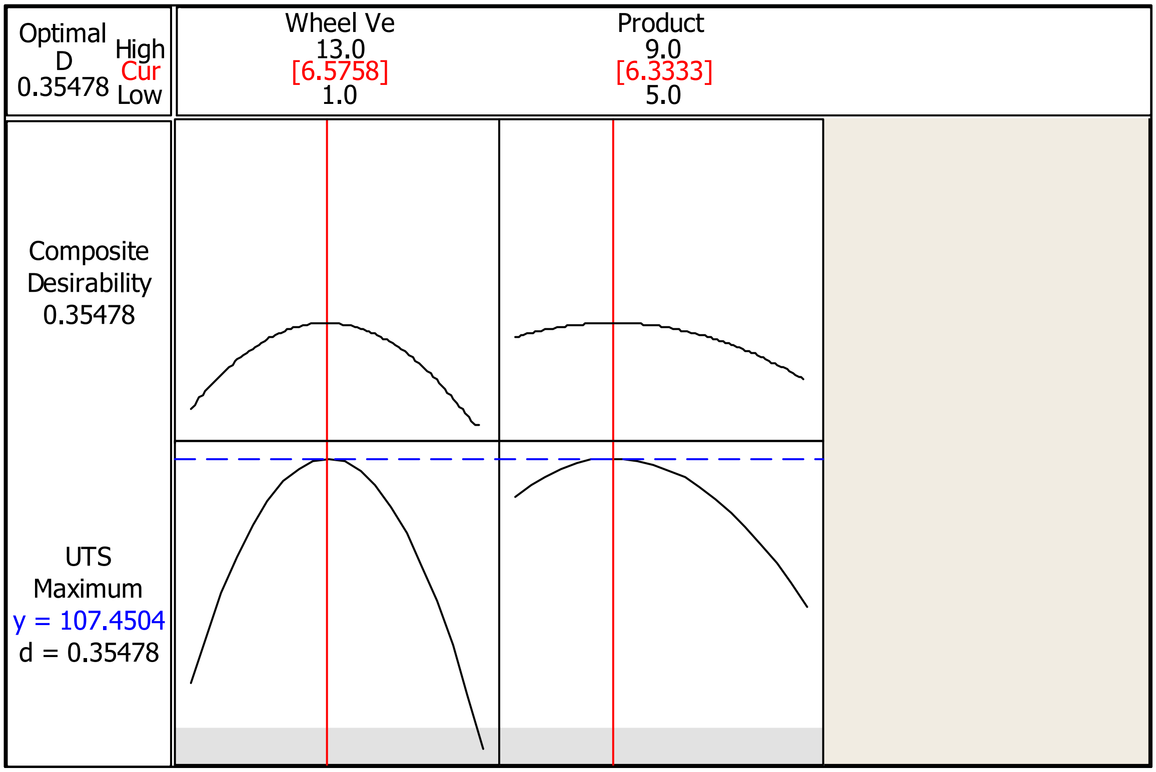

Figure 6.4: Optimization plot for UTS with respect to wheel velocity and product diameter

Modeling and Optimization (Hardness)

Table 6.5 shows the experimental plan and result for modeling and optimization of Hardness based on Central Composite second order rotatable design. The test of significance of Hardness has been carried out in Table 6.6 using the quadratic model and statistically significant analysis**.** The results of the quadratic model for Hardness are given below in Table 6.6. Analysis of Variance (ANOVA) test for modeling and optimization of Hardness has been carried out as shown in Table 6.7.

Table 6.5: Experimental plan and result for Hardness based on Central Composite second order rotatable design

Experiment No.	Wheel Velocity (RPM)		Product diameter (mm)		Hardness (HV)
	Coded	Actual	Coded	Actual	
1	-2	1	0	7	29.000
2	1	10	1	8	32.100
3	-1	4	1	8	31.200
4	0	7	0	7	35.000
5	0	7	-2	5	28.500
6	0	7	0	7	35.000
7	2	13	0	7	29.000
8	-1	4	-1	6	30.500
9	0	7	0	7	35.000
10	1	10	-1	6	29.400

11	0	7	2	9	30.000
12	0	7	0	7	35.000
13	0	7	0	7	35.000

Table 6.6: Test for significance of Hardness

Term	**Coefficient**	***t***	***p***
Constant	34.6517	98.690	0.000
Wheel Velocity	-0.0167	-0.068	0.947
Product diameter	0.5333	2.185	0.065
Wheel Velocity *Wheel Velocity	-1.5218	-8.614	0.000
Product diameter* Product diameter	-1.4593	-8.261	0.000
Wheel Velocity*Product diameter	0.5000	1.183	0.276

The test of significance of Hardness has been carried out using the quadratic model. The results of the quadratic model for Hardness are given in Table 6.6.The value of R^2 and adjusted R^2 are 94.36 %and 90.34%.This means that the regression model provides a complete relationship between the dependent and independent variables. The associated p value for the model is <0.05(i.e., $\alpha=0.05$ or 95% confidence), indicating that the model is considered to be statistically significant. It can be seen from the Table 6.7 that the standard F value for 95% confidence limit is 5.05. Hence the model is found to be adequate and statistically significant. It is also seen from Table 6.6 that from the p values, the main effect X_2 and second order effect of X_1 and X_2 is much more significant. The other model terms can be regarded as insignificant due to their probabilities values being more than 0.05.Figure 6.5 depicts the normal probability of residuals for Hardness. It is observed from the plot that residuals values are distributed normally and in a straight line and hence the model is adequate.

Using the results presented in Table 6.6, the derived model is shown as:

$$H= 34.6517- 0.0167X_1 + 0.5333 X_2- 1.5218 X_1^2- 1.4593 X_2^2+ 0.5000 X_1X_2 \qquad (6.4)$$

The adequacy of Hardness models is also tested through analysis of variance (ANOVA).The results of the analysis justifies the closeness of the fit of the derived mathematical model. It has been concluded that the evolved mathematical models given by Eqs (6.4) is quite adequate at 95% confidence level.

Table 6.7: Test result of ANOVA for Hardness

Source	DF	Sum of Squares	Mean Sum of squares	*F*-value	*P*-value
Regression	5	83.7747	16.7549	23.43	0.000
Linear	2	3.4167	1.7083	2.39	0.162
Square	2	79.3580	39.6790	55.49	0.000
Interaction	1	1.000	1.000	1.400	0.276
Residual error	7	5.0053	0.7150		
Lack of fit	3	5.0053	1.6684		
Pure error	4	0.0000	0.0000	0.0000	
Total	12	88.7800			

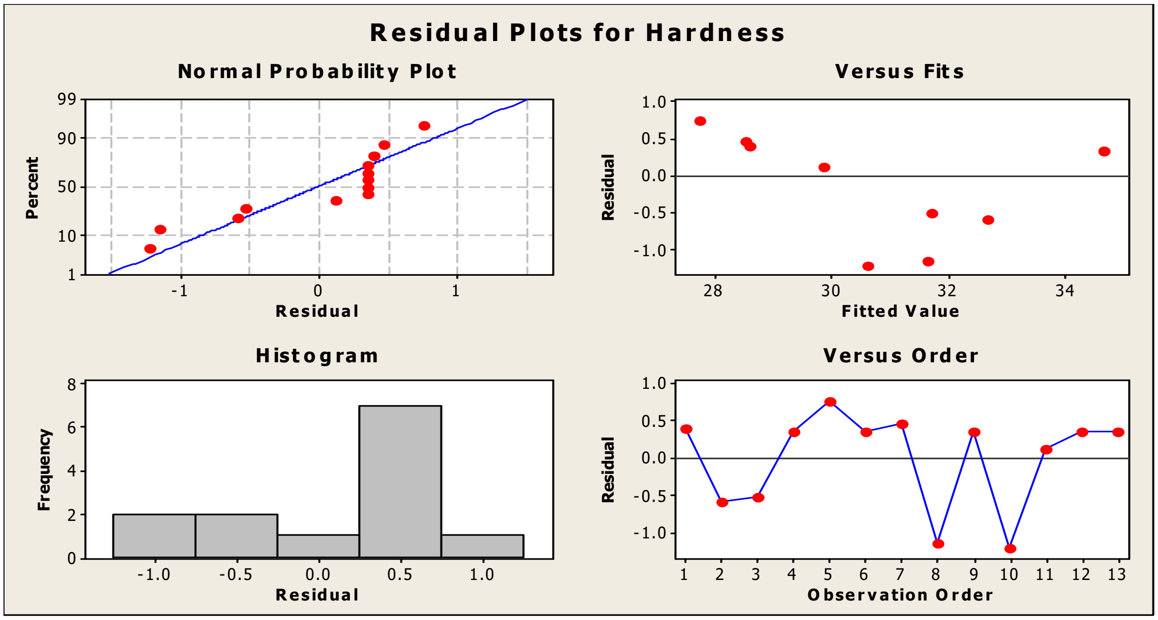

Figure 6.5: Residual plots for Hardness

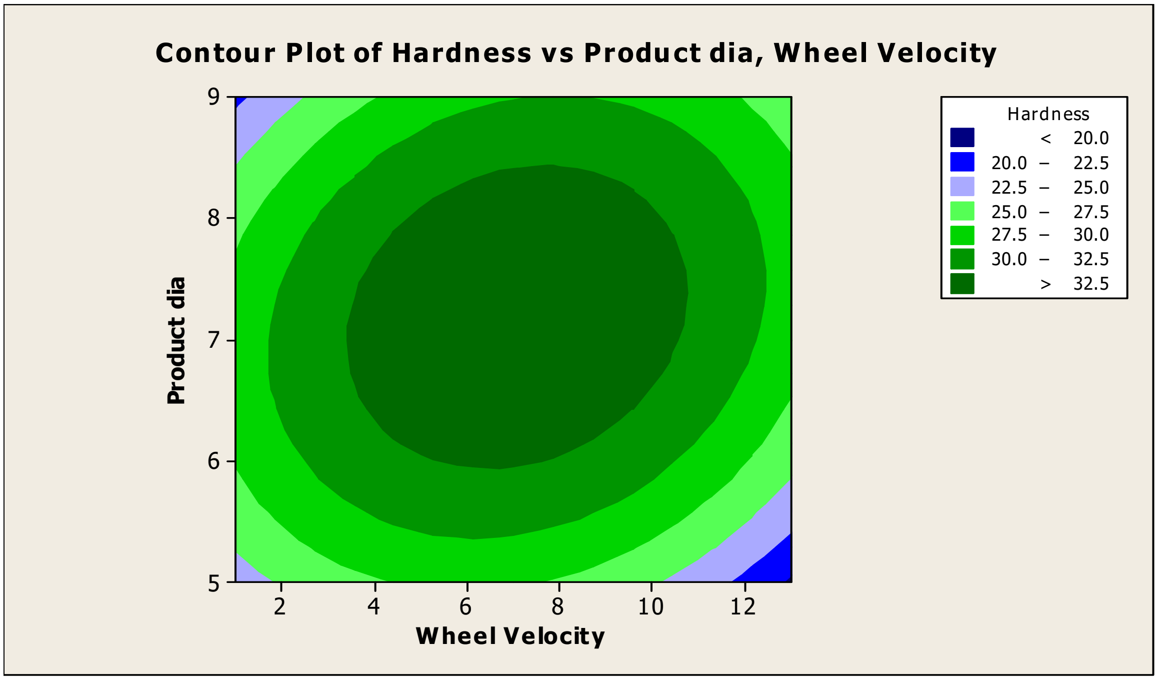

Figure 6.6: Contour plot of Hardness with variation of wheel velocity and product diameter

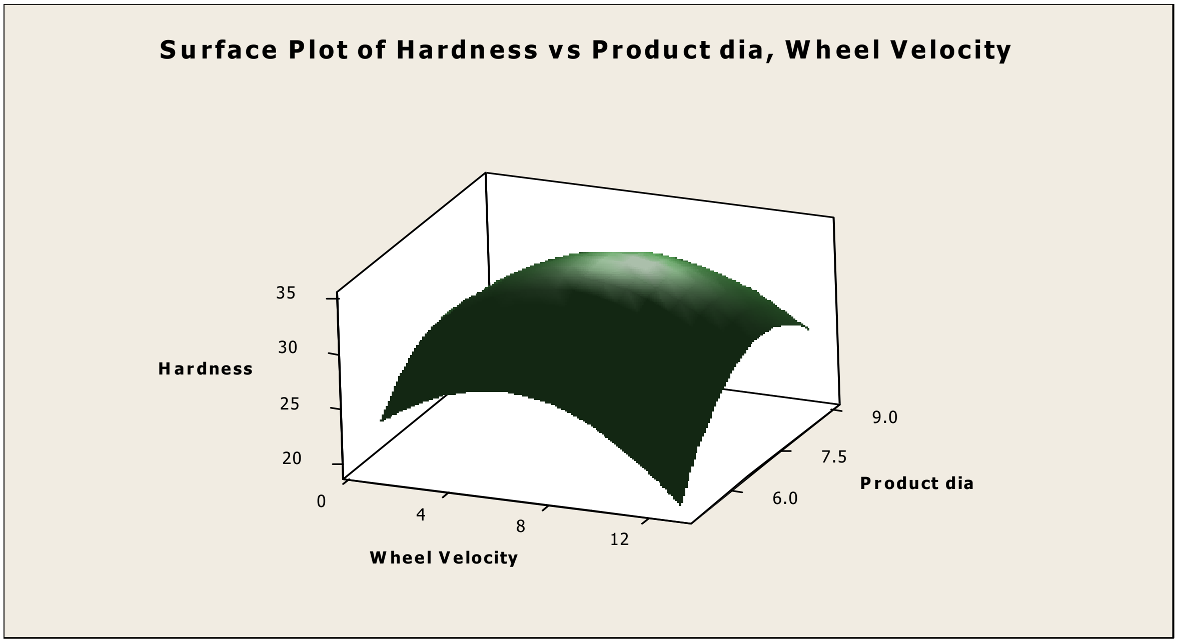

Figure 6.7: Surface plot of Hardness with variation in wheel velocity and product diameter

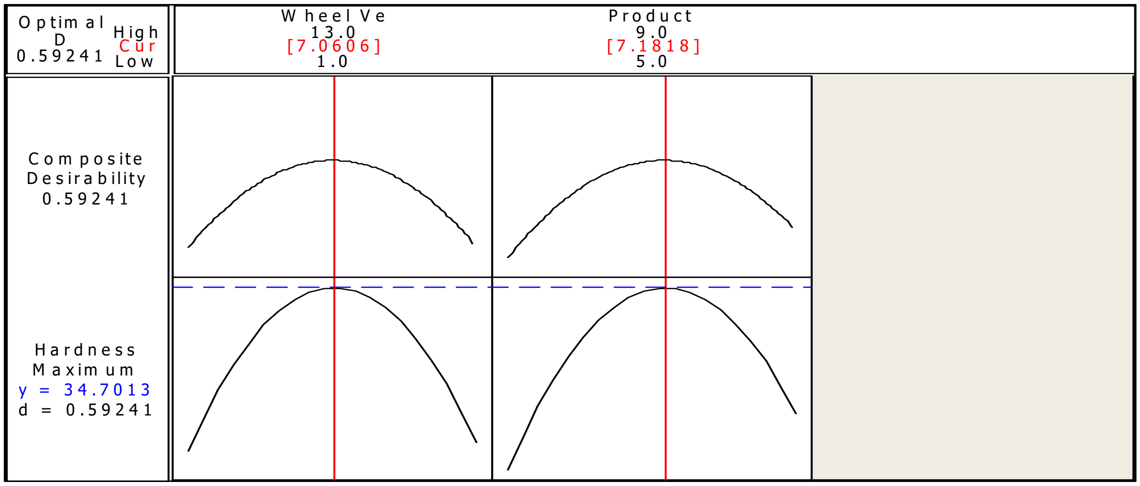

Figure 6.8: Optimization plot for Hardness with respect to wheel velocity and product diameter

Figure 6.6, 6.7 and 6.8 shows the contour plot, surface plot and optimization plot respectively for modeling and optimization of Hardness of the extruded Aluminum feedstock material. The effect of extrusion wheel velocity and product diameter or extrusion ratio on Hardness of the extruded feedstock through Continuous Extrusion process can be clearly seen through contour and surface plot.The Hardness value of the extruded feedstock also follows the same trend like that of Ultimate Tensile Strength with the variation of extrusion wheel velocity and product diameter. So, with the increase in extrusion wheel velocity and product diameter increases the Hardness of the extruded product to a certain maximum value and then decreases with further increase in the value of extrusion wheel velocity and product diameter.

Based on the developed second order response surface equations correlating the various Continuous Extrusion process parameters affecting Hardness of the extruded product, the optimal value of extrusion wheel velocity and product diameter or extrusion ratio has been determined. An analysis for the optimization of process parameter has been carried out using response surface methodology (RSM) optimization technique. The main aim has been to maximize the Hardness of the extruded product at some specific combination of extrusion wheel velocity and product diameter or extrusion ratio. The optimum values of the input process parameters in Continuous Extrusion process for Aluminum feedstock material are 6.57 rpm as wheel velocity and 6 mm as product diameter respectively. For Continuous Extrusion process of Pure Aluminum feedstock with optimum parametric combination, the Hardness can be achieved as high as HV 34.6.

Modeling and Optimization (Yield Strength)

Table 6.8 shows the Experimental plan and result for Yield Strength based on Central Composite second order rotatable design. The test of significance of Yield Strength has been carried out using the quadratic model as shown in Table 6.9.The value of R^2 and adjusted R^2 are 95.96 %and 93.07%.This means that the regression model provides a complete relationship between the dependent and independent variables. The associated p value for the model is <0.05(i.e., $\alpha=0.05$ or 95% confidence), indicating that the model is considered to be statistically significant. It is also seen from Table 6.9 that from the *p* values, the main effect X_2 and second order effect of X_1 and X_2 is much more significant. The other model terms can be regarded as insignificant due to their probabilities values being more than 0.05. The analysis of variance test (ANOVA) has been carried out as shown in Table 6.10.The standard F value for 95% confidence limit is 5.05. Hence the model is found to be adequate and statistically significant. Figure 6.9 depicts the normal probability of residuals for Yield Strength. It is observed from the plot that residuals values are distributed normally and in a straight line and hence the model is adequate.

Using the results presented in Table 6.9, the derived model is shown as:

$$\sigma = 34.5255 - 0.9350X_1 - 1.4483X_2 - 3.6547X_1^2 - 8.0953X_2^2 + 1.5700X_1X_2 \qquad (6.5)$$

Table 6.8: Experimental plan and result for Yield Strength based on Central Composite second order rotatable design

Experiment No.	**Wheel Velocity (RPM)**		**Product diameter (mm)**		**YS (MPa)**
	Coded	**Actual**	**Coded**	**Actual**	
1	-2	1	0	7	18.000
2	1	10	1	8	51.500
3	-1	4	1	8	42.750
4	0	7	0	7	33.000
5	0	7	-2	5	65.000
6	0	7	0	7	33.000
7	2	13	0	7	18.000
8	-1	4	-1	6	36.200
9	0	7	0	7	33.000
10	1	10	-1	6	39.670
11	0	7	2	9	65.000
12	0	7	0	7	33.000
13	0	7	0	7	33.000

Table 6.9: Test for significance of Yield Strength

Term	**Coefficient**	***t***	***p***
Constant	34.5255	16.631	0.000
Wheel Velocity	0.9350	0.687	0.514
Product diameter	1.4483	1.064	0.323

Wheel Velocity *Wheel Velocity	-3.6547	-3.710	0.008
Product diameter* Product diameter	8.0953	8.217	0.000
Wheel Velocity*Product diameter	1.5700	0.666	0.527

Table 6.10: Test result of ANOVA for Yield Strength

Source	**DF**	**Sum of Squares**	**Mean Sum of squares**	***F*-value**	***P*-value**
Regression	5	2430.48	486.10	21.86	0.000
Linear	2	35.66	35.66	0.80	0.486
Square	2	2384.96	1192.48	53.62	0.000
Interaction	1	9.86	9.86	0.44	0.527
Residual error	7	155.69	22.24		
Lack of fit	3	155.69	51.90		
Pure error	4	0.00	0.00	0.00	
Total	12	2586.17			

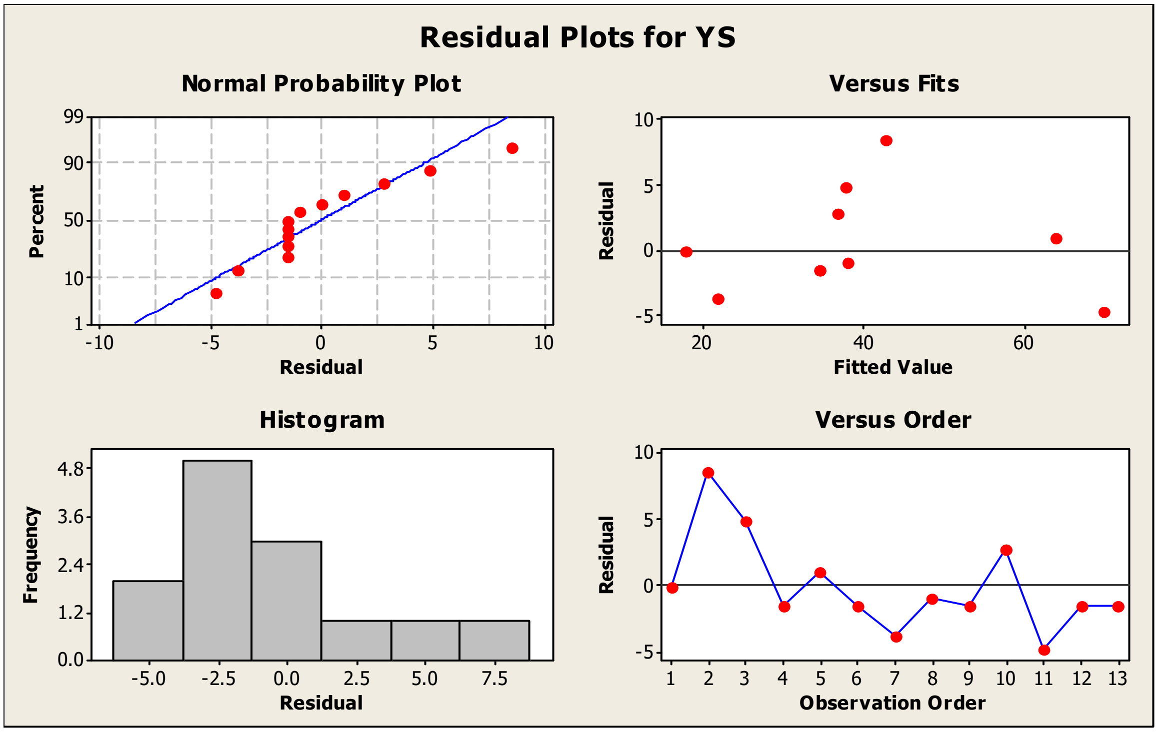

Figure 6.9: Residual plots for Yield Strength

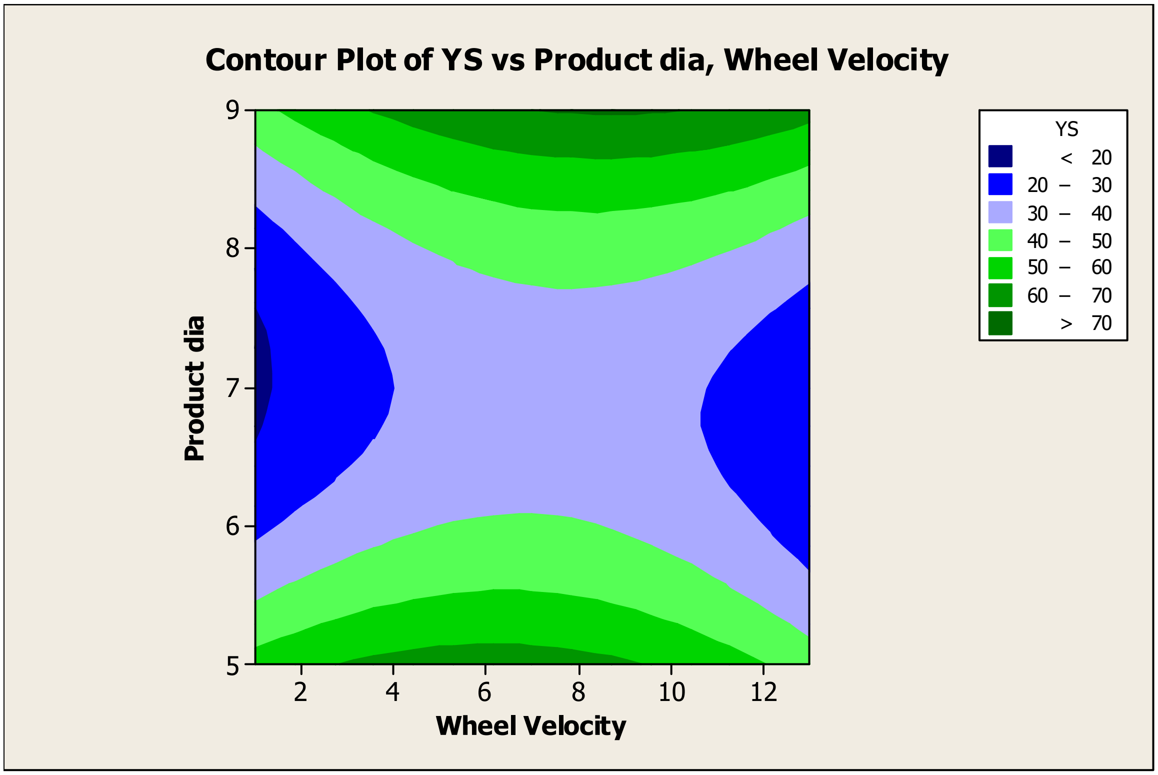

Figure 6.10: Contour plot of YS with variation in wheel velocity and product diameter

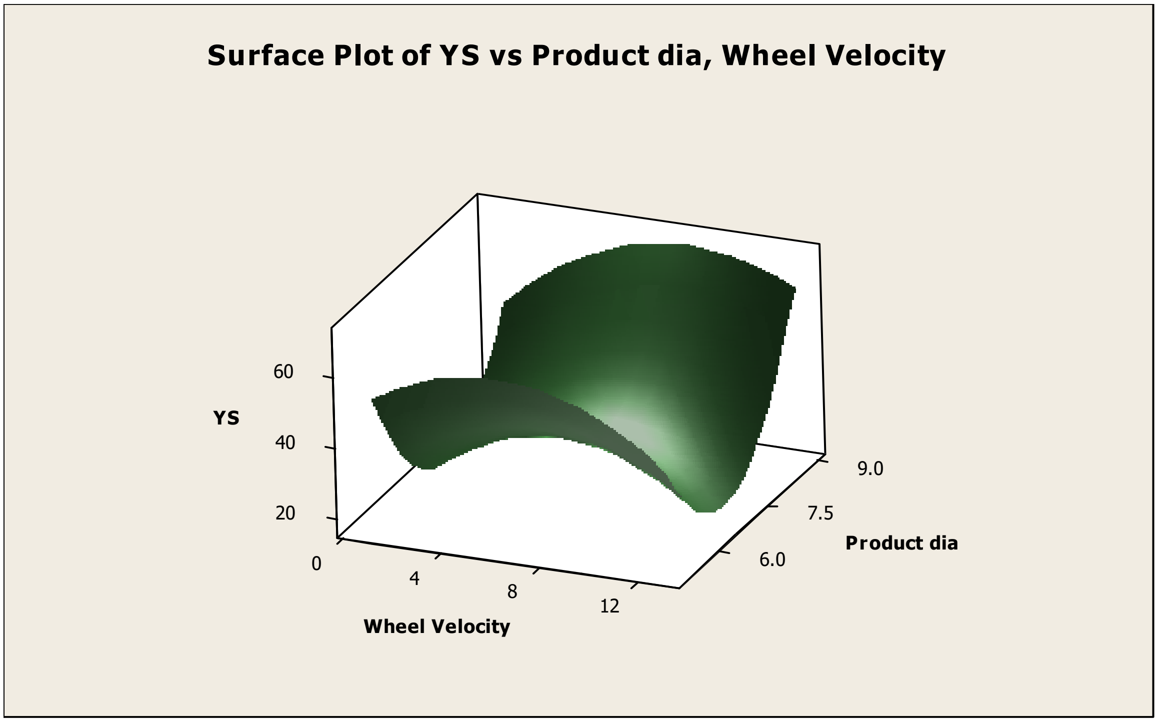

Figure 6.11: Surface plot of YS with variation in wheel velocity and product diameter

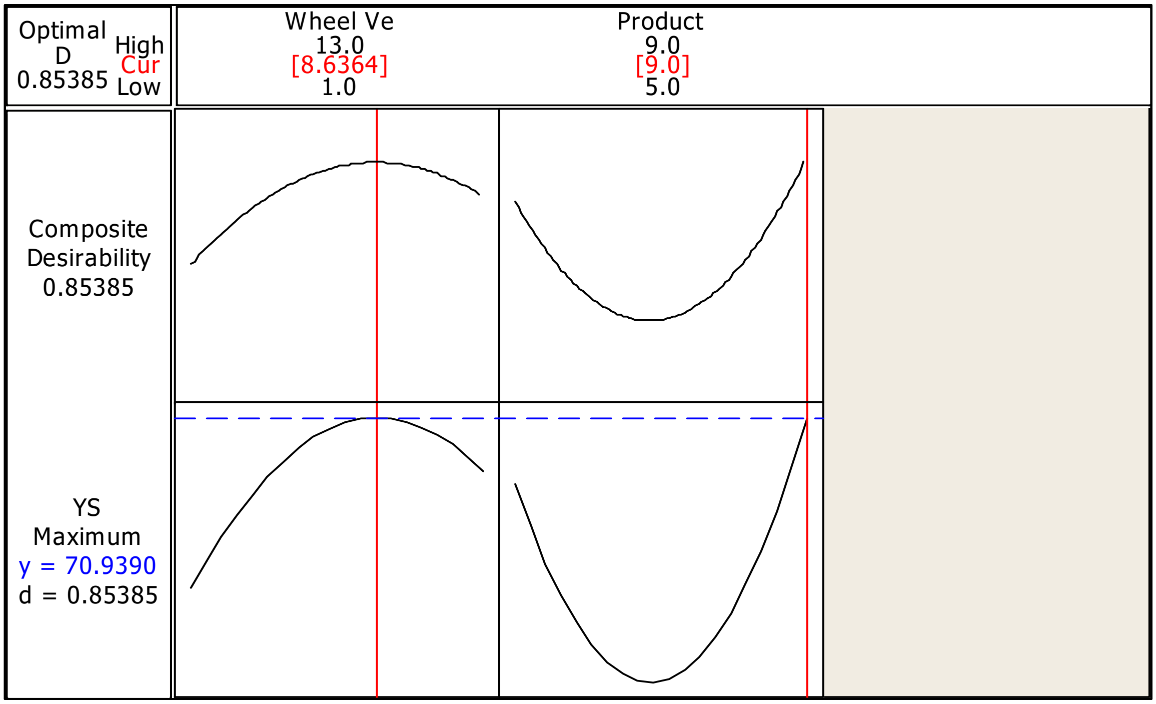

Figure 6.12: Optimization plot for Yield Strength with respect to wheel velocity and product diameter

Figure 6.10, 6.11 and 6.12 shows the contour plot, surface plot and optimization plot respectively for modeling and optimization of Yield Strength for Continuous Extrusion of Aluminum feedstock material. The effect of extrusion wheel velocity and product diameter on Yield Strength can be seen through these plots. It can be inferred from the surface plot and contour plot that as the extrusion wheel velocity and product diameter or extrusion ratio increases, the Yield Strength of the of the extruded feedstock through Continuous Extrusion process increases and reaches to a maximum value and then further increase in the value of extrusion wheel velocity and product diameter or extrusion ratio results in decrease in the value of Yield Strength.

Based on the developed second order response surface equations correlating the various Continuous Extrusion process parameters affecting Yield Strength of the extruded product, the optimal value of extrusion wheel velocity and product diameter or extrusion ratio has been determined. An analysis for the optimization of process parameter has been carried out using response surface methodology (RSM) optimization technique .The main aim has been to maximize the Yield Strength of the extruded product at some specific combination of extrusion wheel velocity and product diameter or extrusion ratio. The optimum values of the input process parameters in Continuous Extrusion process for extrusion wheel velocity and product diameter are 6.06 (RPM) and 6.18 (mm) respectively.

For Continuous Extrusion process of Pure Aluminum feedstock with optimum parametric combination, Yield Strength can be achieved as high as 70.939 M Pa.

The mechanical properties of Aluminum extrusions are greatly affected by the presence of Mg-Si particles. For the mechanical properties, it was generally observed that deformation speed did not itself have a dominant effect, and could simply be regarded as a tool for affecting maximum extrusion temperature [Zhao et al. (2013)]. As the extrusion wheel velocity increases, the maximum extrusion temperature increases which leads to increase of mechanical properties such as tensile strength and hardness. But increase of extrusion temperature beyond a limit results in the presence of coarse grain particles which results in decrease of mechanical properties such as UTS, YS and hardness.

Modeling and Optimization (% Elongation)

Table 6.11, 6.12 and 6.13 shows the Experimental plan and result, the test of significance and analysis of variance test (ANOVA) respectively for modeling and optimization of % Elongation.

Table 6.11: Experimental plan and result % Elongation based on central composite second order rotatable design

Experiment No.	**Wheel velocity (RPM)**		**Product diameter (mm)**		**% ELONGATION**
	Coded	**Actual**	**Coded**	**Actual**	
1	-2	1	0	7	25.500
2	1	10	1	8	43.000
3	-1	4	1	8	34.000
4	0	7	0	7	46.500
5	0	7	-2	5	32.000
6	0	7	0	7	46.500
7	2	13	0	7	21.000
8	-1	4	-1	6	36.000
9	0	7	0	7	46.500
10	1	10	-1	6	34.000
11	0	7	2	9	35.000
12	0	7	0	7	46.500
13	0	7	0	7	46.500

Table 6.12: Test for significance of % Elongation

Term	**Coefficient**	***t* - value**	***P* - value**
Constant	46.3793	65.424	0.000
Wheel Velocity	-0.2500	-0.507	0.628
Product diameter	1.0000	2.029	0.082
Wheel Velocity *Wheel Velocity	-5.8200	-16.318	0.000
Product diameter* Product diameter	-3.2575	-9.134	0.000
Wheel Velocity*Product diameter	3.0000	3.514	0.010

The test of significance for % Elongation has been carried out using the quadratic model. The results of the quadratic model for % Elongation are given in Table 6.12.The value of R^2 and adjusted R^2 are 96.76 %and 96.15%.This means that the regression model provides a complete relationship between the dependent and independent variables. The associated p value for the model is <0.05(i.e., $\alpha=0.05$ or 95% confidence), indicating that the model is considered to be statistically significant. It is also seen from Table 6.12 that from the p values, the main effect X_2 and second order effect of X_1 and X_2 is much more significant. The other model terms can be regarded as insignificant due to their probabilities values being more than 0.05.The standard F value for 95% confidence limit is 5.05 as shown in Table 6.13. Hence the model is found to be adequate. Figure 6.13 depicts the normal probability of residuals for % Elongation. It is observed from the plot that residuals values are distributed normally and in a straight line and hence the model is adequate.

Using the results presented in Table 6.12, the derived model is shown as:

$$L = 46.3973 - 0.2500X_1 + 1.0000\,X_2 - 5.8200\,X_1^2 - 3.2575\,X_2^2 + 3.0000\,X_1X_2 \qquad (6.6)$$

The adequacy of % Elongation models is tested through analysis of variance (ANOVA).The results of the analysis justifies the closeness of the fit of the derived mathematical model. It is concluded that the evolved mathematical models given by Eqs (6.6) is quite adequate at 95% confidence level.

Table 6.13: Test result of ANOVA for % Elongation

Source	**DF**	**Sum of Squares**	**Mean Sum of squares**	***F*-value**	***P*-value**
Regression	5	889.097	176.819	61.01	0.000
Linear	2	12.750	6.375	2.19	0.183
Square	2	840.347	420.173	144.16	0.000
Interaction	1	36.000	36.000	12.35	0.010
Residual error	7	20.403	2.915		
Lack of fit	3	20.403	6.801		
Pure error	4	0.000	0.000		
Total	12	909.500			

At lower wheel rpm, the elongation is mainly dependent on the size and homogeneity of grains. Thus, the effect of deformation speed on the elongation is same as the effect of deformation on the size and homogeneity of grains**.** The experiments showed that as the wheel speed increases, grain size decreases and grain uniformity increases causing higher ductility and elongation. Too high wheel speed produces coarse grains resulting in lower ductility and elongations.

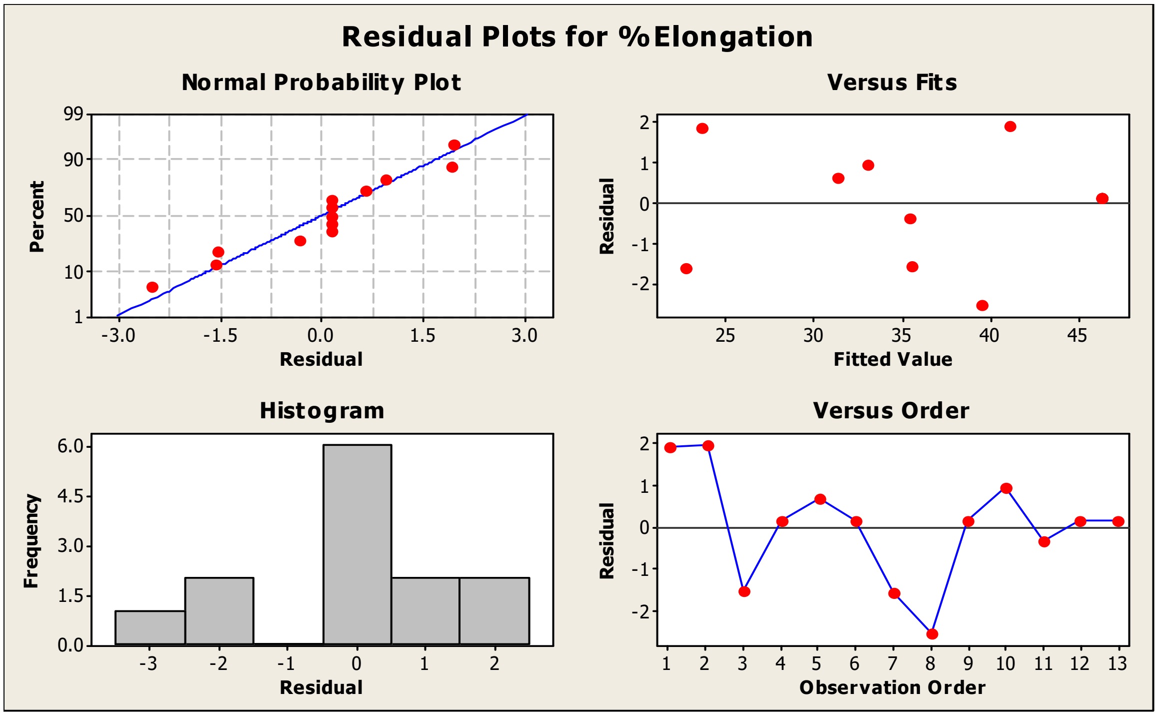

Figure 6.13: Residual plots for % Elongation

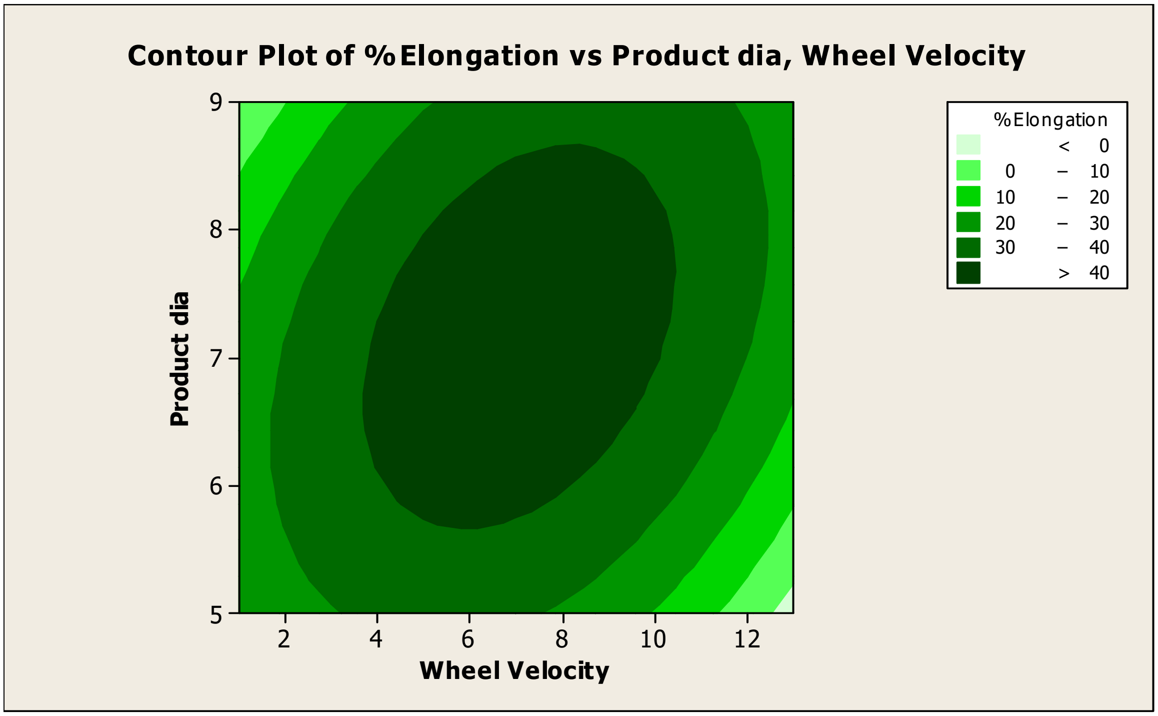

Figure 6.14: Contour plot of % Elongation with variation in wheel velocity and product diameter

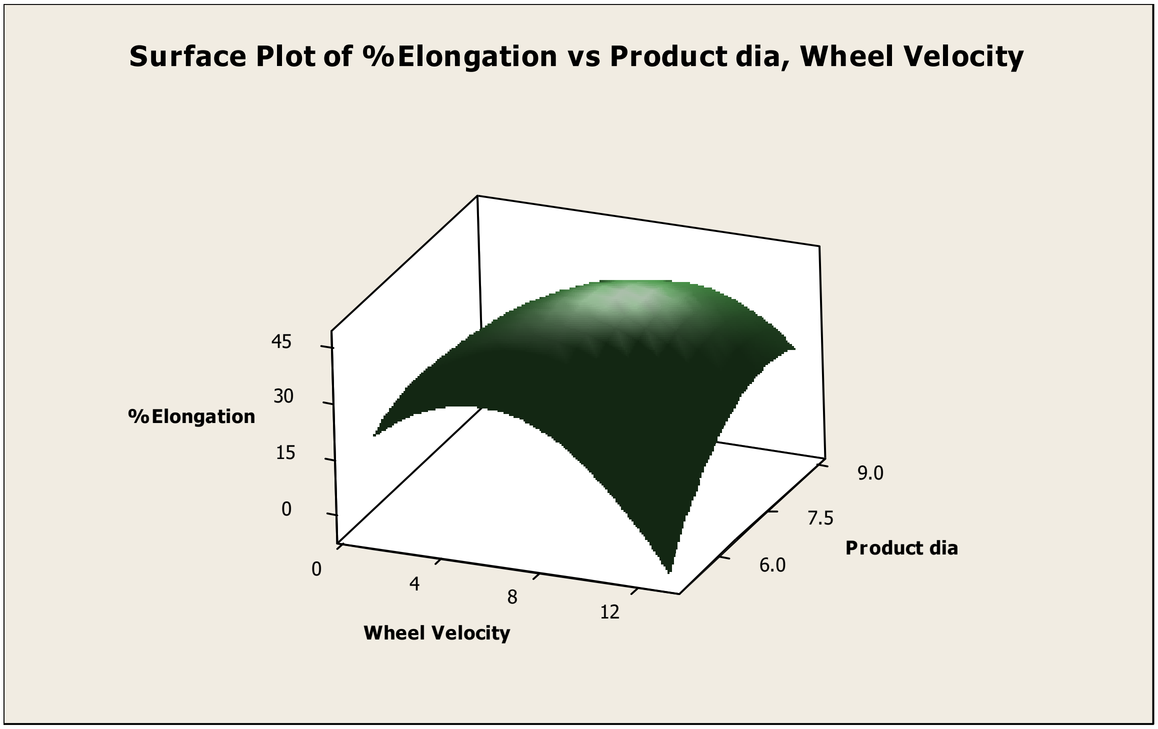

Figure 6.15: Surface plot of % Elongation with variation in wheel velocity and product diameter

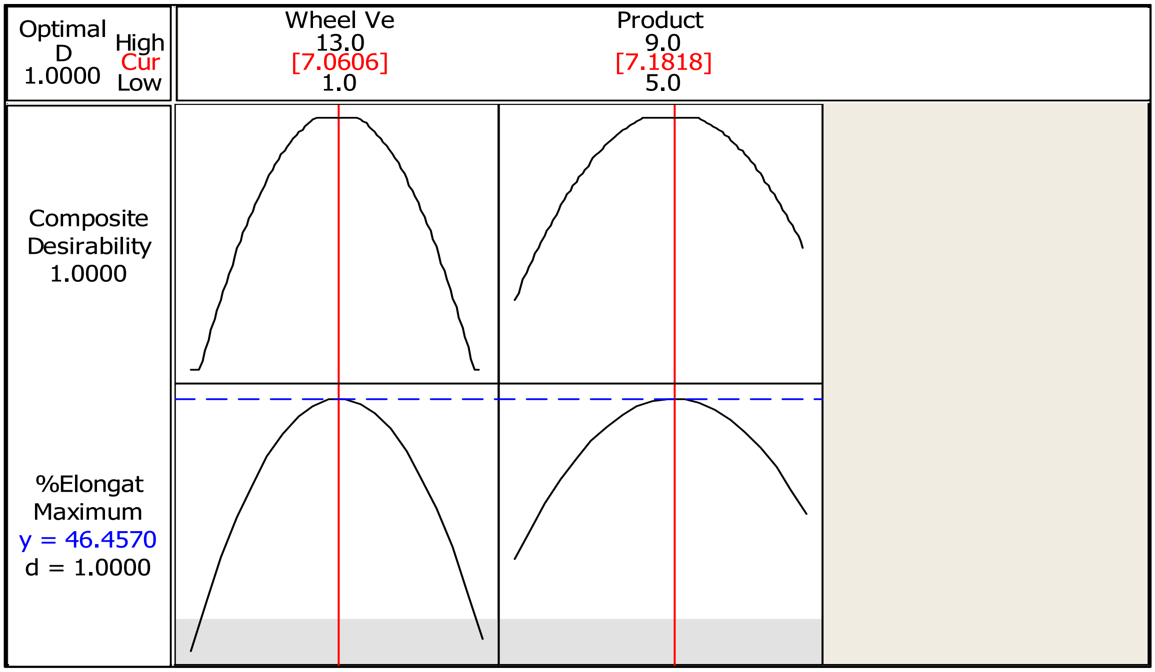

Figure 6.16: Optimization plot for % Elongation with respect to wheel velocity and product diameter

Figure 6.14, 6.15 and 6.16 shows the contour, surface and optimization plots for modeling and optimization of % Elongation in Continuous Extrusion of Aluminum feedstock material. The effect of extrusion wheel velocity and product diameter or extrusion ratio on % Elongation of the extruded feedstock through Continuous Extrusion process can be clearly seen through contour and surface plot. The % Elongation value of the extruded feedstock also follows the same trend like that of Yield Strength with the variation of extrusion wheel velocity and product diameter. So, with the increase in extrusion wheel velocity and product diameter increases the % Elongation of the extruded product to a certain maximum value and then decreases with further increase in the value of extrusion wheel velocity and product diameter.

Based on the developed second order response surface equations correlating the various Continuous Extrusion process parameters affecting % Elongation of the extruded product, the optimal value of extrusion wheel velocity and product diameter or extrusion ratio has been determined. An analysis for the optimization of process parameter has been carried out using response surface methodology (RSM) optimization technique .The main aim has been to maximize the %Elongation of the extruded product at some specific combination of extrusion wheel velocity and product diameter or extrusion ratio. The optimum values of the extrusion wheel velocity and product diameter are 6.06 (RPM) and 6.18 (mm) respectively. The maximum % Elongation is found to be 46.4570 respectively.

Case II

6.3 Optimization of Extrusion process by Response Surface Methodology for Pure Copper (C 101) feedstock material

In this section, Continuous Extrusion forming of the Pure Copper (C 101) rod has been carried out on commercially available set up at different extrusion wheel velocity and extrusion ratios. A mathematical model through Response Surface Methodology has been developed to analyze the influence of wheel velocity and extrusion ratio on mechanical properties such as Ultimate Tensile Strength, Yield Strength, Percentage Elongation and Hardness. An optimum value of the extrusion wheel velocity and extrusion ratio has been determined to predict the best mechanical properties of the

Continuous Extrusion forming products at different wheel velocities and extrusion ratio. The adequacy of the model has also been tested by the Analysis of Variance test (ANOVA).

The Copper rod has been subjected to Continuous Extrusion under the extrusion wheel velocities of 4, 6, 8 and 10 RPM. The Copper rod has also been subjected to Continuous Extrusion under different extrusion ratios .The Continuous Extrusion products have not been subjected to any artificial aging and treatments. The tensile testing of the copper samples has been performed on INSTRON machine. The Hardness test of the extruded products samples has been performed on Vickers Hardness testing machine and the Hardness values has been obtained.

For finding out the relationship between the process parameters and the mechanical properties, second order polynomial response surface mathematical models can be considered as [Vettivel et al.,(2013)], [Seeman et al.,(2009)].

$$Y_u = b_0 + \sum b_i x_{iu} + \sum b_{ii} x^2_{iu} + \sum b_{ij} x_{iu} x_{ju} \tag{6.7}$$

Where Y_u is the corresponding response; x_{iu} is the coded values of the i-th Continuous Extrusion parameters for the u-th experiments; and b_i, b_{ii}, bij are the second order regression coefficients. The second term under the summation sign of this polynomial equation corresponds to linear effect, while the third term denotes to the higher order effect. The fourth term of the equation includes the interactive effects of the process parameters.

The two factors with three levels and Central Composite rotatable design matrix has been chosen to optimize the experimental conditions. The main aim of the factorial experiments depicts the relationship between the response as a dependent variable and the parameter levels. This approach helps to understand how the change in levels of parameters affects the response. The combination of different levels of the parameters leads to certain optimum response.

The investigation made in this section deals with the effects of factors such as extrusion wheel velocity and extrusion ratio.

The response parameters are Ultimate Tensile Strength (UTS), Yield Strength (YS), Percentage Elongation and Hardness of the extruded Copper (C 101) feedstock during Continuous Extrusion process. A 2^k factorial with Central Composite second order design has been used (in this case k=2).

Table 6.14: Experimental parameter and levels

Factor	**Levels**		
	-1	**0**	**1**
Wheel Velocity	4	6	8
Extrusion Ratio	4.34	3.18	2.44

Modeling and optimization (Ultimate Tensile Strength)

Table 6.15, 6.16 and 6.17 shows the Experimental plan and result, test of significance and analysis of variance (ANOVA) of respectively for modeling and optimization of UTS in Continuous Extrusion process of Copper feedstock material. The results of the quadratic model for UTS has been given in Table 6.16.The value of R^2 and adjusted R^2 are 99.51% and 99.26%.This means that the regression model provides a complete relationship between the dependent and independent variables. The associated p value for the model is <0.05(i.e., $\alpha=0.05$ or 95% confidence), indicating that the model is considered to be statistically significant. The standard F value for 95% confidence limit is 5.05 as shown in Table 6.17. Using the results presented in Table 6.16, the derived model is shown as:

$$\sigma = 248.069+0.750X_1 -2.250X_2 -0.121X_1^2 - 0.871X_2^2 \qquad (6.8)$$

Table 6.15: Experimental plan and result for Ultimate Tensile Strength based on Central Composite second order rotatable design

Experiment No.	**Wheel Velocity (RPM)**		**Product Diameter (mm)**		**UTS (MPa)**
	Coded	**Actual**	**Coded**	**Actual**	
1	0	6	0	7	248
2	-2	2	0	7	246
3	0	6	0	7	248
4	0	6	2	9	240
5	-1	4	1	8	244
6	1	8	-1	6	250
7	0	6	0	7	248
8	-1	4	-1	6	249
9	0	6	0	7	248
10	0	6	-2	5	249
11	0	6	0	7	248
12	2	10	0	7	249
13	1	8	1	8	246

Table 6.16: Test for significance of UTS

Term	**Coefficient**	***t***	***p***
Constant	248.069	2599.934	0.000
Wheel Velocity	0.750	11.306	0.000
Product diameter	-2.250	-33.919	0.000
Wheel Velocity *Wheel Velocity	-0.121	-2.574	0.000
Product diameter* Product diameter	-0.871	-18.138	0.000

Table 6.17: Test result of ANOVA for UTS

Source	DF	Sum of Squares	Mean Sum of squares	*F*-value	*P*-value
Regression	4	85.2699	21.3175	403.73	0.000
Linear	2	66.5000	33.7500	639.18	0.000
Square	2	16.7699	8.8849	168.27	0.000
Residual error	8	0.4224	0.0528		
Lack of fit	4	0.4224	0.1056		
Pure error	4	0.0000	0.0000		
Total	12	85.6923			

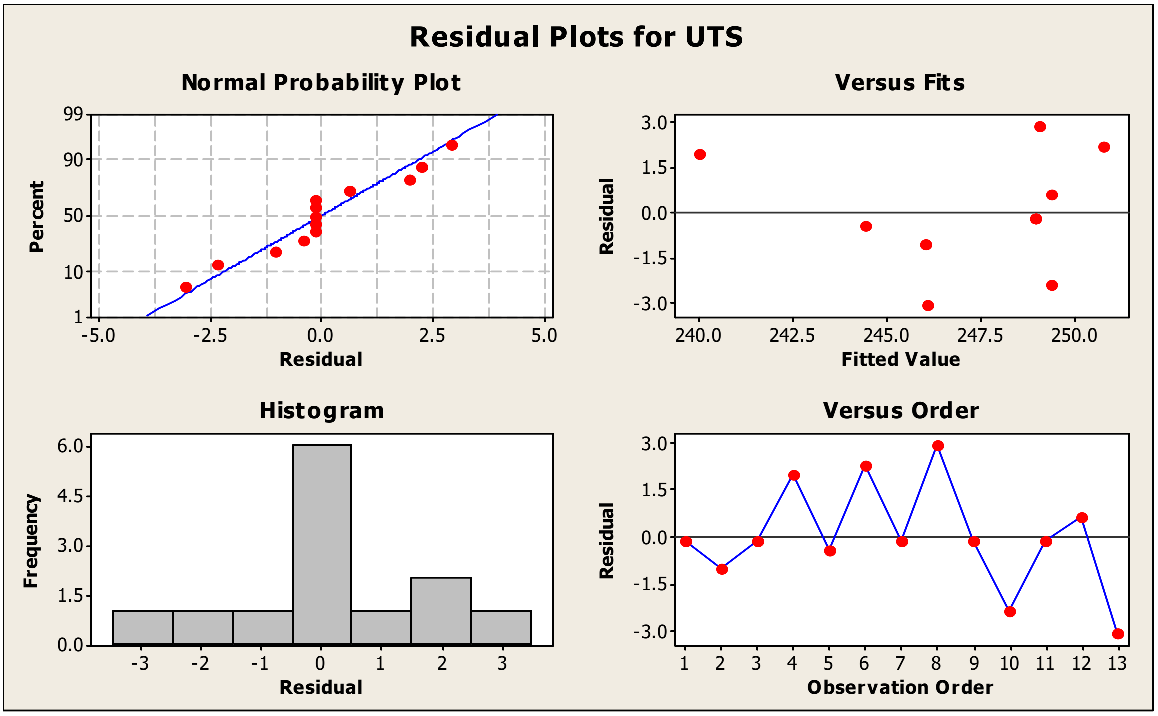

Figure 6.17: Residual plots for UTS

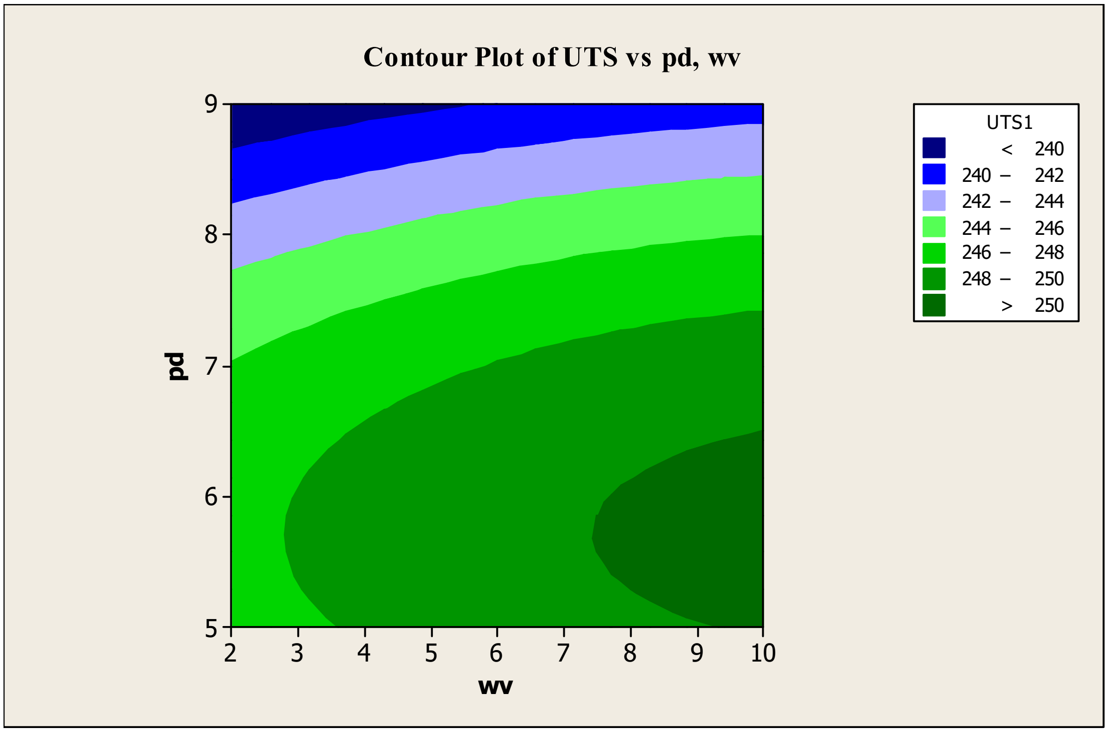

Figure 6.18: Contour plot for UTS

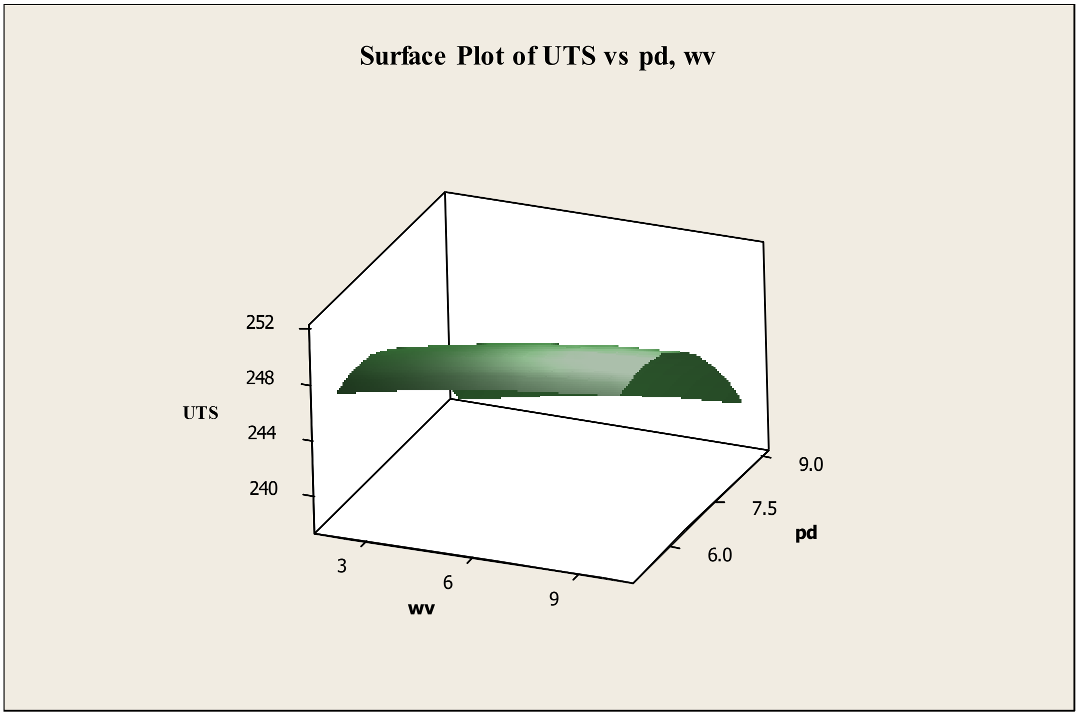

Figure 6.19: Contour plot for UTS

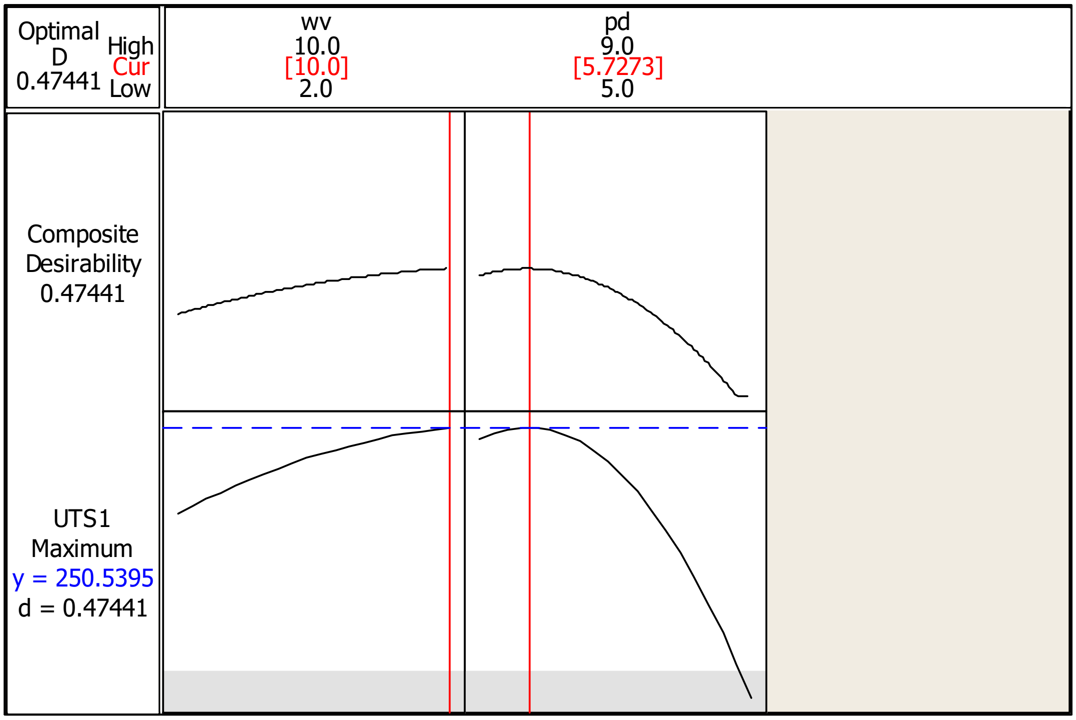

Figure 6.20: Optimization plot for UTS

Figure 6.17, 6.18, 6.19 and 6.20 shows the residual, contour, surface and optimization plot for modeling and optimization of UTS in continuous Extrusion of Copper feedstock material. It can be inferred from the surface plot and contour plot that as the extrusion wheel velocity and product diameter or extrusion ratio increases, the Ultimate Tensile Strength of the of the extruded feedstock through Continuous Extrusion process increases and reaches to a maximum value and then further increase in the value of extrusion wheel velocity and product diameter or extrusion ratio results in decrease in the value of Ultimate Tensile Strength.

Based on the developed second order response surface equations correlating the various Continuous Extrusion process parameters affecting Ultimate Tensile Strength of the extruded product, the optimal value of extrusion wheel velocity and product diameter or extrusion ratio has been determined. An analysis for the optimization of process parameter has been carried out using Response Surface Methodology (RSM) optimization technique. The main aim has been to maximize the Ultimate Tensile Strength of the extruded product at some specific combination of extrusion wheel velocity and product diameter or extrusion ratio. The optimum values of the input process parameters such as extrusion wheel velocity and product diameter are 10 (RPM) and 5.72 (mm) respectively.

For Continuous Extrusion process of Pure Copper feedstock with optimum parametric combination, Ultimate Tensile Strength can be achieved as high as 250.5 MPa.

Modeling and Optimization (Hardness)

Table 6.18, 6.19 and 6.20 shows the Experimental plan, test of significance and analysis of variance test (ANOVA) for modeling and optimization of Hardness in Continuous Extrusion of copper feedstock material.

Table 6.18: Experimental plan and result for Hardness based on Central Composite second order rotatable design

Experiment No.	Wheel Velocity (RPM)		Product Diameter (mm)		Hardness (HV)
	Coded	Actual	Coded	Actual	
1	0	6	0	7	66.0
2	-2	2	0	7	68.0
3	0	6	0	7	66.1
4	0	6	2	9	59.0
5	-1	4	1	8	61.0
6	1	8	-1	6	79.0
7	0	6	0	7	66.3
8	-1	4	-1	6	69.0
9	0	6	0	7	66.2
10	0	6	-2	5	74.0
11	0	6	0	7	66.2
12	2	10	0	7	89.0

13	1	8	1	8	71.0

Table 6.19: Test for significance of Hardness

Term	**Coefficient**	***t* – value**	***P* - value**
Constant	66.2069	671.304	0.000
Wheel Velocity	5.1667	74.232	0.000
Product diameter	-3.8333	-55.076	0.000
Wheel Velocity *Wheel Velocity	2.8379	56.343	0.000
Product diameter* Product diameter	-0.1621	-3.218	0.012

The test of significance of Hardness has been carried out using the quadratic model. The results of the quadratic model for Hardness are given in Table 6.19.The value of R^2 and adjusted R^2 are 99.93 %and 99.90%.This means that the regression model provides a complete relationship between the dependent and independent variables. The associated p value for the model is <0.05(i.e., $\alpha=0.05$ or 95% confidence), indicating that the model is considered to be statistically significant. . It is also seen from Table 6.19 that from the p values, the main effect X_1, X_2 and second order effect of X_1 and X_2 is much more significant. The other model terms can be regarded as insignificant due to their probabilities values being more than 0.05.The standard F value for 95% confidence limit is 5.05. Hence the model is found to be adequate. Figure 6.21 depicts the normal probability of residuals for Hardness. It is observed from the plot that residuals values are distributed normally and in a straight line and hence the model is adequate.

Using the results presented in Table 6.19, the derived model is shown as:

$$H= 66.2069+5.1667X_1-3.8333 X_2+2.8379 X_1^2- 0.1621 X_2^2 \tag{6.9}$$

The adequacy of both UTS and Hardness models has been also tested through Analysis of Variance (ANOVA).The results of the analysis justifies the closeness of the fit of the derived mathematical model. It has been concluded that the evolved mathematical models given by Eqs (1) and (2) are quite adequate at 95% confidence level.

Table 6.20: Test result of ANOVA for Hardness

Source	**DF**	**Sum of Squares**	**Mean Sum of squares**	***F*-value**	***P*-value**
Regression	4	704.558	176.140	3029.98	0.000
Linear	2	496.667	248.333	4271.87	0.000
Square	2	206.891	103.946	1788.09	0.000
Residual error	8	0.465	0.058		
Lack of fit	4	0.413	0.103	6.94	0.035
Pure error	4	0.000	0.000		
Total	12	705.023			

Figure 6.22, 6.23 and 6.24 shows the contour, surface and optimization plot for modeling and optimization of hardness in Continuous extrusion of Copper feedstock material. It can be inferred from the surface plot and contour plot that as the extrusion wheel velocity and product diameter or extrusion ratio increases, the hardness of the of the extruded feedstock through Continuous Extrusion process increases and reaches to a maximum value and then further increase in the value of extrusion wheel velocity and product diameter or extrusion ratio results in decrease in the value of hardness. The optimum values of the input process parameters such as extrusion wheel velocity and

product diameter are 10 (RPM) and 5 (mm) respectively. For Continuous Extrusion process of Pure Copper feedstock with optimum parametric combination, hardness can be achieved as high as 95.9.

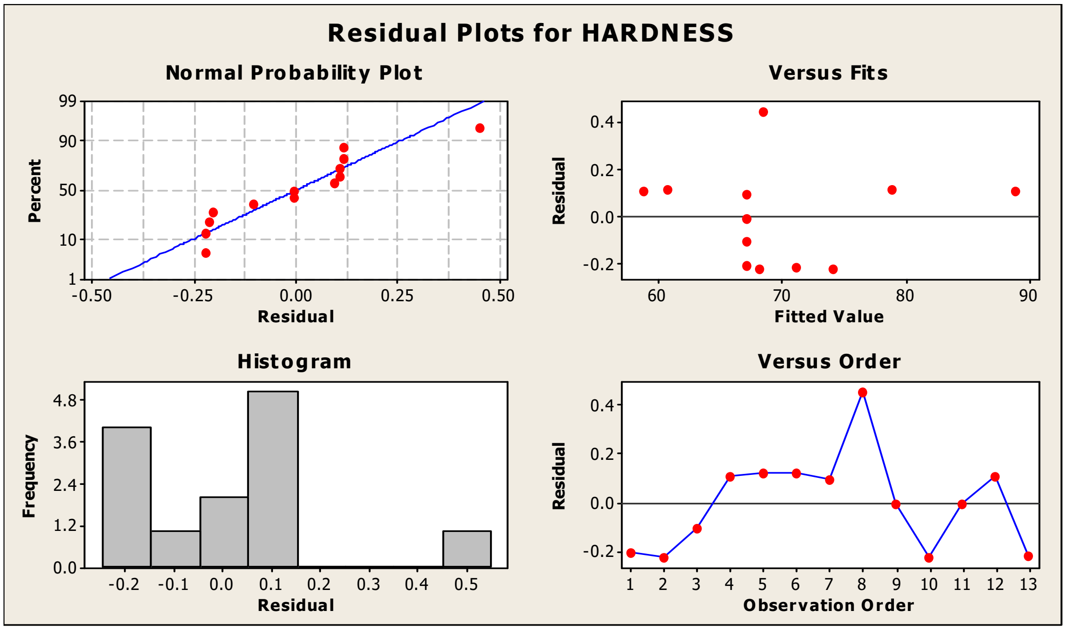

Figure 6.21: Residual plots for Hardness

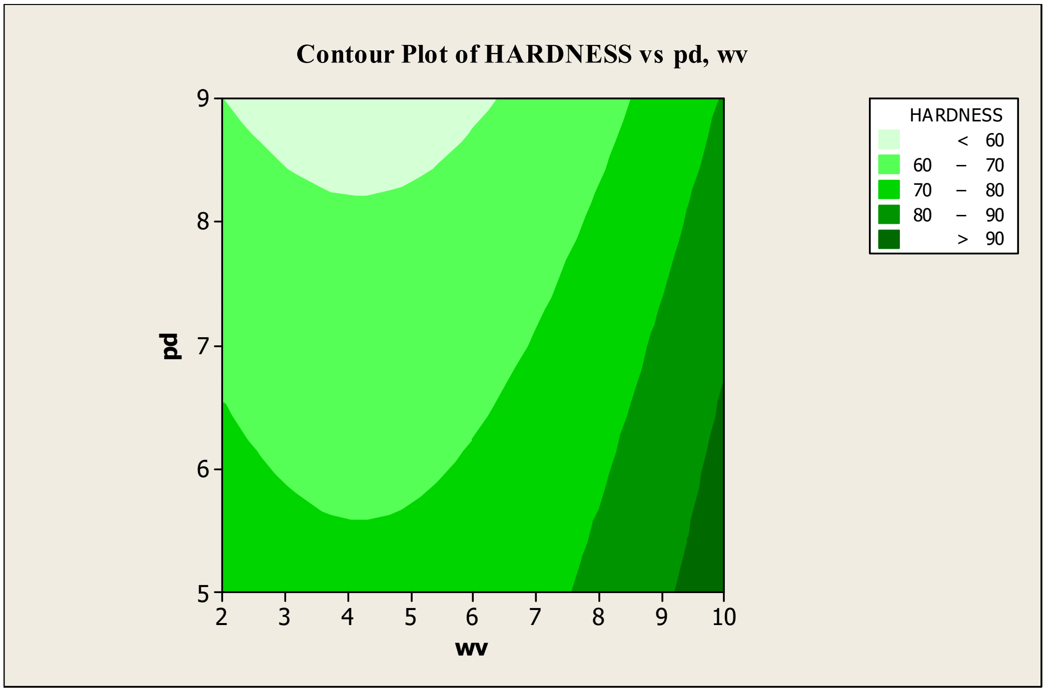

Figure 6.22: Contour plot for Hardness

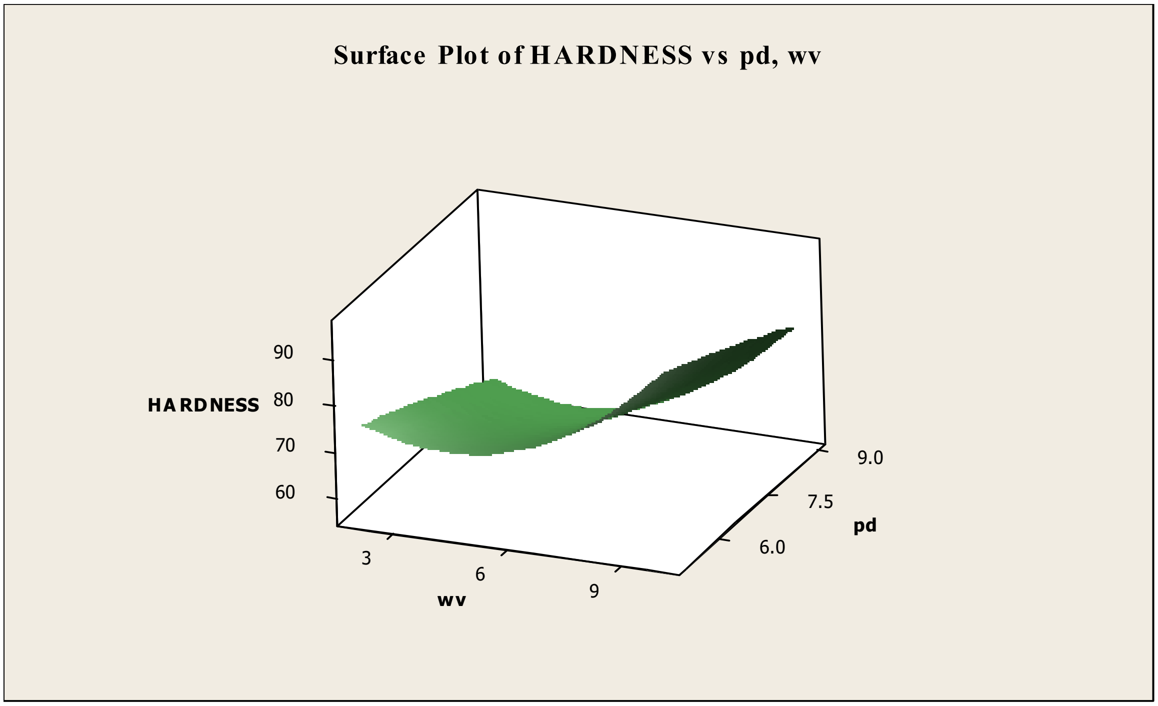

Figure 6.23: Surface plot for Hardness

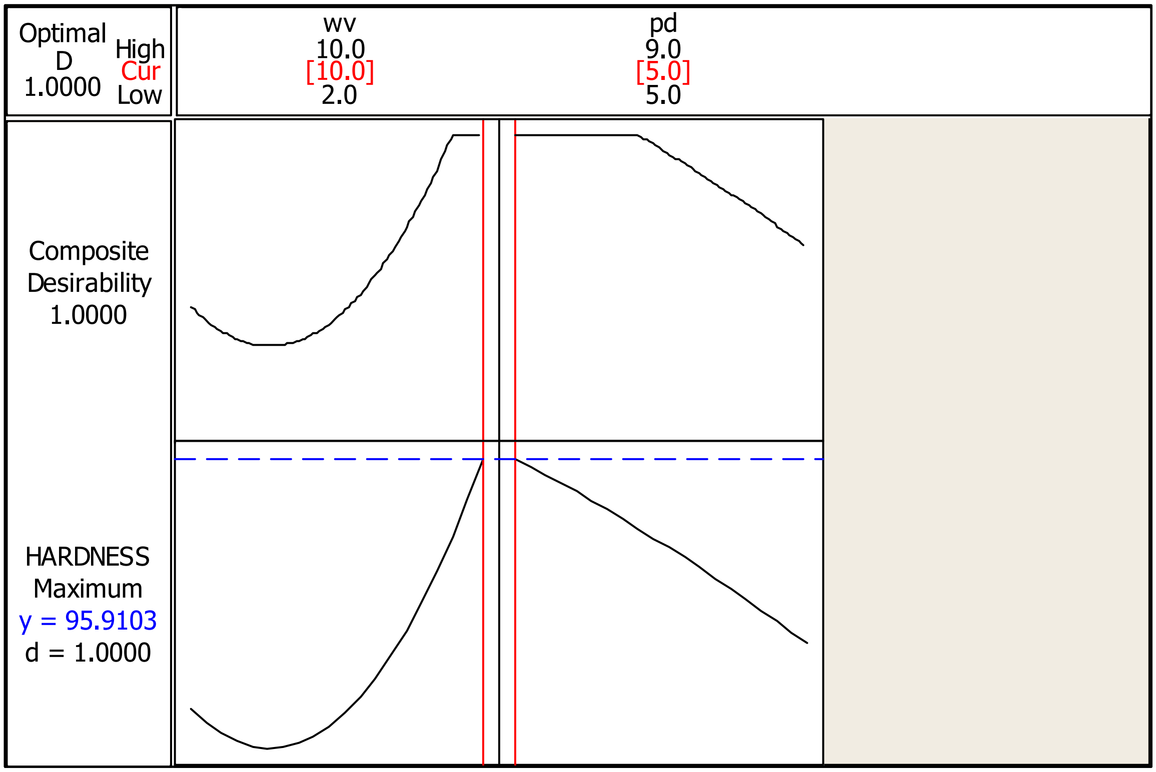

Figure 6.24: Optimization plot for Hardness

Modeling and Optimization (Yield Strength of Copper)

Table 6.21, 6.22 and 6.23 shows the Experimental plan and result, the test of significance and analysis of variance (ANOVA) test for modeling and optimization of Yield Strength respectively. The results of the quadratic model for Yield Strength has been given in Table 6.22.The value of R^2 and adjusted R^2 are 99.74 %and 99.61%.This means that the regression model provides a complete relationship between the dependent and independent variables. The associated p value for the model is <0.05(i.e., $\alpha=0.05$ or 95% confidence), indicating that the model is considered to be statistically significant. It is also seen from Table 6.22 that from the p values, the main effect X_1, X_2 and second order effect of X_1 and X_2 is much more significant. The other model terms can be regarded as insignificant due to their probabilities values being more than 0.05.The standard F- value for 95% confidence limit is 5.05. Hence the model has been found to be adequate. Figure 6.25 depicts the normal probability of residuals for Yield Strength. It is observed from the plot that residuals values are distributed normally and in a straight line and hence the model is adequate.

Table 6.21: Experimental plan and result for Yield Strength based on Central Composite second order rotatable design

Experiment No.	Wheel Velocity (RPM)		Product Diameter (mm)		Yield Strength (MPa)
	Coded	Actual	Coded	Actual	
1	0	6	0	7	59
2	-2	2	0	7	49
3	0	6	0	7	59
4	0	6	2	9	53
5	-1	4	1	8	54

6	1	8	-1	6	57
7	0	6	0	7	59
8	-1	4	-1	6	55
9	0	6	0	7	59
10	0	6	-2	5	54
11	0	6	0	7	59
12	2	10	0	7	52
13	1	8	1	8	56

Table 6.22: Test for significance of Yield Strength

Term	**Coefficient**	***t* – value**	***P* - value**
Constant	59.0000	696.100	0.000
Wheel Velocity	0.8333	14.142	0.000
Product diameter	-0.3333	-5.657	0.000
Wheel Velocity *Wheel Velocity	-2.1250	-49.832	0.000
Product diameter* Product diameter	-1.3750	-32.244	0.000

Using the results presented in Table 6.22, the derived model is shown as:

$$\sigma=59.0000+0.8333X_1-0.3333X_2-2.1250X_1^2-1.3750X_2^2 \tag{6.10}$$

Table 6.23: Test result of ANOVA for Yield Strength

Source	**DF**	**Sum of Squares**	**Mean Sum of squares**	***F*-value**	***P*-value**
Regression	4	126.974	31.994	766.85	0.000
Linear	2	9.667	4.833	116.00	0.000
Square	2	118.308	59.154	1419.69	0.000
Residual error	8	0.333	0.042		
Lack of fit	4	0.333	0.083		
Pure error	4	0.000	0.000		
Total	12	128.308			

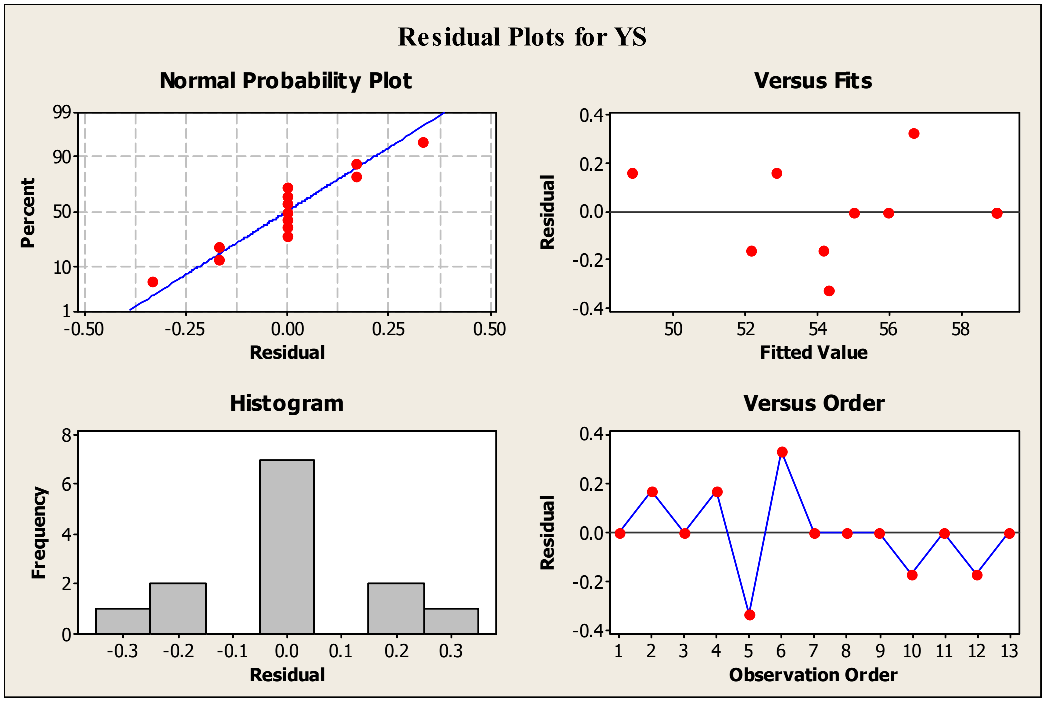

Figure 6.25: Residual plots for Yield Strength

Figure 6.26, 6.27 and 6.28 shows the contour, surface and optimization plot for modeling and optimization of Yield Strength. It can be seen that trend of Yield strength varies in the same manner as that of hardness of copper feedstock material. The optimum values of wheel velocity and product diameter are 6.36 rpm and 6.87 mm respectively for maximum yield strength of 59.10 MPa as observed from the optimization plot (Figure 6.28).

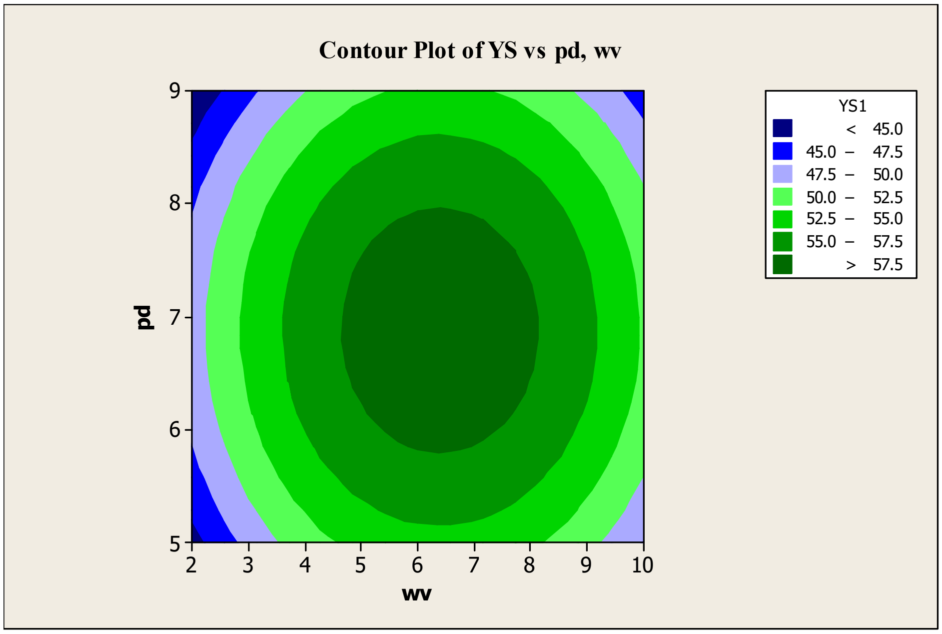

Figure 6.26: Contour plot for Yield Strength

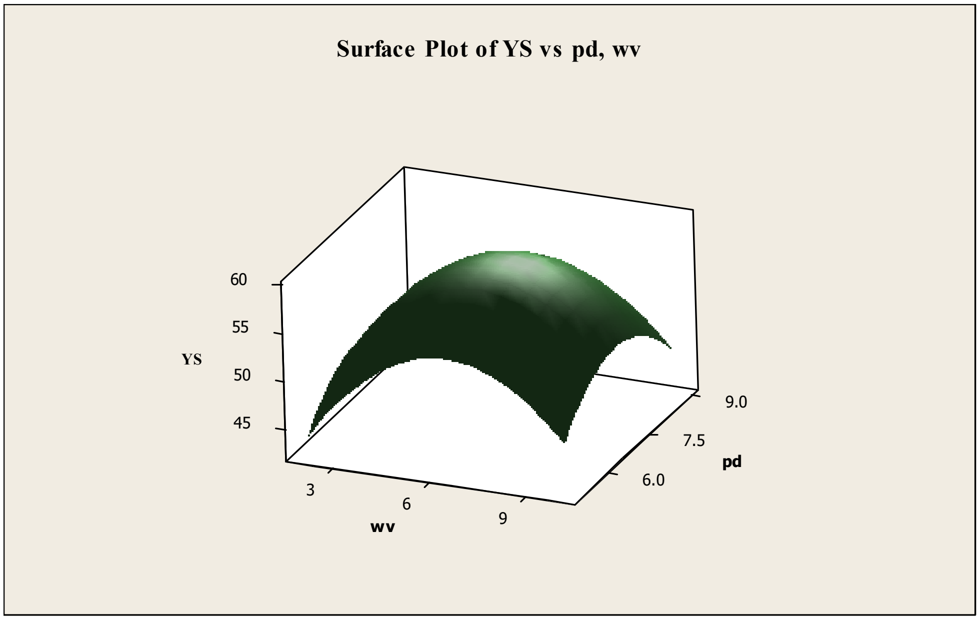

Figure 6.27: Surface plot for YS

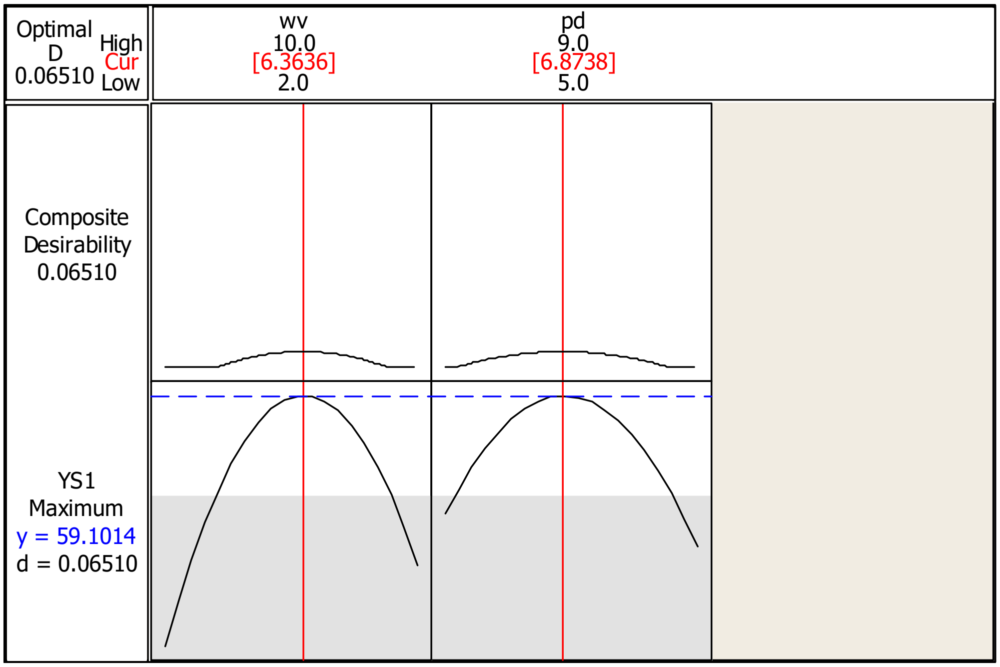

Figure 6.28: Optimization plot for Yield Strength

Modeling and Optimization (% Elongation of Copper samples)

Table 6.24, 6.25 and 6.26 shows the Experimental plan and result, test of significance and analysis of variance test for modeling and optimization of % Elongation of copper feedstock material in continuous Extrusion process.

Table 6.24: Experimental plan and result for % Elongation based on central composite second order rotatable design

Experiment No.	Wheel Velocity (RPM)		Product Diameter (mm)		% Elongation
	Coded	Actual	Coded	Actual	
1	0	6	0	7	59.85
2	-2	2	0	7	51.00
3	0	6	0	7	59.85
4	0	6	2	9	54.00
5	-1	4	1	8	52.00

6	1	8	-1	6	56.00
7	0	6	0	7	59.85
8	-1	4	-1	6	55.14
9	0	6	0	7	59.85
10	0	6	-2	5	55.00
11	0	6	0	7	59.85
12	2	10	0	7	52.46
13	1	8	1	8	56.00

Table 6.25: Test for significance of % Elongation

Term	**Coefficient**	***t* – value**	***P* - value**
Constant	59.3817	111.741	0.000
Wheel Velocity	0.6483	1.755	0.117
Product diameter	-0.4283	-1.159	0.280
Wheel Velocity *Wheel Velocity	-2.0593	-6.702	0.000

The test of significance of % Elongation has been carried out using the quadratic model. The results of the quadratic model for % Elongation are given in Table 6.25.The value of R^2 and adjusted R^2 are 90.13 %and 85.19%.This means that the regression model provides a complete relationship between the dependent and independent variables. The associated p value for the model is <0.05(i.e., α=0.05 or 95% confidence), indicating that the model is considered to be statistically significant. It is also seen from Table 6.25 that from the p values, the main effect X_2 and second order effect of X_1 and X_2 is much more significant. The other model terms can be regarded as insignificant due to their probabilities values being more than 0.05The standard F value for 95% confidence limit is 5.05. Hence the model is found to be adequate..Figure 6.29 depicts the normal probability of residuals for % Elongation. It is observed from the plot that residuals values are distributed normally and in a straight line and hence the model is adequate.

Using the results presented in Table 6.25, the derived model is shown as:

$$L= 59.3817+ 0.6483X_1 -0.4283 X_2- 2.0593 X_1^2- 1.3668 X_2^2 \qquad (6.11)$$

The adequacy of both Yield Strength and % Elongation models has been also tested through Analysis of Variance (ANOVA).The results of the analysis justifies the closeness of the fit of the derived mathematical model. It has been concluded that the evolved mathematical models given by Eqs (1) and (2) are quite adequate at 95% confidence level.

Table 6.26: Test result of ANOVA for % Elongation

Source	**DF**	**Sum of Squares**	**Mean Sum of squares**	***F*-value**	***P*-value**
Regression	4	119.655	29.9138	18.26	0.000
Linear	2	6.246	3.6228	2.21	0.172
Square	2	112.409	56.2047	34.31	0.000
Residual error	8	13.104	1.6380		
Lack of fit	4	13.104	3.2760		
Pure error	4	0.000	0.000		
Total	12	132.759			

Figure 6.30, 6.31 and 6.32 shows the contour, surface and optimization plots respectively in modeling and optimization of % Elongation for copper feedstock material. The trend of % Elongation is same that of Yield Strength. The optimum value of wheel velocity and

product diameter is 6.28 rpm and 6.84 mm respectively for maximum % Elongation of 59.46 which can be seen from optimization plot (Figure 6.32).

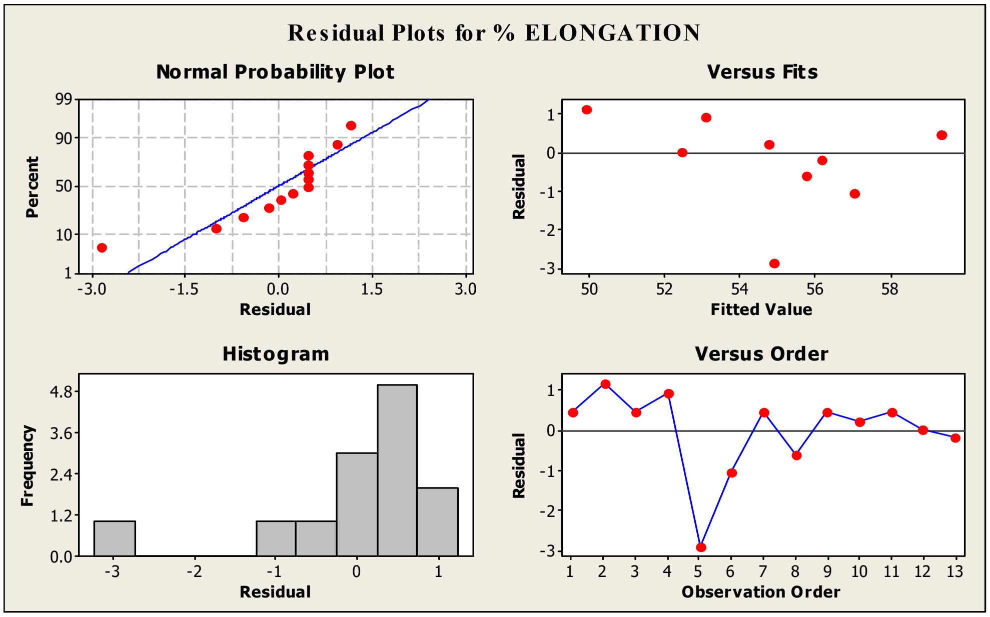

Figure 6.29: Residual plots for % Elongation

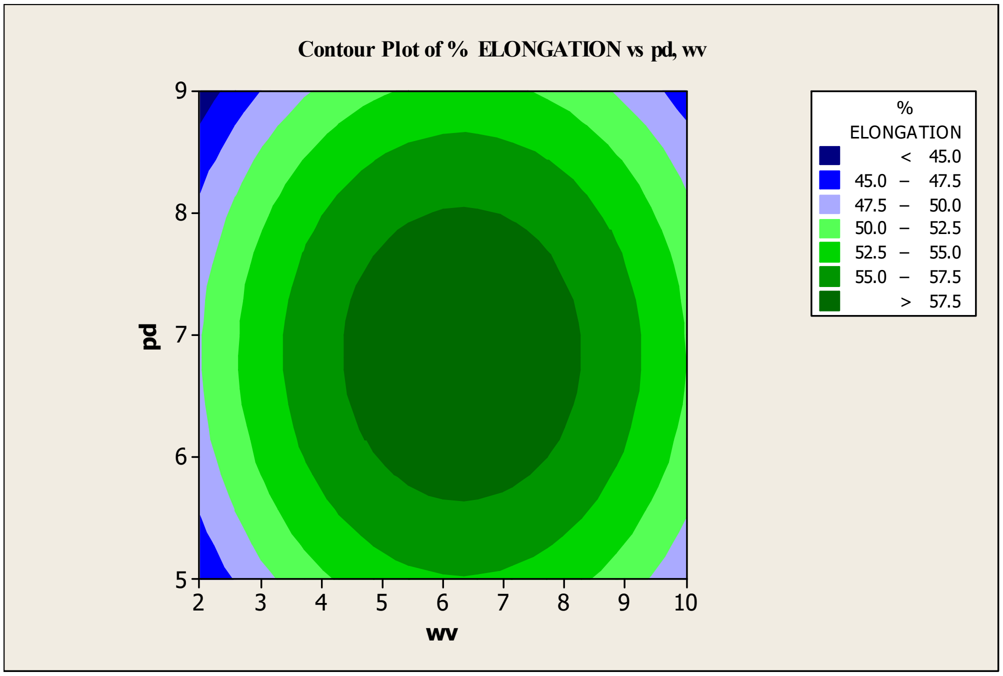

Figure 6.30: Contour plot for % Elongation

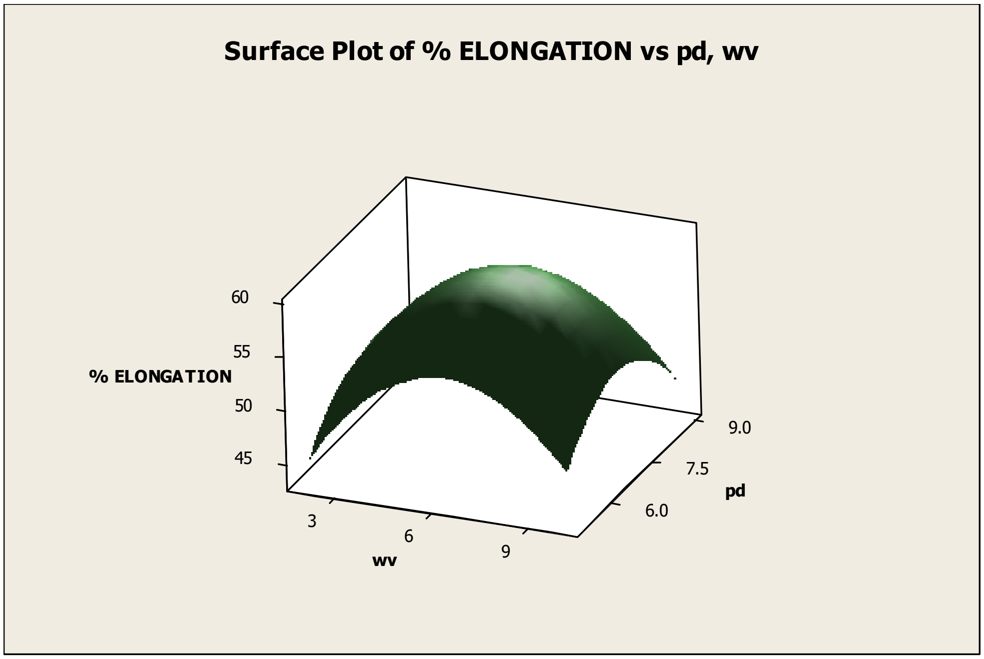

Figure 6.31: Surface plot for % Elongation

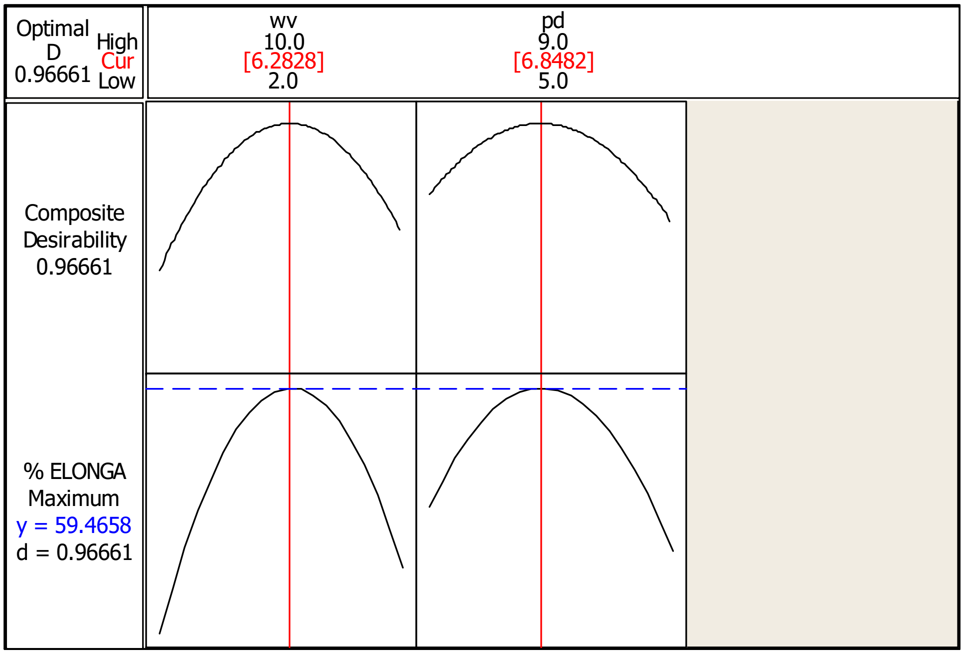

Figure 6.32: Optimization plot for % Elongation

Case III

6.4 Numerical Modeling and Optimization of CE process parameters for Aluminum feedstock

In this section Numerical Modeling and Optimization of the CE process parameters has been carried out. The simulation of Aluminum (AA 1100) feedstock material at different wheel velocities, product diameter, feedstock temperature, and die temperature and friction condition has been carried out using simulation tool DEFORM-3D. A mathematical model through Response Surface Methodology has been developed to analyze the influence of wheel velocity, extrusion ratio, and feedstock temperature, die temperature and friction conditions on CE response process parameters such as load required, torque required, effective stress, Damage value of product and Product Temperature. An optimum value of the extrusion wheel velocity, extrusion ratio, feedstock temperature, and die temperature and friction condition has been determined to predict the best output response variables in Continuous Extrusion forming process. The adequacy of the model has also been tested by the analysis of variance test (ANOVA).

For finding out the relationship between the input process parameters and the response variables of Continuous Extrusion process, second order polynomial response surface mathematical models can be considered as

$$Y_u = b_0 + \sum b_i x_{iu} + \sum b_{ii} x^2_{iu} + \sum b_{ij} x_{iu} x_{ju} \qquad (6.12)$$

Where Y_u is the corresponding response; x_{iu} is the coded values of the i-th Continuous Extrusion parameters for the u-th experiments; and b_i, b_{ii}, b_{ij} are the second order regression coefficients. The second term under the summation sign of this polynomial equation corresponds to linear effect, while the third term denotes to the higher order effect. The fourth term of the equation includes the interactive effects of the process parameters.

The five factors with three levels, and central composite rotatable design matrix has been chosen to optimize the experimental conditions. The main aim of the factorial experiments depicts the relationship between the response as a dependent variable and the parameter levels. This approach helps to understand how the change in levels of parameters affects the response. The combination of different levels of the parameters leads to certain optimum response.

The sectional investigation studied the effects of factors such as extrusion wheel velocity, extrusion ratio, feedstock temperature, die temperature and frictional conditions in Continuous Extrusion process.

The response parameters are load required, torque required, effective stress, and Damage value and Product Temperature of the extruded Aluminum feedstock during Continuous Extrusion process. A 2^k factorial with central composite second order design has been used (in this case k=5).

Table 6.27 shows the Experimental parameter and levels. Table 6.28 shows the experimental plan and design matrix for input process parameters based on CCD. Table 6.29 shows experimental plan and result based on CCD. For analysis of the data, fitness of the model is necessary and well required. For checking accuracy of the model includes test for significance of the regression model, test for significance of model coefficient and test for lack of fit. For this purpose, analysis of variance (ANOVA) has been performed.

Table 6.27: Experimental parameter and levels

Factors	**Levels**		
	-1	**0**	**1**
Wheel Velocity	2	7	12
Product diameter	6	7	8
Frictional conditions	0.6	0.8	1.0
Feedstock temperature	100	300	500
Die temperature	400	450	500

By applying the empirical data recorded from each 32 sets of experiments to the PBD, the predicted values of the Continuous Extrusion response process parameters has been obtained as shown in Table 6.28. On the basis of analysis of variance (ANOVA) and values of coefficients for significance ($P<0.05$), all five factors i.e. wheel velocity, product diameter, frictional condition, feedstock temperature and die temperature has been found to have significant effect. The P value has been the probability of magnitude of a contrast coefficient due to random process variability.

Table 6.28: Development of experimental plan design matrix based on CCD

Run	Block	Wheel Velocity (RPM)		Product diameter (mm)		Friction Condition		Feedstock Temperature (˚C)		Die Temperature (˚C)	
		Coded	**Actual**	**Coded**	**Actual**	**Coded**	**Actual**	**Coded**	**Actual**	**Coded**	**Actual**
1	1	-1	2	1	8	1	1.0	1	500	-1	400
2	1	1	10	1	8	-1	0.6	1	500	-1	400
3	1	0	6	0	7	0	0.8	0	300	0	450
4	1	0	6	0	7	0	0.8	0	300	0	450
5	1	-1	2	-1	6	-1	0.6	-1	100	1	500
6	1	-1	2	-1	6	1	1.0	-1	100	-1	400
7	1	-1	2	1	8	1	1.0	-1	100	1	500
8	1	0	6	0	7	0	0.8	-2	-100	0	450
9	1	0	6	2	9	0	0.8	0	300	0	450
10	1	1	10	1	8	1	1.0	-1	100	-1	400
11	1	0	6	0	7	0	0.8	0	300	2	550
12	1	0	6	0	7	0	0.8	0	300	0	450
13	1	0	6	0	7	0	0.8	0	300	-2	350
14	1	0	6	0	7	0	0.8	0	300	0	450
15	1	-1	2	1	8	-1	0.6	-1	100	-1	400
16	1	1	10	-1	6	-1	0.6	-1	100	-1	400
17	1	-1	2	1	8	-1	0.6	1	500	1	500
18	1	0	6	0	7	0	0.8	0	300	0	450
19	1	-1	2	-1	6	-1	0.6	1	500	-1	400
20	1	2	14	0	7	0	0.8	0	300	0	450
21	1	0	6	-2	5	0	0.8	0	300	0	450
22	1	1	10	1	8	1	1.0	1	500	1	500
23	1	1	10	-1	6	-1	0.6	1	500	1	500
24	1	0	6	0	7	2	1.2	0	300	0	450
25	1	0	6	0	7	-2	0.4	0	300	0	450
26	1	-2	-2	0	7	0	0.8	0	300	0	450
27	1	0	6	0	7	0	0.8	0	300	0	450
28	1	1	10	1	8	-1	0.6	-1	100	1	500
29	1	1	10	-1	6	1	1.0	1	500	-1	400
30	1	0	6	0	7	0	0.8	2	700	0	450
31	1	1	10	-1	6	1	1.0	-1	100	1	500
32	1	-1	2	-1	6	1	1.0	1	500	1	500

Table 6.29: Experimental plan and result based on CCD

Trial	Wheel Velocity (RPM)	Product diameter (MM)	Friction Condition	Feedstock Temperature (ºC)	Die Temperature (°C)	Total Load (kN)	Torque (kN-m)	Effective stress (MPa)	Damage value	Product Temperature (ºC)
1	2	8	1.0	500	400	89.60	3.920	132	1.750	400
2	10	8	0.6	500	400	108.30	4.490	137	1.030	528
3	6	7	0.8	300	450	102.79	4.850	141	0.970	359
4	6	7	0.8	300	450	102.79	4.850	141	0.970	359
5	2	6	0.6	100	500	113.93	5.220	133	0.943	158
6	2	6	1.0	100	400	158.70	6.860	132	0.843	223
7	2	8	1.0	100	500	91.49	4.050	133	1.390	150
8	6	7	0.8	-100	450	110.96	4.910	133	0.872	239
9	6	9	0.8	300	450	63.54	0.966	132	0.847	313
10	10	8	1.0	100	400	113.60	4.260	146	1.320	238
11	6	7	0.8	300	550	89.80	4.370	135	0.900	449
12	6	7	0.8	300	450	102.79	4.850	141	0.970	359
13	6	7	0.8	300	350	102.53	4.700	135	0.920	463
14	6	7	0.8	300	450	102.79	4.850	141	0.970	359
15	2	8	0.6	100	400	85.20	3.960	130	1.070	170
16	10	6	0.6	100	400	144.27	5.800	141	1.470	167
17	2	8	0.6	500	500	89.20	3.920	134	1.300	435
18	6	7	0.8	300	450	102.79	4.850	141	0.970	359
19	2	6	0.6	500	400	182.11	6.020	124	0.889	350
20	14	7	0.8	300	450	102.53	5.110	140	1.550	449
21	6	5	0.8	300	450	188.59	6.530	148	0.483	324
22	10	8	1.0	500	500	113.60	4.260	151	1.660	531
23	10	6	0.6	500	500	154.70	5.910	128	1.760	500
24	6	7	1.2	300	450	110.41	4.910	137	0.800	336
25	6	7	0.4	300	450	89.93	4.200	133	0.800	382
26	-2	7	0.8	300	450	91.93	4.290	137	0.875	309
27	6	7	0.8	300	450	102.79	4.850	141	0.970	359
28	10	8	0.6	100	500	108.30	4.490	142	1.240	237
29	10	6	1.0	500	400	228.30	5.930	130	0.850	550
30	6	7	0.8	700	450	111.95	4.830	136	0.861	701
31	10	6	1.0	100	500	92.70	4.530	142	0.500	270
32	2	6	1.0	500	500	165.40	6.820	130	0.752	450

Modeling and Optimization (Load required)

Table 6.30 and 6.31 shows the test of significance and analysis of variance (ANOVA) test respectively for extrusion load required in continuous extrusion of Aluminum feedstock material. The results of the quadratic model for load required are given in Table 6.30.The value of R^2 and adjusted R^2 are 91.49 %and 76.07%.This means that the regression model provides a complete relationship between the dependent and independent variables. The associated p value for the model is <0.05(i.e., $\alpha=0.05$ or 95% confidence), indicating that the model is considered to be statistically significant. It is also seen from Table 6.30 that from the p values, the main effect X_2, second order effect of X_2, interactive effects of X_2, X_4, X_5 is much more significant. The other model terms can be regarded as insignificant due to their probabilities values being more than 0.05.Figure 6.33 depicts the normal probability of residuals for load required for extrusion. It is observed from the plot that residuals values are distributed normally and in a straight line and hence the model is adequate.

Using the results presented in Table 6.30, the derived model is shown as:

$$Load\ (P) = 0.011+0.002X_2+0.012X_2^2+ 0.009X_4X_2 +0.018X_2X_5 \tag{6.13}$$

Figure 6.34, 6.35 and 6.36 shows the surface, contour and optimization plots respectively for modeling and optimization of load required in Continuous Extrusion process of Aluminum feedstock material. It can be inferred from the surface and contour plots that the minimum load required for extrusion of Aluminum feedstock materials can be visualized by the area shaded between interactive effects of different process parameters as mentioned and plotted. Further the minimum value of extrusion load can achieved at optimum combination of different process parameters which can be observed from the optimization plot (Figure 6.36). The optimization plot reveals that the minimum extrusion load can be achieved at wheel velocity of 2 RPM, product diameter of 9 mm (Extrusion ratio of 1.11), frictional coefficient as 0.4 and die temperature of 350 ◦C.

Table 6.30: Test of significance for Extrusion Load

Terms	**Coefficients**	**Standard Error Coefficient**	***t* - value**	***P* - value**
Constant	1469.76	479.113	3.068	0.011
wv	-1.98	13.399	-0.148	0.885
pd	-250.04	62.482	-4.002	0.002
fc	152.18	281.769	0.54	0.6
ft	0.3	0.268	1.138	0.279
dt	-2.12	1.358	-1.559	0.147
wv*wv	0.14	0.199	0.726	0.483
pd*pd	9.53	3.191	2.985	0.012
fc*fc	76.3	79.785	0.956	0.359
ft*ft	0	0	1.84	0.093
dt*dt	0	0.001	0.643	0.534
wv*pd	1.38	1.08	1.28	0.227
wv*fc	-0.17	5.401	-0.031	0.976
wv*ft	0.01	0.005	0.999	0.339
wv*dt	-0.02	0.022	-1.006	0.336
pd*fc	-10.25	21.606	-0.474	0.644
pd*ft	-0.07	0.022	-3.165	0.009
pd*dt	0.24	0.086	2.785	0.018
fc*ft	0.09	0.108	0.836	0.421
fc*dt	-0.46	0.432	-1.06	0.312
ft*dt	0	0	0.144	0.888

R-Sq = 91.49% , R-Sq(adj) = 76.03%
Abbreviations: wv-wheel velocity; pd-product diameter; ft- feedstock temperature; fc-friction condition; dt- die temperature

Table 6.31: Test of ANOVA for Extrusion Load

Source	**Degree of freedom**	**Sequential sum of square**	**Adjusted sum of square**	**Adjusted mean of square**	***F*-value**	***P*-value**
Regression	20	35346.4	35346.4	1766.32	5.92	0.002
Linear	5	24759	5922	1184.39	3.96	0.027
wv	1	498.1	6.5	6.51	0.02	0.885
pd	1	19890.4	4784.3	4784.32	16.01	0.002
fc	1	489.1	86.2	86.15	0.29	0.6
ft	1	2109.4	386.6	386.61	1.29	0.279
dt	1	1771.9	725.9	725.91	2.43	0.147
Square	5	3570.6	3570.6	714.12	2.39	0.106
wv*wv	1	26.1	156.5	156.48	0.53	0.483
pd*pd	1	2282.4	2661.7	2661.73	8.91	0.012
fc*fc	1	176.3	273.2	273.24	0.91	0.359
ft*ft	1	960.5	1011.9	1011.87	3.39	0.093
dt*dt	1	123.4	123.4	123.37	0.41	0.534
Interaction	10	7016.8	7016.8	701.68	2.35	0.089
wv*pd	1	489.3	489.3	489.29	1.64	0.227
wv*fc	1	0.3	0.3	0.28	0	0.976
wv*ft	1	296.9	296.9	296.91	1	0.339
wv*dt	1	302.6	302.6	302.59	1.01	0.336
pd*fc	1	66.2	66.2	66.24	0.23	0.644
pd*ft	1	2992.1	2992.1	2992.09	10.01	0.009
pd*dt	1	2317	2317	2316.98	6.76	0.018
fc*ft	1	208.8	208.8	208.8	0.7	0.421
fc*dt	1	335.4	335.4	335.44	1.12	0.312
ft*dt	1	6.2	6.2	6.18	0.02	0.888
Residual Error	11	3286.4	3286.4	298.76		
Lack-of-Fit	6	3286.4	3286.4	546.73		
Pure Error	5	0	0	0		
Total	31	38632.7				

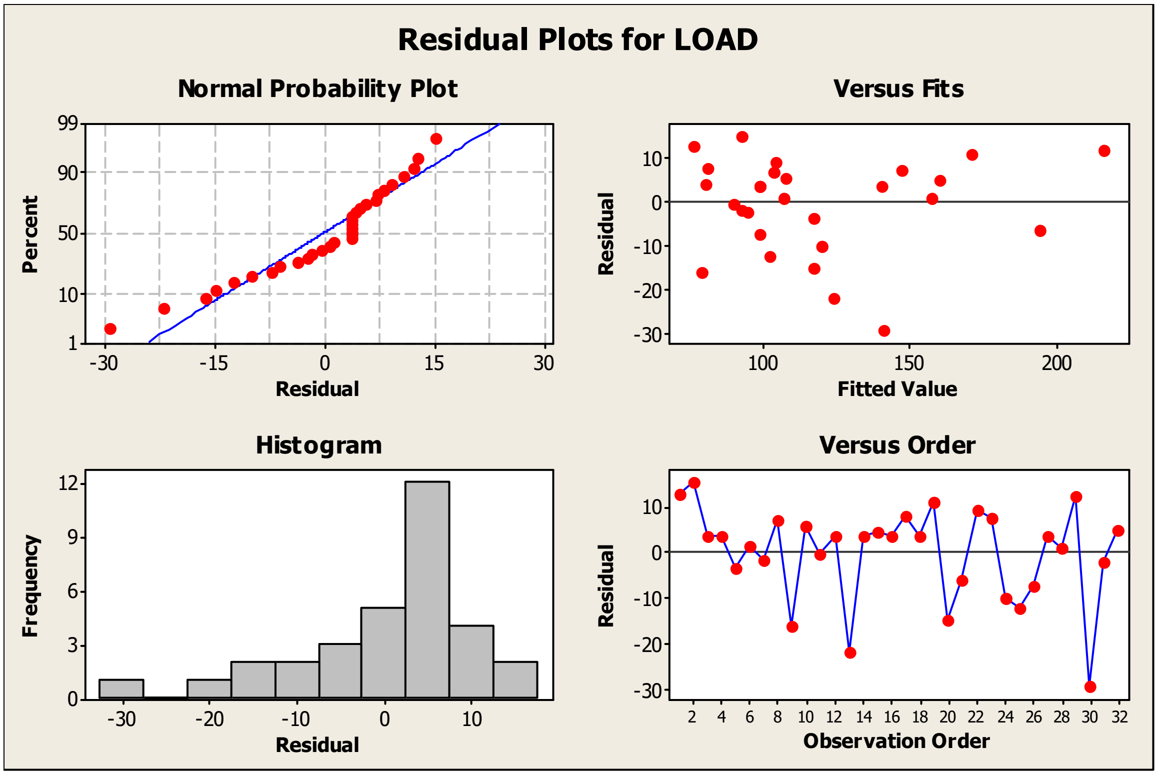

Figure 6.33: Residual plots for load required

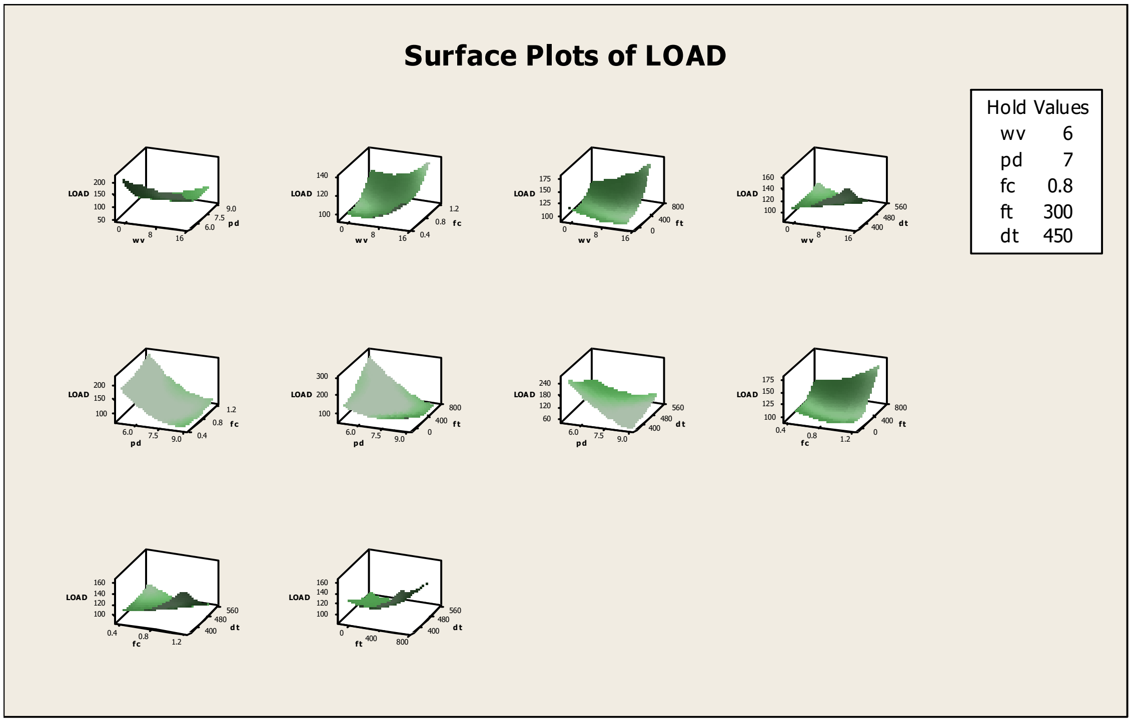

Figure 6.34: Surface plots for load required

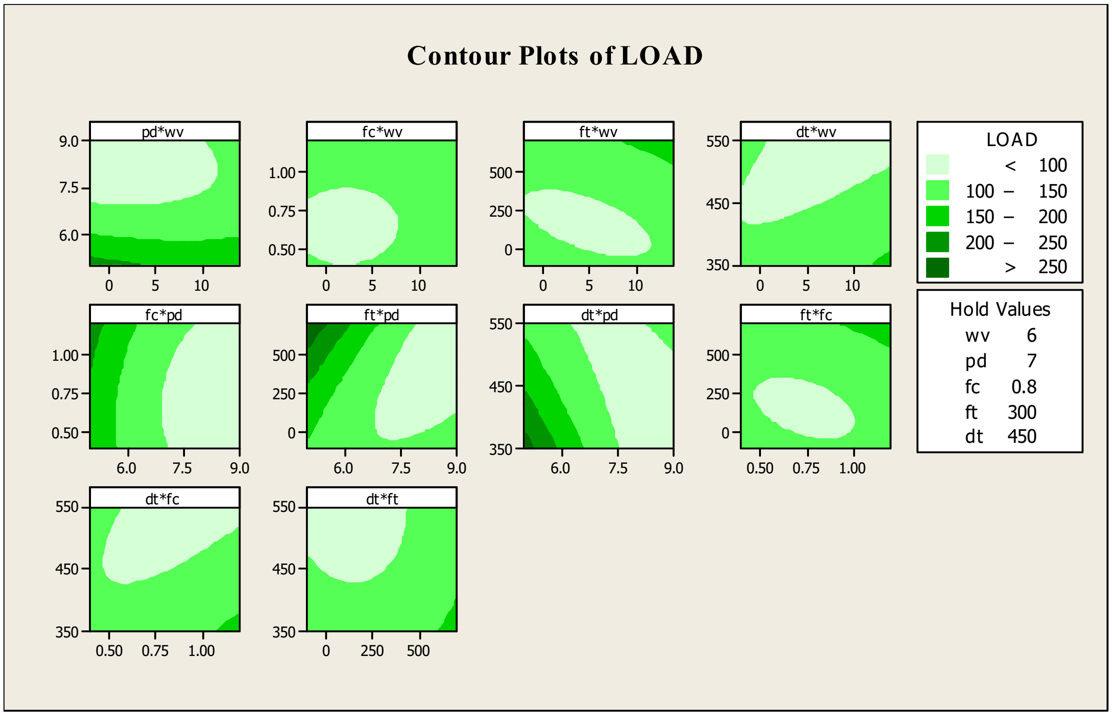

Figure 6.35: Contour plots for load required

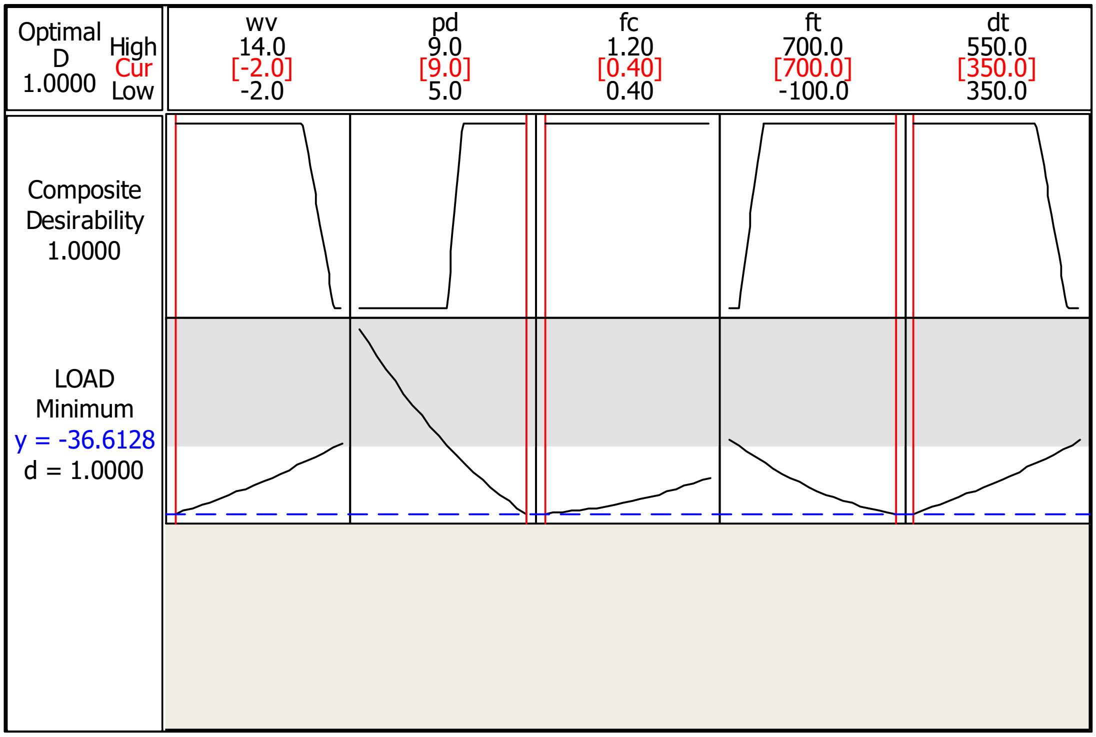

Figure 6.36: Optimization plot for load required

Modeling and Optimization (Torque required)

Table 6.32 and 6.33 shows the test of significance and analysis of variance (ANOVA) respectively for modeling and optimization of torque required for Aluminum feedstock material. The results of the quadratic model for torque required are given in Table 6.32.The value of R^2 and adjusted R^2 are 99.29 %and 98.01%.This means that the regression model provides a complete relationship between the dependent and independent variables. The associated p value for the model is <0.05(i.e., $\alpha=0.05$ or 95% confidence), indicating that the model is considered to be statistically significant. It is also seen from Table 6.32 that from the p values, the main effect X_1, X_2, X_5 and second order effect of X_1 and X_2, interactive effects of $X_1$and X_2, $X_1$and X_3, X_2 and X_4, X_2 and X_5 is much more significant. The other model terms can be regarded as insignificant due to their probabilities values being more than 0.05.Figure 6.37 depicts the normal probability of residuals for Torque required. It is observed from the plot that residuals values are distributed normally and in a straight line and hence the model is adequate.

Using the results presented in Table 6.32, the derived model is shown as:

$$\sigma=29.7621-0.5489X_1-1.5436X_2-0.0609X_5+0.0064X_1^2-0.1169X_2^2+0.0.093X_1X_2-0.2367X_1X_3-0.1969X_2X_4+0.0046X_2X_5 \quad (6.14)$$

Figure 6.38, 6.39 and 6.40 shows the surface, contour and optimization plot respectively for modeling and optimization of torque required. It can be inferred from the surface and contour plots (Figures 6.38 to 6.39) that the minimum torque required for extrusion of Aluminum feedstock materials can be visualized by the area shaded between interactive effects of different process parameters as mentioned and plotted. Further the minimum value of extrusion torque can achieved at optimum combination of different process parameters which can be observed from the optimization plot (Figure 6.40). The optimization plot reveals that the minimum extrusion torque can be achieved at wheel velocity of 2 RPM, product diameter of 9 mm (Extrusion ratio of 1.11), frictional coefficient as 0.5 and die temperature of 350 °C.

Table 6.32: Test of significance for Torque required

Estimated Regression Coefficients for Torque required				
Terms	**Coefficients**	**Standard Error Coefficient**	***t* - value**	***P* - value**
Constant	29.7621	4.15898	6.156	0.000
wv	-0.5489	0.11631	-4.719	0.001
pd	-1.5436	0.54238	-2.846	0.016
fc	-0.5186	2.44592	-0.212	0.836
ft	0.001	0.00233	0.412	0.688
dt	-0.0609	0.01179	-5.17	0.000
wv*wv	0.0064	0.00173	3.675	0.004
pd*pd	-0.1169	0.0277	-4.221	0.001
fc*fc	1.1392	0.69258	1.645	0.128
ft*ft	0.00	0.00	4.442	0.001
dt*dt	0.00	0.00001	1.735	0.111
wv*pd	0.093	0.00938	9.914	0.000
wv*fc	-0.2367	0.04689	-5.049	0.000
wv*ft	0.00	0.00005	0.816	0.432
wv*dt	0.00	0.00019	-0.05	0.961
pd*fc	-0.1969	0.18755	-1.05	0.316
pd*ft	-0.0012	0.00019	-6.515	0.000
pd*dt	0.0046	0.00075	6.148	0.000
fc*ft	-0.0014	0.00094	-1.483	0.166
fc*dt	0.0047	0.00375	1.25	0.237
ft*dt	0.000	0.00	4.249	0.001
R-Sq = 99.29% , R-Sq(adj) = 98.01% Abbreviations: wv-wheel velocity; pd- product diameter; fc- friction condition; ft-feedstock temperature; dt- die temperature				

Table 6.33: Test of ANOVA for Torque required

Source	**Degree of freedom**	**Sequential sum of square**	**Adjusted sum of square**	**Adjusted mean of square**	***F*-value**	***P*-value**
Regression	20	34.7976	34.7976	1.73988	76.29	0.000
Linear	5	28.3606	1.0907	0.21814	9.69	0.001
wv	1	0.0057	0.5014	0.5014	22.27	0.001
pd	1	26.5418	0.1823	0.18234	8.1	0.016
fc	1	0.0376	0.001	0.00101	0.04	0.836
ft	1	0.2795	0.0038	0.00382	0.17	0.688
dt	1	0.4959	0.6017	0.6017	26.73	0.000
Square	5	1.3131	1.3131	0.26261	11.67	0.000
wv*wv	1	0.2717	0.3041	0.3041	13.51	0.004
pd*pd	1	0.5236	0.4011	0.40108	16.82	0.001
fc*fc	1	0.0312	0.0609	0.06091	2.71	0.128
ft*ft	1	0.4187	0.4443	0.44428	19.73	0.001
dt*dt	1	0.0678	0.0678	0.06778	3.01	0.111
Interaction	10	5.1239	5.1239	0.51239	22.76	0.000
wv*pd	1	2.2127	2.2127	2.21266	98.29	0.000
wv*fc	1	0.5738	0.5738	0.57381	25.49	0.000
wv*ft	1	0.015	0.015	0.01501	0.67	0.432
wv*dt	1	0.0001	0.0001	0.00006	0.00	0.961
pd*fc	1	0.0248	0.0248	0.02481	1.1	0.316
pd*ft	1	0.9555	0.9555	0.95551	42.44	0.000
pd*dt	1	0.851	0.851	0.85101	36.8	0.000
fc*ft	1	0.0495	0.0495	0.04951	2.2	0.166
fc*dt	1	0.0352	0.0352	0.03516	1.56	0.237
ft*dt	1	0.4064	0.4064	0.40641	18.05	0.001
Residual Error	11	0.2476	0.2476	0.02251		
Lack-of-Fit	6	0.2043	0.2043	0.03405	3.93	0.077
Pure Error	5	0.0433	0.0433	0.00867		
Total	31	35.0452				

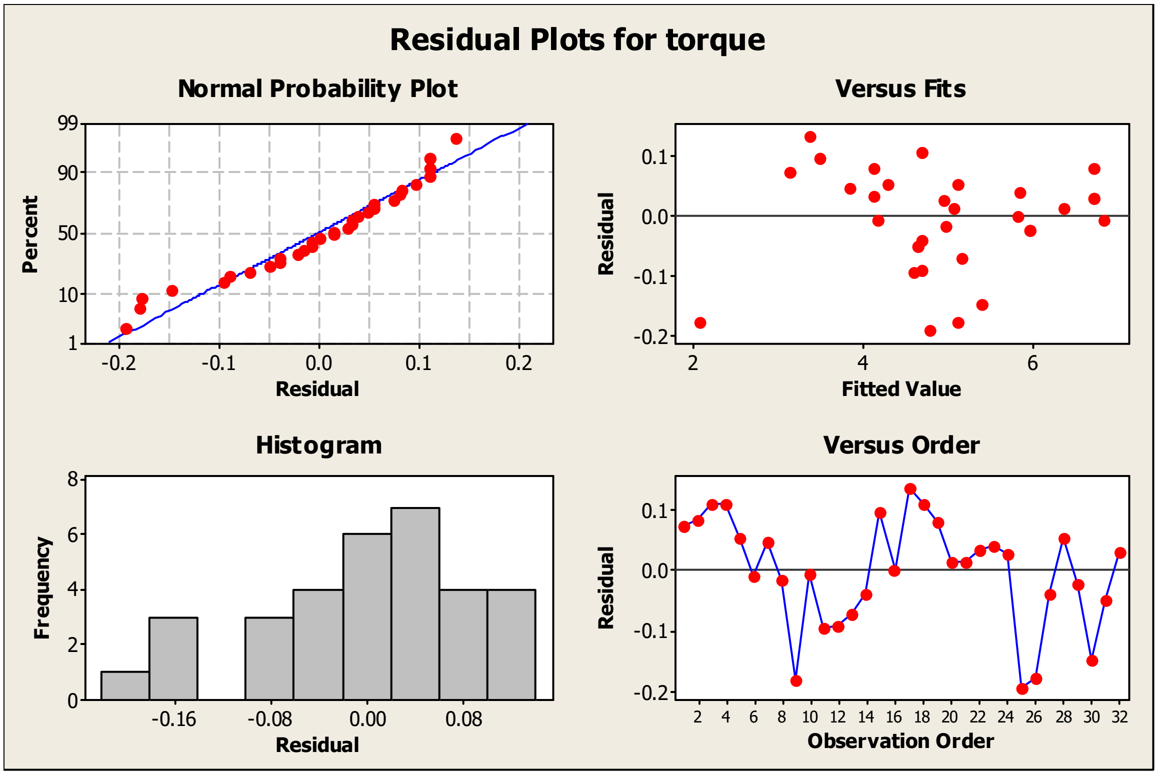

Figure 6.37: Residual plots for torque required

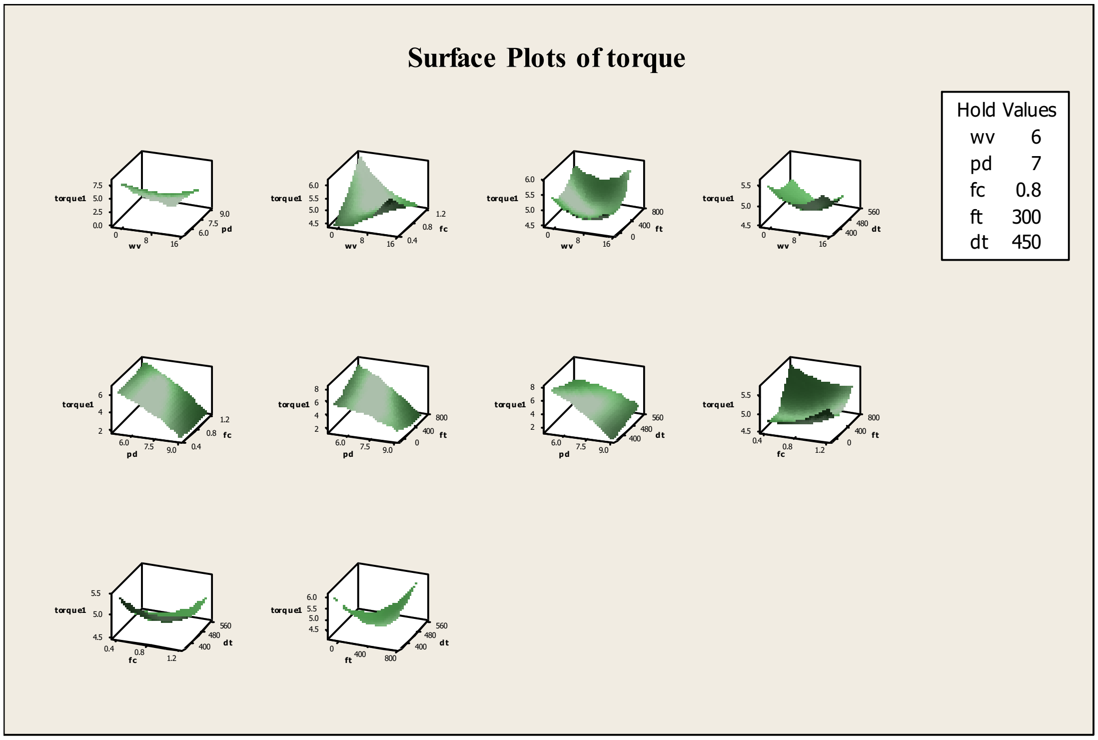

Figure 6.38: Surface plots for torque required

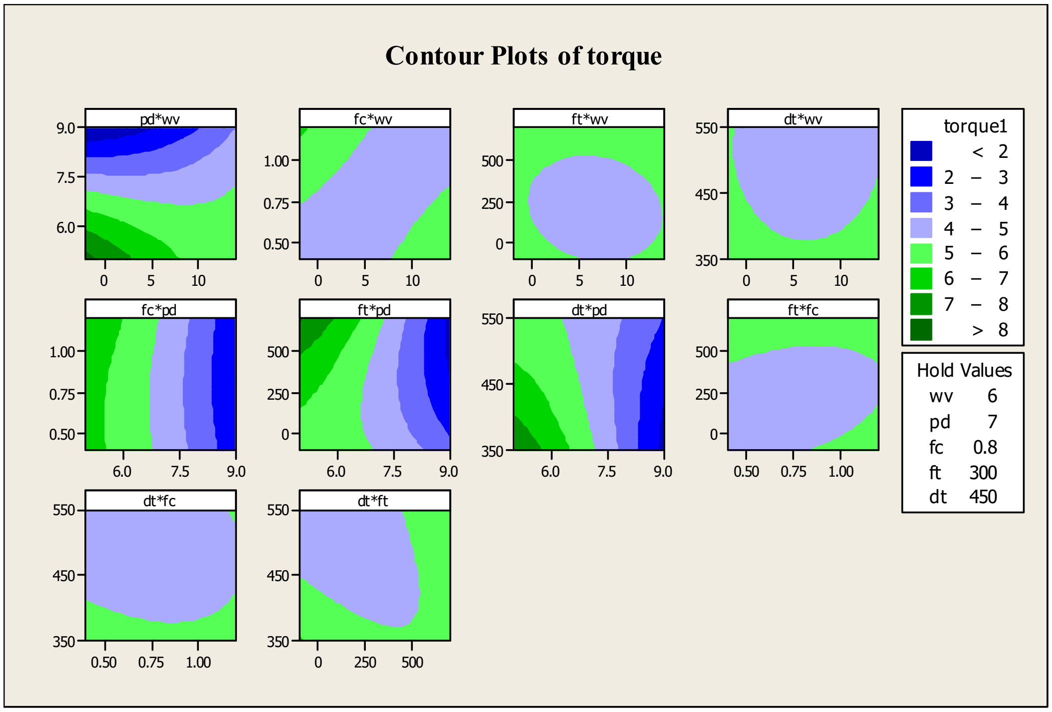

Figure 6.39: Contour plots for torque required

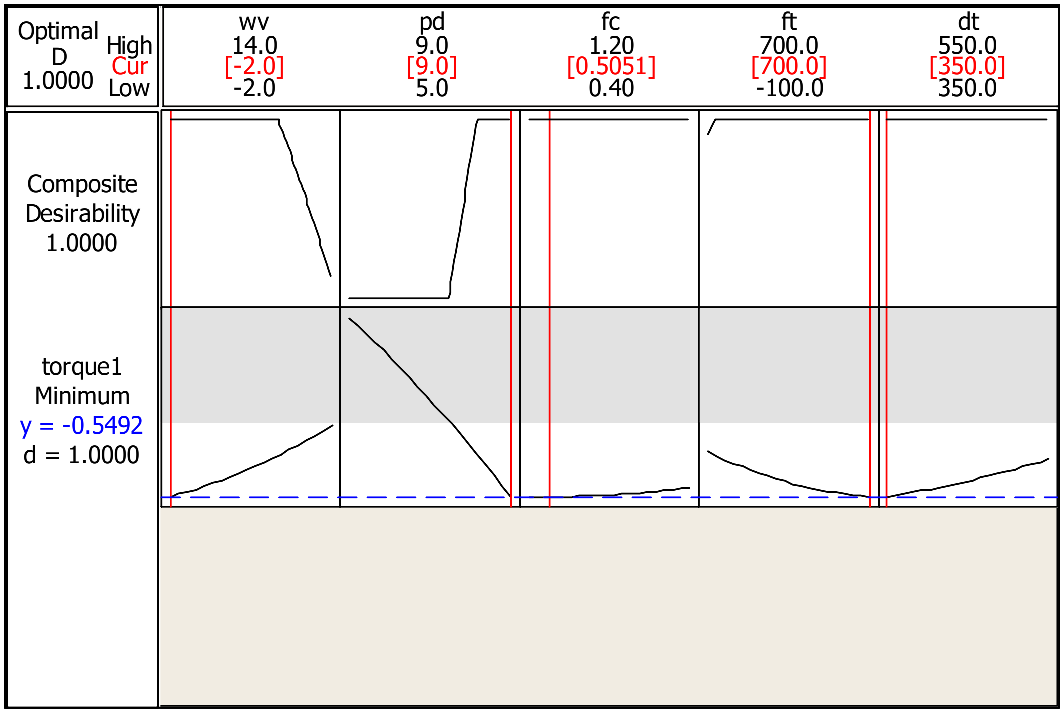

Figure 6.40: Optimization plot for torque required

Modeling and Optimization (Effective stress)

Table 6.34 and 6.35 shows the test of significance and analysis of variance (ANOVA) test for modeling and optimization of Effective stresses in continuous extrusion of Aluminum feedstock material. The results of the quadratic model for Effective stresses are given in Table 6.34.The value of R^2 and adjusted R^2 are 99.896 %and 99.70%.This means that the regression model provides a complete relationship between the dependent and independent variables. The associated p value for the model is <0.05(i.e., $\alpha=0.05$ or 95% confidence), indicating that the model is considered to be statistically significant. It is also seen from Table 6.34 that from the p values, the main effect X_1, X_2, X_5 and second order effect of X_1 and X_2, interactive effects of X_1 and X_2, X_1 and X_3, X_2 and X_4, X_2 and X_5 is much more significant The other model terms can be regarded as insignificant due to their probabilities values being more than 0.05.Figure 6.41 depicts the normal probability of residuals for Effective stresses. It is observed from the plot that residuals values are distributed normally and in a straight line and hence the model is adequate.

Using the results presented in Table 6.34, the derived model is shown as:

$$\sigma_{effective} = 73.2921 - 1.4218X_1 - 6.6018X_2 - 0.1130X4 + 0.4397X5 - 0.0462X_1^2 - 0.4886X_2^2 - 43.4659X^2{}_3 - 0.0006X_5 + 0.3891X_1X_2 + 1.0937X_1X_3 - 0.0013X_1X_4 - 0.0011X_1X_5 + 3.5313X_2X_3 + 0.0123X_2X_4 + 0.0115X_2X_5 + 0.0192X_3X_4 + 0.0706X_3X_5 + 0.0001X_4X_5 \tag{6.15}$$

Figures 6.42, 6.43 and 6.44 show the surface, contour and optimization plot for modeling and optimization of effective stresses. It can be inferred from the surface and contour plots (Figures 6.42 to 6.43) that the minimum effective stresses developed during extrusion of Aluminum feedstock materials can be visualized by the area shaded between interactive effects of different process parameters as mentioned and plotted. Further the minimum value of effective stresses can achieved at optimum combination of different process parameters which can be observed from the optimization plot (Figure 6.44). The optimization plot reveals that the minimum effective stresses can be achieved at wheel velocity of 2 RPM, product diameter of 6 mm (Extrusion ratio of 2.5), frictional coefficient as 0.6, feedstock temperature of 100 ◦C and die temperature of 350 ◦C.

Table 6.34: Test of significance for Effective stress

Estimated Regression Coefficients for effective stress				
Terms	**Coefficients**	**Standard Error Coefficient**	***t* - value**	***P* - value**
Constant	73.2921	6.50613	9.764	0.000
wv	-1.4218	0.20991	-6.773	0.000
pd	-6.6018	0.97889	-6.744	0.000
fc	8.1132	4.4144	1.838	0.093
ft	-0.113	0.0042	-26.906	0.000
dt	0.4397	0.02127	20.667	0.000
wv*wv	-0.0462	0.00312	-14.773	0.000
pd*pd	-0.4886	0.05	-9.773	0.000
fc*fc	-43.4659	1.24997	-34.774	0.000
ft*ft	0.00	0.00	-34.524	0.000
dt*dt	-0.0006	0.00002	-32.273	0.000
wv*pd	0.3891	0.01692	22.988	0.000
wv*fc	1.0937	0.08462	12.925	0.000
wv*ft	-0.0013	0.00008	-14.864	0.000
wv*dt	-0.0011	0.00034	-3.231	0.008
pd*fc	3.5313	0.33849	10.432	0.000
pd*ft	0.0123	0.00034	36.19	0.000
pd*dt	0.0115	0.00135	8.494	0.000
fc*ft	0.0192	0.00169	11.355	0.000
fc*dt	0.0706	0.00677	10.432	0.000
ft*dt	0.0001	0.00001	13.11	0.000
R-Sq = 99.89%, R-Sq(adj) = 99.70% Abbreviations: wv-wheel velocity; pd- product diameter; fc- friction condition; ft-feedstock temperature; dt- die temperature				

Table 6.35: Test of ANOVA for Effective stress

Source	**Degree of freedom**	**Sequential sum of square**	**Adjusted sum of square**	**Adjusted mean of square**	***F*-value**	***P*-value**
Regression	20	748.661	748.661	36.433	510.47	0.000
Linear	5	320.369	108.398	21.6795	295.64	0.000
wv	1	216.804	3.364	3.3643	45.88	0.000
pd	1	6.707	3.335	3.3353	45.48	0.000
fc	1	52.51	0.248	0.2477	3.38	0.093
ft	1	26.307	53.086	53.0856	723.93	0.000
dt	1	15.042	31.32	31.3201	426.11	0.000
Square	5	220.975	220.975	44.195	602.69	0.000
wv*wv	1	4.052	16.004	16.0038	218.24	0.000
pd*pd	1	0.473	6.004	6.0038	95.51	0.000
fc*fc	1	65.238	88.67	88.6705	1209.2	0.000
ft*ft	1	74.833	86.4	86.4	1191.87	0.000
dt*dt	1	76.379	76.379	76.3788	1041.58	0.000
Interaction	10	206.316	206.316	20.7316	282.72	0.000
wv*pd	1	38.751	38.751	38.7506	528.44	0.000
wv*fc	1	12.25	12.25	12.25	166.05	0.000
wv*ft	1	16.201	16.201	16.2006	220.93	0.000
wv*dt	1	0.766	0.766	0.7656	10.44	0.008
pd*fc	1	6.981	6.981	6.9806	108.83	0.000
pd*ft	1	96.04	96.04	96.04	1309.7	0.000
pd*dt	1	5.29	5.29	5.29	72.14	0.000
fc*ft	1	9.456	9.456	9.4556	128.95	0.000
fc*dt	1	6.981	6.981	6.9806	108.83	0.000
ft*dt	1	12.603	12.603	12.6025	171.86	0.000
Residual Error	11	0.807	0.807	0.0733		
Lack-of-Fit	6	0.473	0.473	0.0789	1.18	0.436
Pure Error	5	0.333	0.333	0.0667		
Total	31	749.467				
Abbreviations: wv-wheel velocity; pd- product diameter; fc- friction condition; ft- feedstock temperature; dt- die temperature						

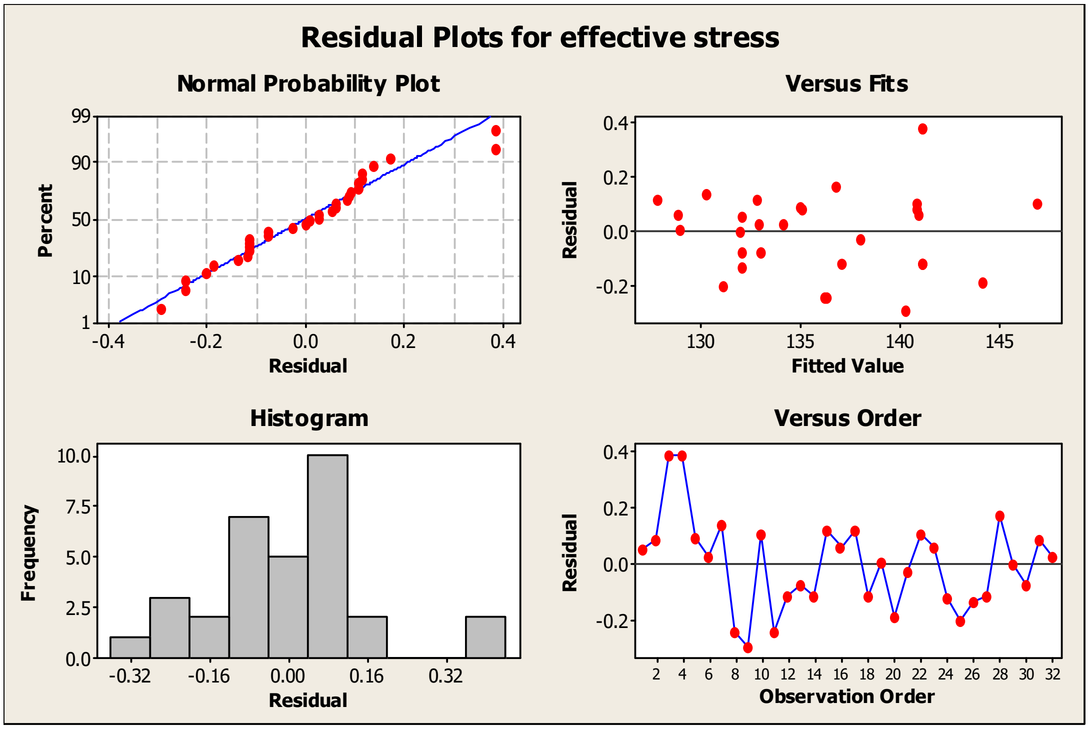

Figure 6.41: Residual plots for effective stresses

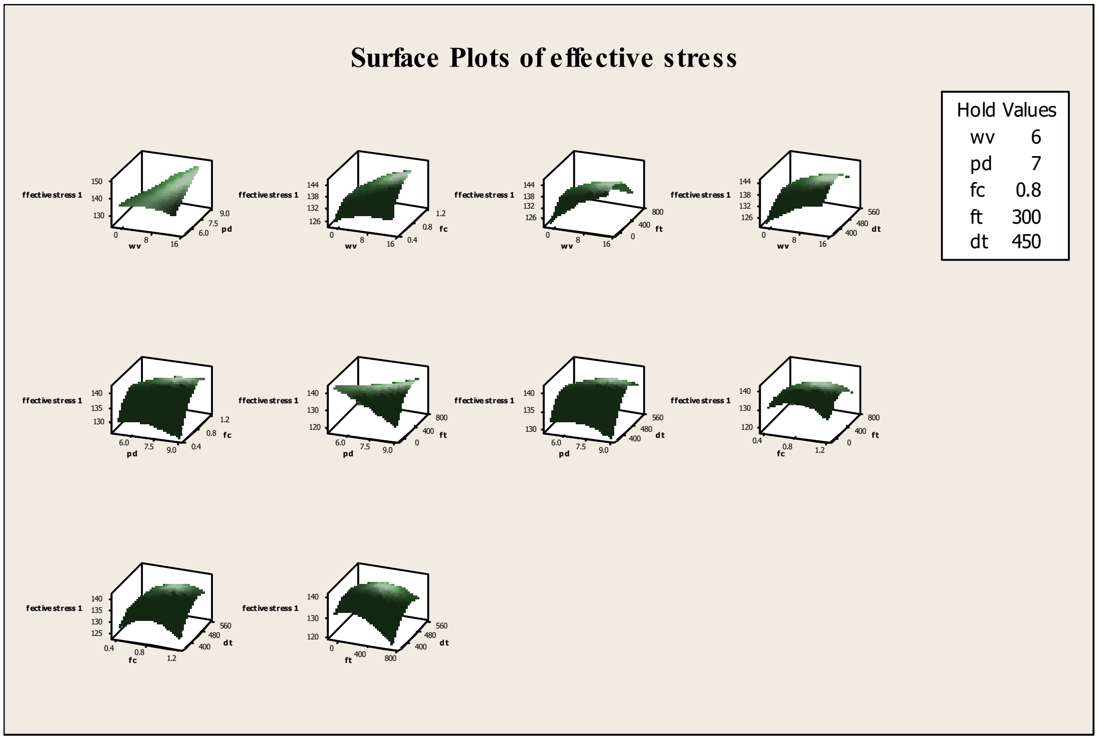

Figure 6.42: Surface plots for effective stresses

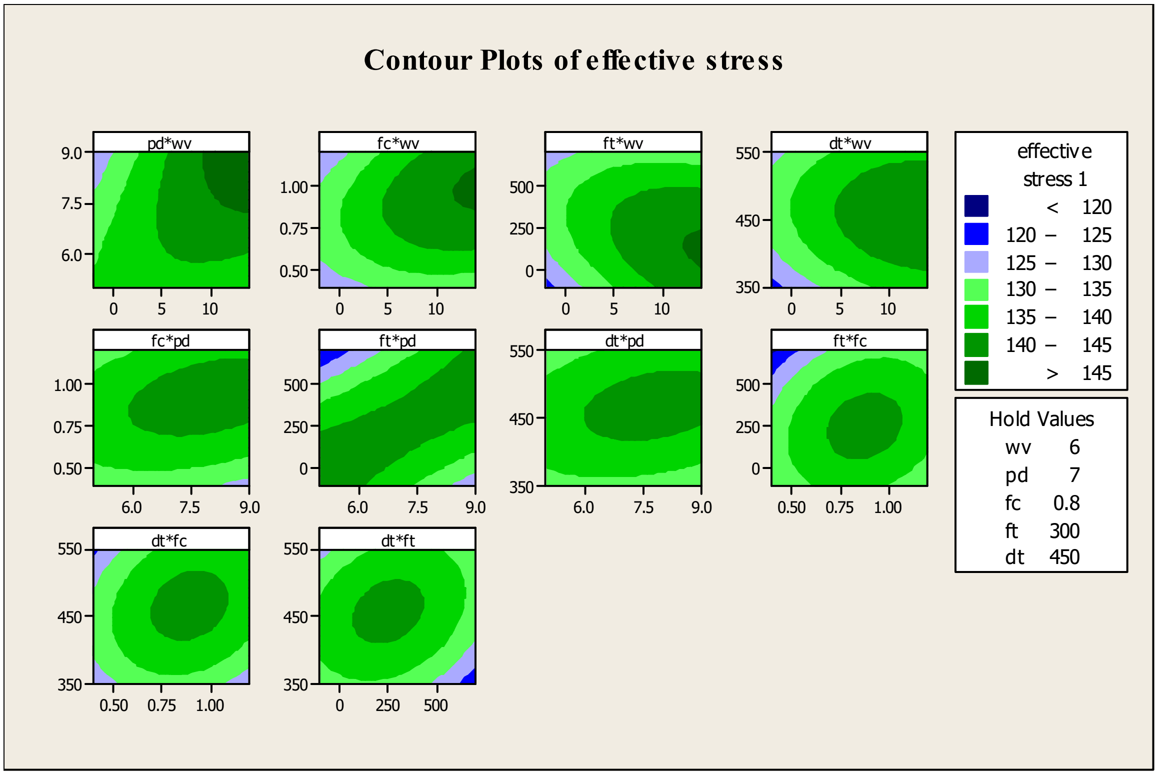

Figure 6.43: Contour plots for effective stresses

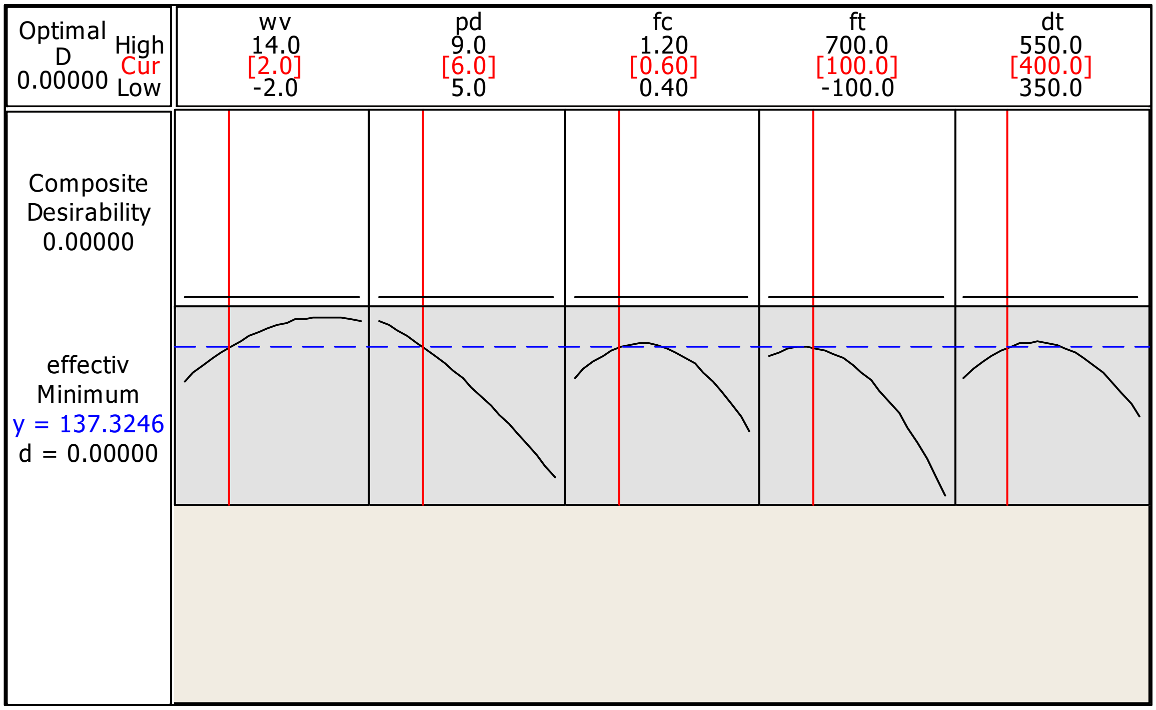

Figure 6.44: Optimization plot for effective stresses

Modeling and Optimization (Damage value)

Table 6.36 and 6.37 shows the test of significance and analysis of variance (ANOVA) test for modeling and optimization of Damage value. The results of the quadratic model for Damage value are given in Table 6.36.The value of R^2 and adjusted R^2 are 94.69 %and 85.04%.This means that the regression model provides a complete relationship between the dependent and independent variables. The associated p value for the model is <0.05(i.e., $\alpha=0.05$ or 95% confidence), indicating that the model is considered to be statistically significant. It is also seen from Table 6.36 that from the p values, second order effect of the second order effect of X_1, interactive effects of X_1 and X_3, X_1 and X_5, X_2 and X_3, X_2 and X_5, X_3 and X_4, X_3 and X_5 is much more significant is much more significant. The other model terms can be regarded as insignificant due to their probabilities values being more than 0.05.Figure 6.45 depicts the normal probability of residuals for Damage value. It is observed from the plot that residuals values are distributed normally and in a straight line and hence the model is adequate.

Using the results presented in Table 6.36, the derived model is shown as:

$$\sigma=8.56282+0.0072X_1^2+0.19492X_1X_3+ 0.0045X_1X_5+0.91719X_2X_3+0.00147X_2X_5+0.00235X_3X_4-0.011 \qquad (6.16)$$

Figures 6.46, 6.47 and 6.48 show the surface, contour and optimization plot respectively for modeling and optimization of damage value. It can be inferred from the surface and contour plots (Figures 6.46 to 6.47) that the damage value of extruded Aluminum feedstock materials can be visualized by the area shaded between interactive effects of different process parameters as mentioned and plotted. Further the minimum value of extrusion load can achieved at optimum combination of different process parameters which can be observed from the optimization plot (Figure 6.48). The optimization plot reveals that the minimum damage value can be achieved at wheel velocity of 4 RPM, product diameter of 5 mm (Extrusion ratio of 1.11), frictional coefficient as 0.4, and feedstock temperature of 100 ◦C and die temperature of 550 ◦C.

Table 6.36: Test of significance for Damage value

Estimated Regression Coefficients for Damage value				
Terms	**Coefficients**	**Standard Error Coefficient**	***t* - value**	***P* - value**
Constant	8.56282	3.58843	2.386	0.036
wv	-0.02009	0.10035	-0.2	0.845
pd	-0.83605	0.46797	-1.787	0.102
fc	-1.82507	2.11038	-0.865	0.406
ft	-0.00324	0.00201	-1.612	0.135
dt	-0.01867	0.01017	-1.835	0.094
wv*wv	0.0072	0.00149	4.821	0.001
pd*pd	-0.01976	0.0239	-0.827	0.426
fc*fc	0.60597	0.59757	1.014	0.332
ft*ft	0	0	1.37	0.198
dt*dt	0.00002	0.00001	1.683	0.12
wv*pd	-0.01008	0.00809	-1.246	0.239
wv*fc	-0.19492	0.04046	-4.818	0.001
wv*ft	-0.00002	0.00004	-0.608	0.555
wv*dt	0.00045	0.00016	2.752	0.019
pd*fc	0.91719	0.16182	5.668	0
pd*ft	-0.00018	0.00016	-1.091	0.299
pd*dt	0.00147	0.00065	2.269	0.044
fc*ft	0.00235	0.00081	2.906	0.014
fc*dt	-0.01197	0.00324	-3.698	0.004
ft*dt	0.00001	0	1.593	0.139
R-Sq = 94.69%, R-Sq(adj) = 85.04% Abbreviations: wv-wheel velocity; pd- product diameter; fc- friction condition; ft- feedstock temperature; dt- die temperature				

Table 6.37: Test of ANOVA for Damage value

Source	**Degree of freedom**	**Sequential sum of square**	**Adjusted sum of square**	**Adjusted mean of square**	***F*-value**	***P*-value**
Regression	20	3.28744	3.28744	0.164372	9.81	0.000
Linear	5	1.21322	0.11793	0.023586	1.41	0.295
wv	1	0.41475	0.00067	0.000672	0.04	0.845
pd	1	0.68175	0.05349	0.053491	3.19	0.102
fc	1	0.07809	0.01253	0.012534	0.75	0.406
ft	1	0.00525	0.04354	0.043544	2.6	0.135
dt	1	0.03338	0.05644	0.056443	3.37	0.094
Square	5	0.4681	0.4681	0.093621	5.59	0.008
wv*wv	1	0.3635	0.38955	0.389545	23.24	0.001
pd*pd	1	0.0213	0.01146	0.011455	0.68	0.426
fc*fc	1	0.01031	0.01723	0.017234	1.03	0.332
ft*ft	1	0.02551	0.03144	0.03144	1.88	0.198
dt*dt	1	0.0475	0.0475	0.047495	2.83	0.12
Interaction	10	1.60612	1.60612	0.160612	9.58	0.000
wv*pd	1	0.026	0.026	0.026002	1.55	0.239
wv*fc	1	0.38906	0.38906	0.389064	23.21	0.001
wv*ft	1	0.0062	0.0062	0.006202	0.37	0.555
wv*dt	1	0.12691	0.12691	0.126914	6.57	0.019
pd*fc	1	0.53839	0.53839	0.538389	32.12	0.000
pd*ft	1	0.01995	0.01995	0.019952	1.19	0.299
pd*dt	1	0.08629	0.08629	0.086289	5.15	0.044
fc*ft	1	0.14156	0.14156	0.141564	8.45	0.014
fc*dt	1	0.2292	0.2292	0.229202	13.68	0.004
ft*dt	1	0.04254	0.04254	0.042539	2.54	0.139
Residual Error	11	0.18435	0.18435	0.016759		
Lack-of-Fit	6	0.18422	0.18422	0.030703	1151.37	0.000
Pure Error	5	0.00013	0.00013	0.000027		
Total	31	3.47179				

Abbreviations: wv-wheel velocity; pd- product diameter; fc- friction condition; ft-feedstock temperature; dt- die temperature

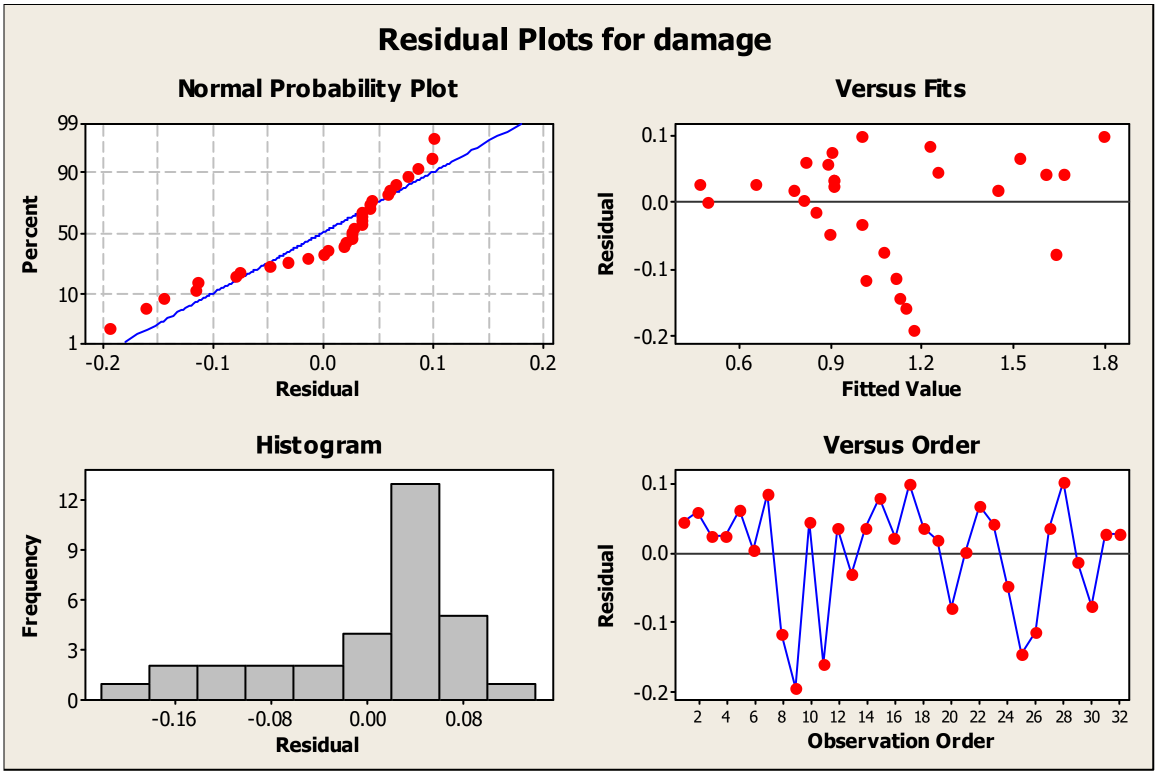

Figure 6.45: Residual plots for Damage value

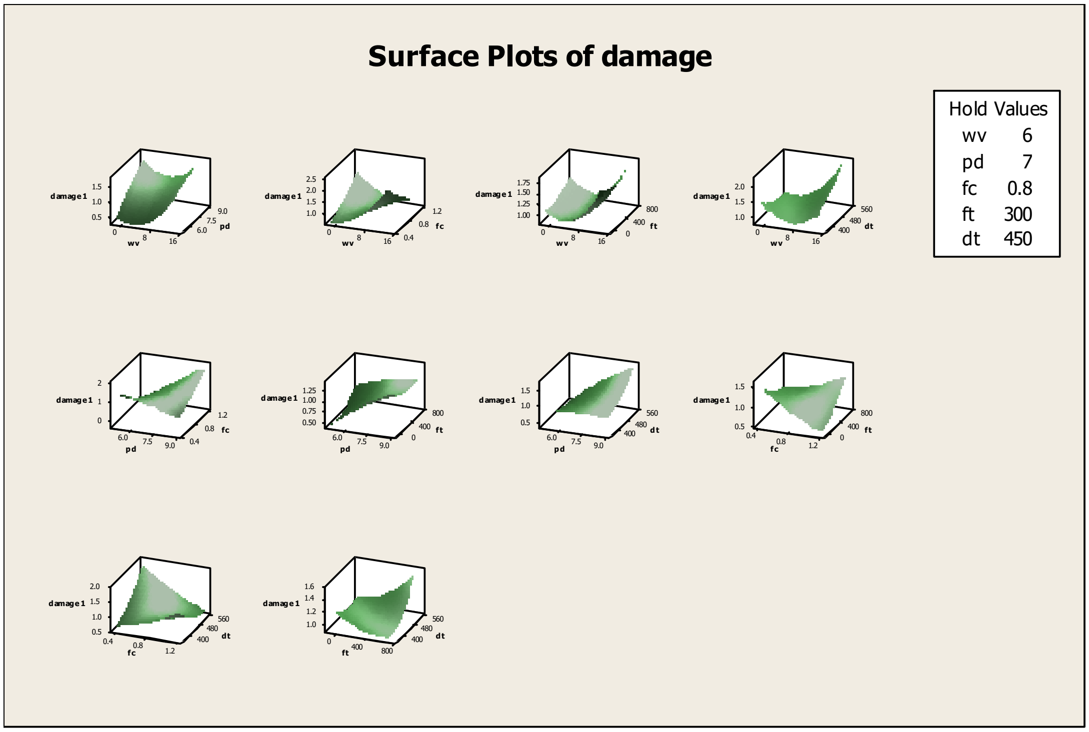

Figure 6.46: Surface plots for Damage value

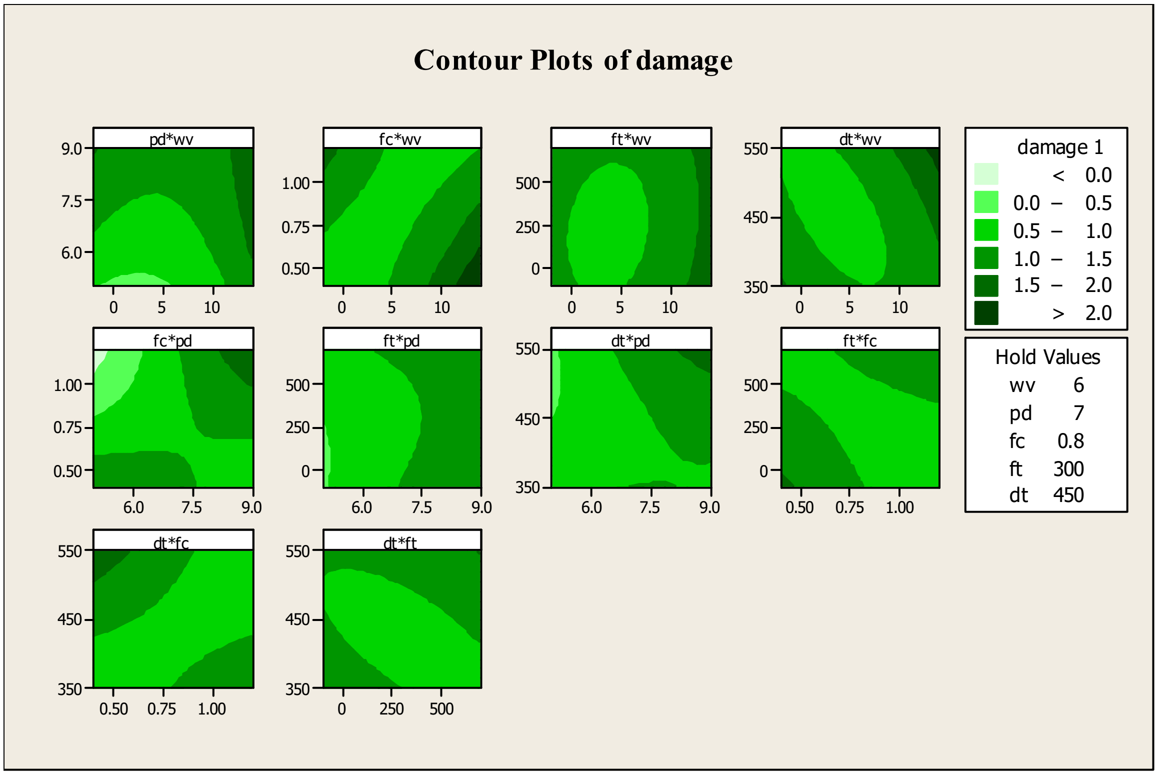

Figure 6.47: Contour plots for Damage value

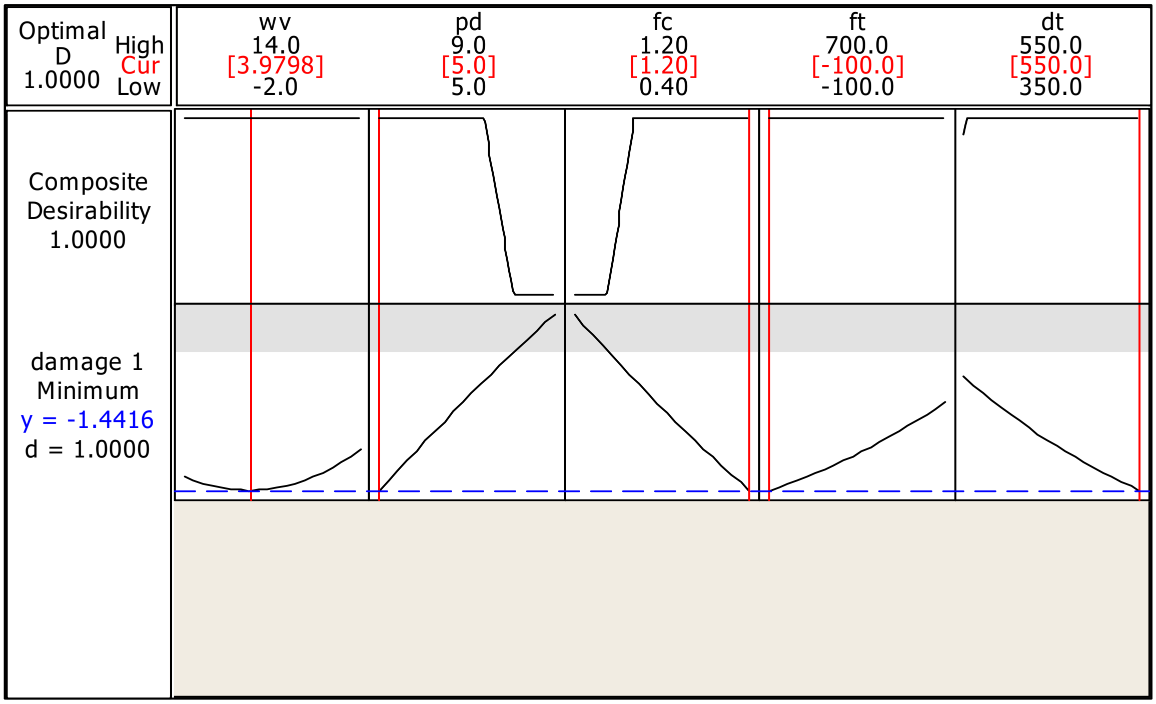

Figure 6.48: Optimization plot for Damage value

Modeling and Optimization (Product Temperature)

Table 6.38 and 6.39 shows the test of significance and analysis of variance (ANOVA) test for modeling and optimization of Product Temperature in continuous extrusion of Aluminum feedstock material. The results of the quadratic model for Product Temperature are given in Table 6.38.The value of R^2 and adjusted R^2 are 94.31 % and 83.98% respectively. This means that the regression model provides a complete relationship between the dependent and independent variables. The associated p value for the model is <0.05(i.e., $\alpha=0.05$ or 95% confidence), indicating that the model is considered to be statistically significant. It is also seen from Table 6.38 that from the p values, the main effect X_2 and second order effect of X_2 is much more significant. The other model terms can be regarded as insignificant due to their probabilities values being more than 0.05.Figure 6.49 depicts the normal probability of residuals for Product Temperature. It is observed from the plot that residuals values are distributed normally and in a straight line and hence the model is adequate.

Using the results presented in Table 6.38, the derived model is shown as:

$$T = 441.66X_2-23.05X_2^2 \tag{6.17}$$

Figures 6.50, 6.51 and 6.52 show the surface, contour and optimization plot for modeling and optimization of product temperature. It can be inferred from the surface and contour plots (Figures 6.50 to 6.51) that the minimum product temperature for extrusion of Aluminum feedstock materials can be visualized by the area shaded between interactive effects of different process parameters as mentioned and plotted. Further the minimum value of product temperature can achieved at optimum combination of different process parameters which can be observed from the optimization plot (Figure 6.52). The optimization plot reveals that the minimum extrusion load can be achieved at wheel velocity of 14 RPM, product diameter of 5 mm (Extrusion ratio of 3.61), frictional coefficient as 0.4, feedstock temperature of 100 ◦C and die temperature of 406 ◦C.

Table 6.38: Test of significance for Product Temperature

Estimated Regression Coefficients for Product Temperature

Terms	**Coefficients**	**Standard Error Coefficient**	***t* - value**	***P* - value**
wv	-1.8	39.73	-0.045	0.965
pd	441.66	185.27	2.384	0.036
fc	1704.53	835.51	2.04	0.066
ft	-0.02	0.79	-0.021	0.984
dt	-2.91	4.03	-0.723	0.485
wv*wv	-0.5	0.59	-0.837	0.42
pd*pd	-23.05	9.46	-2.435	0.033
fc*fc	-323.01	236.58	-1.365	0.199
ft*ft	0.00	0.00	1.567	0.145
dt*dt	0.00	0.00	1.197	0.256
wv*pd	1.14	3.2	0.356	0.729
wv*fc	3.67	16.02	0.229	0.823
wv*ft	0.02	0.02	1.283	0.226
wv*dt	0.00	0.06	0.024	0.981
pd*fc	-115.31	64.07	-1.8	0.099
pd*ft	0.02	0.06	0.327	0.75
pd*dt	-0.09	0.26	-0.346	0.736
fc*ft	-0.05	0.32	-0.151	0.883
fc*dt	-0.78	1.28	-0.61	0.554
ft*dt	0.00	0.00	0.346	0.736

R-Sq = 94.31%   R-Sq(adj) = 83.98%

Abbreviations: wv-wheel velocity; pd- product diameter; fc- friction condition; ft- feedstock temperature; dt- die temperature

Table 6.39: Test of ANOVA for Product Temperature

Source	**Degree of freedom**	**Sequential sum of square**	**Adjusted sum of square**	**Adjusted mean of square**	***F*-value**	***P*-value**
Regression	20	479314	479314	23965.7	9.12	0
Linear	5	429200	31472	6294.3	2.4	0.106
wv	1	38801	5	5.4	0	0.965
pd	1	0	14927	14926.5	5.68	0.036
fc	1	1276	10933	10933.1	4.16	0.066
ft	1	388876	1	1.2	0	0.984
dt	1	247	1373	1372.6	0.52	0.485
Square	5	34861	34861	6972.1	2.65	0.082
wv*wv	1	1584	1840	1840.2	0.7	0.42
pd*pd	1	17190	15579	15578.7	5.93	0.033
fc*fc	1	6626	4897	4896.9	1.86	0.199
ft*ft	1	5695	6451	6450.9	2.46	0.145
dt*dt	1	3765	3765	3765.2	1.43	0.256
Interaction	10	15253	15253	1525.3	0.58	0.8
wv*pd	1	333	333	333.1	0.13	0.729
wv*fc	1	138	138	138.1	0.05	0.823
wv*ft	1	4323	4323	4323.1	1.65	0.226
wv*dt	1	2	2	1.6	0	0.981
pd*fc	1	8510	8510	8510.1	3.24	0.099
pd*ft	1	281	281	280.6	0.11	0.75
pd*dt	1	315	315	315.1	0.12	0.736
fc*ft	1	60	60	60.1	0.02	0.883
fc*dt	1	977	977	976.6	0.37	0.554
ft*dt	1	315	315	315.1	0.12	0.736
Residual Error	11	28895	28895	2626.9		
Lack-of-Fit	6	28895	28895	4815.9		
Pure Error	5	0	0	0		
Total	31	508210				

Abbreviations: wv-wheel velocity; pd- product diameter; fc- friction condition; ft-feedstock temperature; dt- die temperature

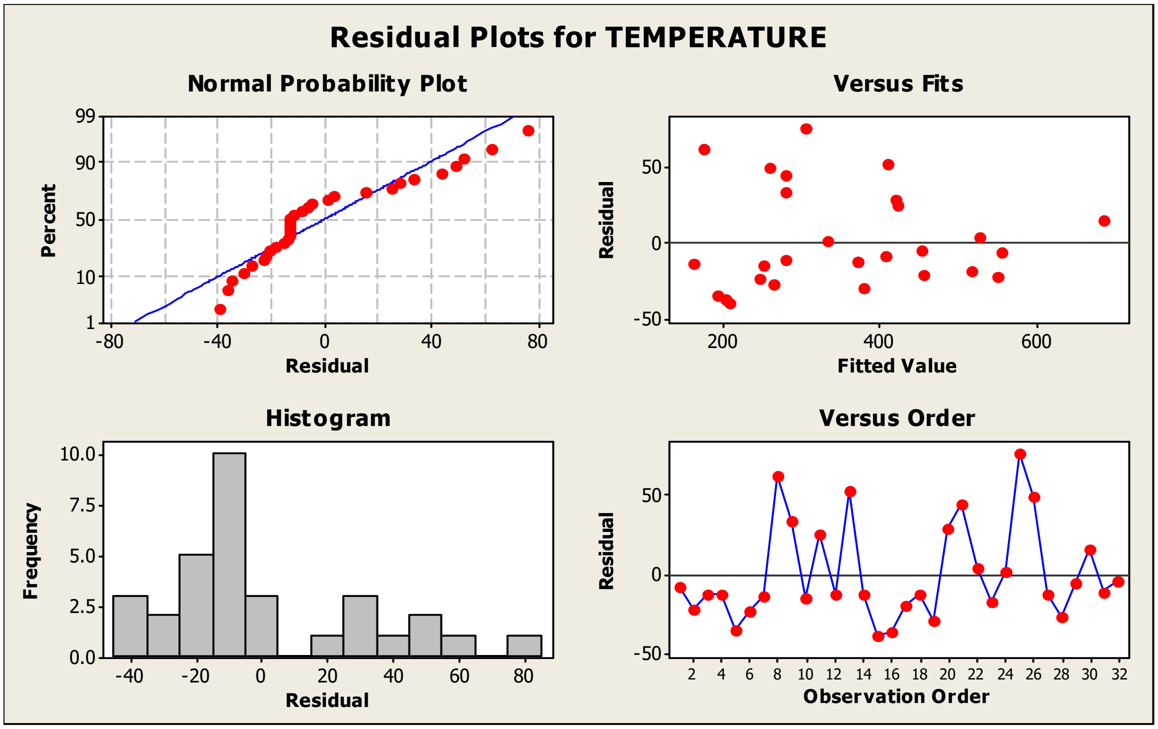

Figure 6.49: Residual plots for Product Temperature

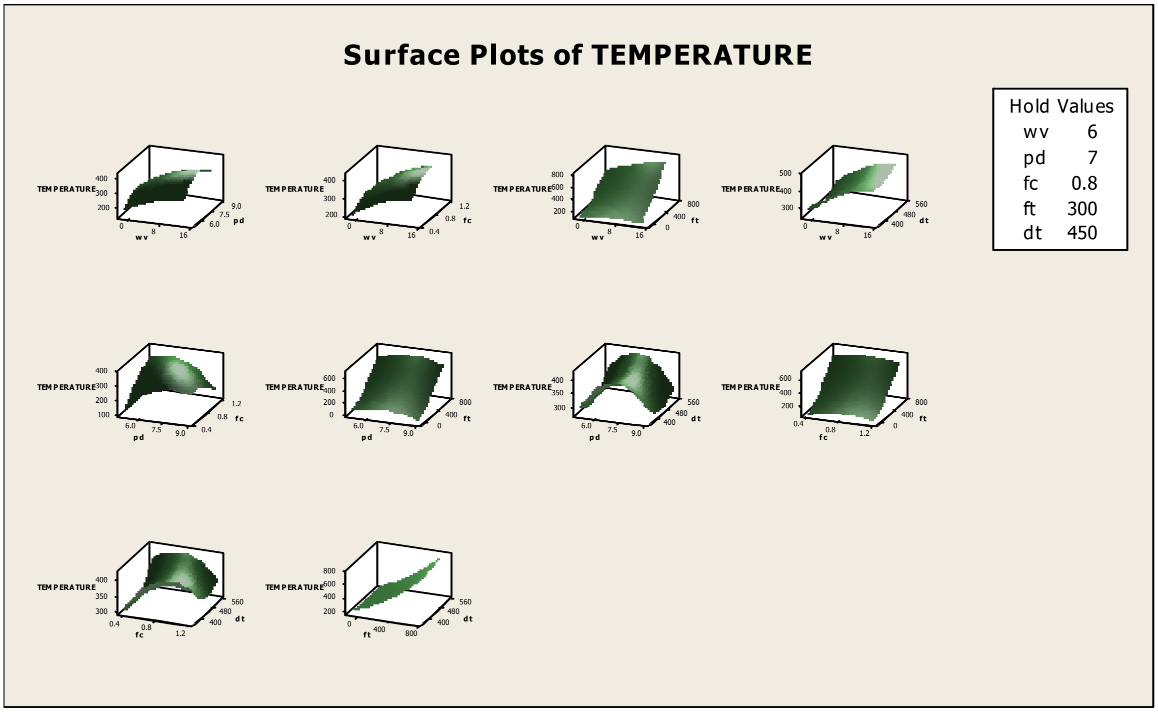

Figure 6.50: Surface plots for Product Temperature

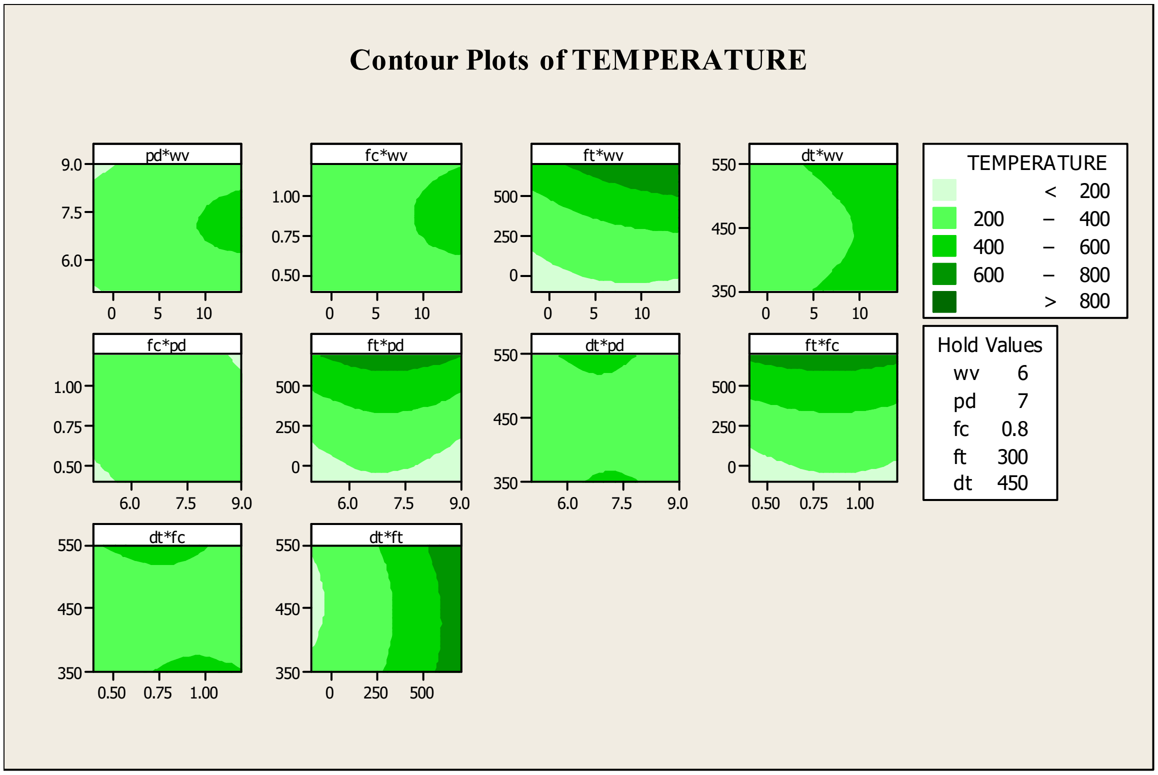

Figure 6.51: Contour plots for Product Temperature

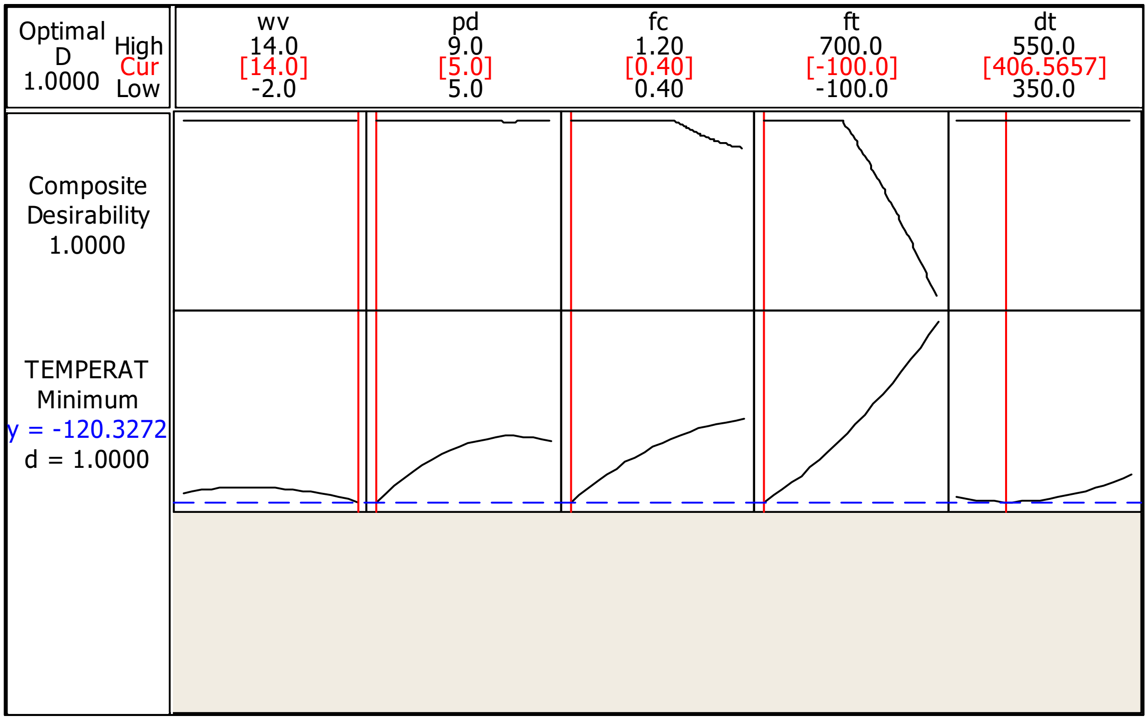

Figure 6.52: Optimization plot for Product Temperature

Case IV

6.5 Modeling of Numerical data for Aluminum feedstock using Artificial Neural Network model

A machine learning approach such as Artificial Neural Networks (ANNs) can be used as an alternative to the polynomial regression based modeling approach that overcomes the non-linearity of bioprocess variables interaction (Franco-Lara et al. 2006).The feed forward neural network (FFNN) paradigm is most widely used ANN configuration, which is composed of input, hidden, and output layers (Zafar et al. 2012). A feed forward architecture of ANN model, which is also known as multilayer perception (MLP), has been used with back propagation (BP) algorithm to build the predictive models with five input variables of the Continuous Extrusion process i.e. wheel velocity, product diameter, frictional conditions, feedstock temperature and die temperature having significant impact on the output responses of the Continuous Extrusion process such as load required, torque required, effective stress, Damage value and Product Temperature. All the inputs and outputs are normalized within a uniform range of (0.1-0.9) to ensure the uniform attention during the training process. The first step in the training of a neural network model is to design the topology of the network. The number of neurons in the input layer is fixed by number of inputs, and in the output layer by the number of outputs (Wang and wan, 2009).The determination of the number of neurons in the hidden layer of the network is the critical step, and is determined by varying the number of nodes from 1 to 6 in the hidden layer. During the training process, the mean square error between the experimental and the corresponding predicted values is calculated and propagated backward through the network using well known Levenberg- Marqardt back propagation algorithm (trainlm). The back propagation algorithm adjusts the weights in each successive layer to reduce the error. This procedure is repeated until the error between the experimental and the corresponding predicted values satisfy the certain error criteria. The number of neurons in the hidden layer plays a vital role in the training time and generalization property of neural networks. Lower number of neurons in the hidden layer would increase the training time whereas higher number of neurons in the hidden layer would cause the over training and saturation of the network which leads to the false results. There are no general rules for selecting the number of neurons in a hidden layer.

The best approach is to find the optimal number of neurons in the hidden layer is by trial and error (Zafar et al., 2012).The MATLAB (Version 6.0, Math works, Inc., MA, USA) has been used to perform artificial neural network based modeling studies.

In present study, 32 data sets generated by RSM has been utilized as the input variables for ANN into two sets. 24 data sets (from run no.1 to 24 as specified in Table 6.30) has been exploited as the training data sets whereas rest 6 data sets (from run no. 25 to 30 as specified in Table6.30) has been utilized for testing the efficiency of the neural network. A network consists of 5 input nodes representing the input process variables of the Continuous Extrusion process, 6 neurons for the hidden layer and one output node for the output response process parameters of the Continuous Extrusion process at the end of training process (topology 5-6-1). The number of neurons in hidden layer has been chosen in a range of 10 to 18 by cross-validating for the lowest values of root mean square error (MSE) and standard error of prediction (SEP). During the supervised training process, the associated learning error rate (mean-squared error) is minimized by increasing the number of training epochs (cycles). However, an optimal number of training epochs need to be determined in order to avoid any possible overtraining of the network. A total number of 434 epochs has been determined to be the optimum number of training cycles for the present ANN structure and the corresponding MSE (relative error between the network output and target value) has been found to be 6.69.

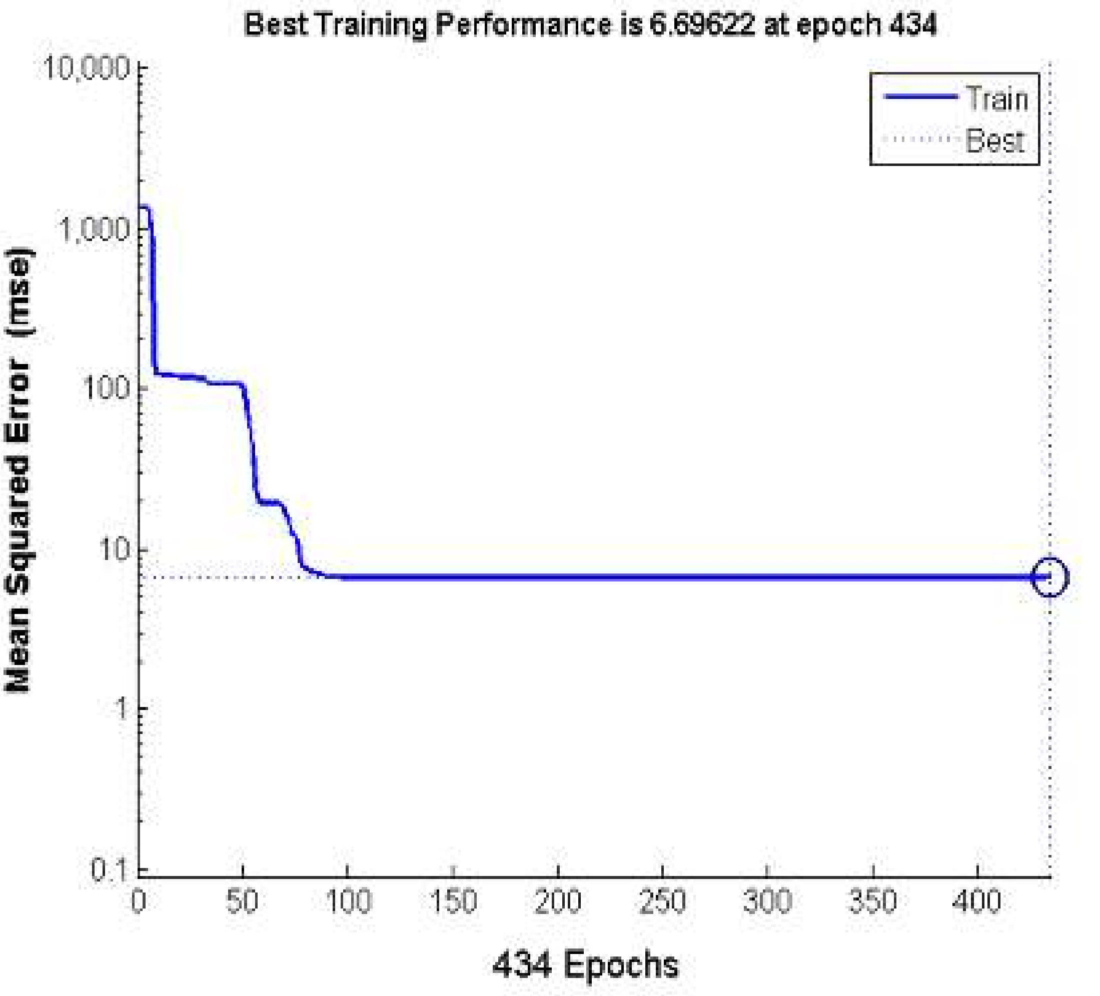

Figure 6.53: Training epoch cycles vs. calculated mean square error of the supervised training for the designed ANN

The Figure 6.53 show the training and test epoch cycles versus the calculated mean squared error of the supervised training and the comparison of input benchmark data and corresponding ANN predictions for the training process.

Further, the fitness of the neural network prediction has been analyzed by calculating the coefficient of R^2 using the experimental and predicted data. The R^2 (entire data including training and testing data) has been found to be 0.972 as shown in Figure 6.54 indicating that the used network is significant and the obtained data is more accurate. This has been further confirmed from the Figure 6.54 that the predictions have been concentrated near the diagonal line on the graph without much scattering.

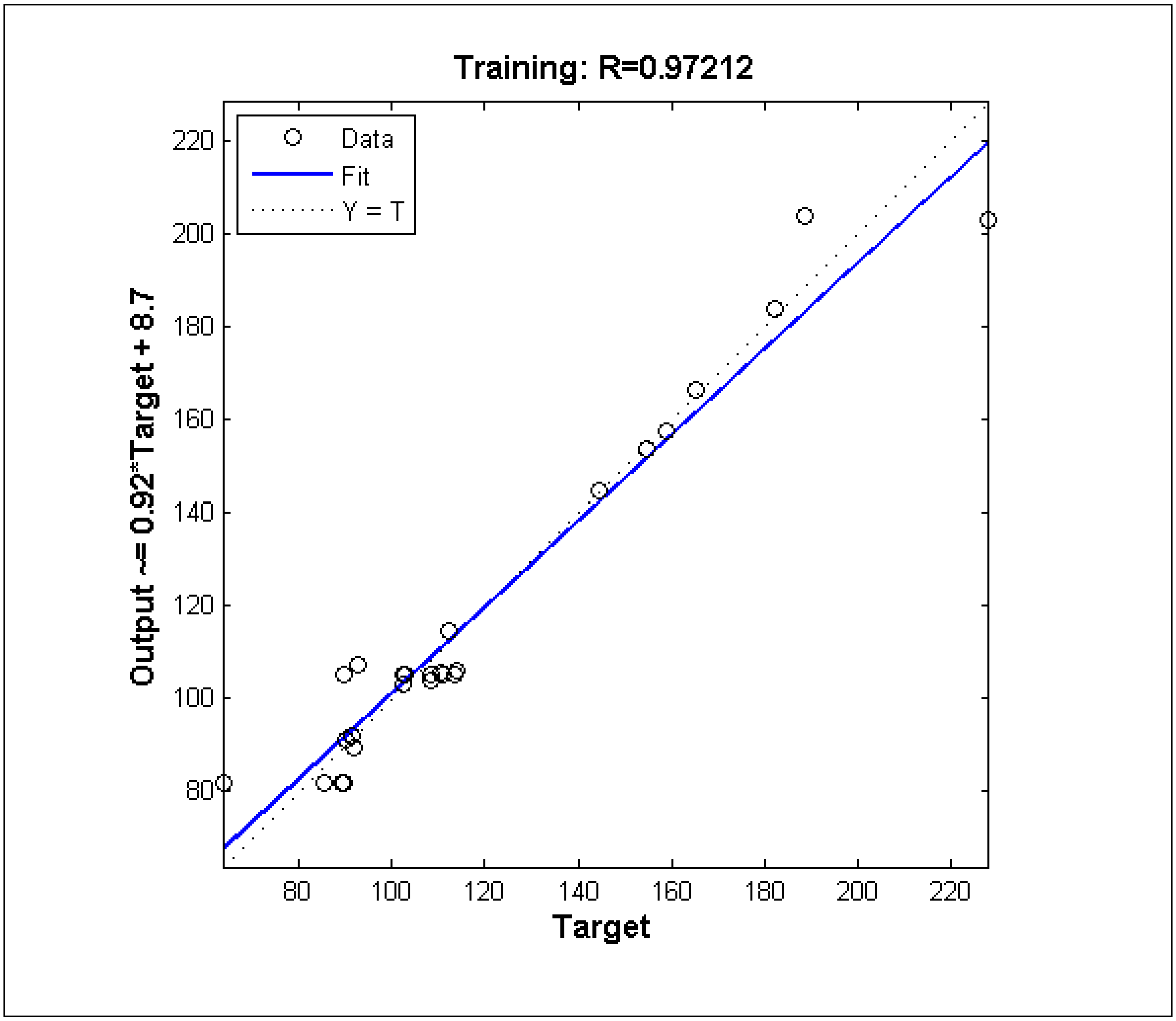

Figure 6.54: Correlation chart for experimental and predicted load required for Continuous Extrusion

6.6 Genetic algorithm based optimization using ANN model

To facilitate a solution for an optimization problem (regression equation), GA creates an initial population of randomly generated individuals called chromosomes, generally represented as strings of binary digits. During successive iterations (generations), the evolved chromosomes acquire better fitness value by reproduction among individuals of the previous generation. In order to create new generations three genetic operators are applied: selection, crossover and mutation. The descendants evolved at each generation, has been subjected to evaluation for their fitness value using the fitness function (regression equation). At each step, the genetic algorithm selects the individuals at random, from current population, to be parents and uses them to produce the offspring for the next generation. Crossover operator combines two parents to form children for upcoming generations. However, mutation rules are concerned with introducing new diversities among individual parents producing children. Point mutations are the most commonly occurring mutations, which are used to avoid any convergence to local maxima. This iterative process continues until a satisfactory solution according to the need of fitness function has been achieved. The MATLAB (Version 6.0, Math works, Inc., MA, USA) has been used to perform genetic algorithm based modeling studies.

To facilitate an optimum solution, genetic algorithm has been employed on the newly generated population (data sets) of independent variables. The CCD and ANN based regression models have been executed as the fitness functions by GA for minimum load required in Continuous Extrusion process. All the five parameters of the model have been represented in terms of chromosomes for GA based optimization technique with the following constraints:

2≤Wheel Velocity≤10

6≤Product Diameter≤8

0.6≤Friction condition≤1.0

100≤Feedstock temperature≤500

400≤Die temperature≤500

The genetic algorithm parameters in the MATLAB software for the optimization of Load required in Continuous Extrusion of Aluminum feedstock has been set as the following: double vector; original population size: 100; cross over probability: 0.8; elite count: 20; crossover function: @crossover single point; migration direction: forward; selection function: @selection Roulette; mutation function: @mutation Gaussian; total generations: 100.

Since genetic algorithms based optimization procedure frequently does not declare the global optimum solution, the process of optimization has been repeated several times by varying the input space parameters (Maiti et al.2011).These re-iterations at different GA input conditions ascertained that the whole searching space has been explored thoroughly to achieve a global optimum solution. Accomplishment of alike optimal solutions for most of the input conditions confirmed that it is a global optimum solution. Figure 6.55 below shows five random trials generated by GA with their model generated predicted values.

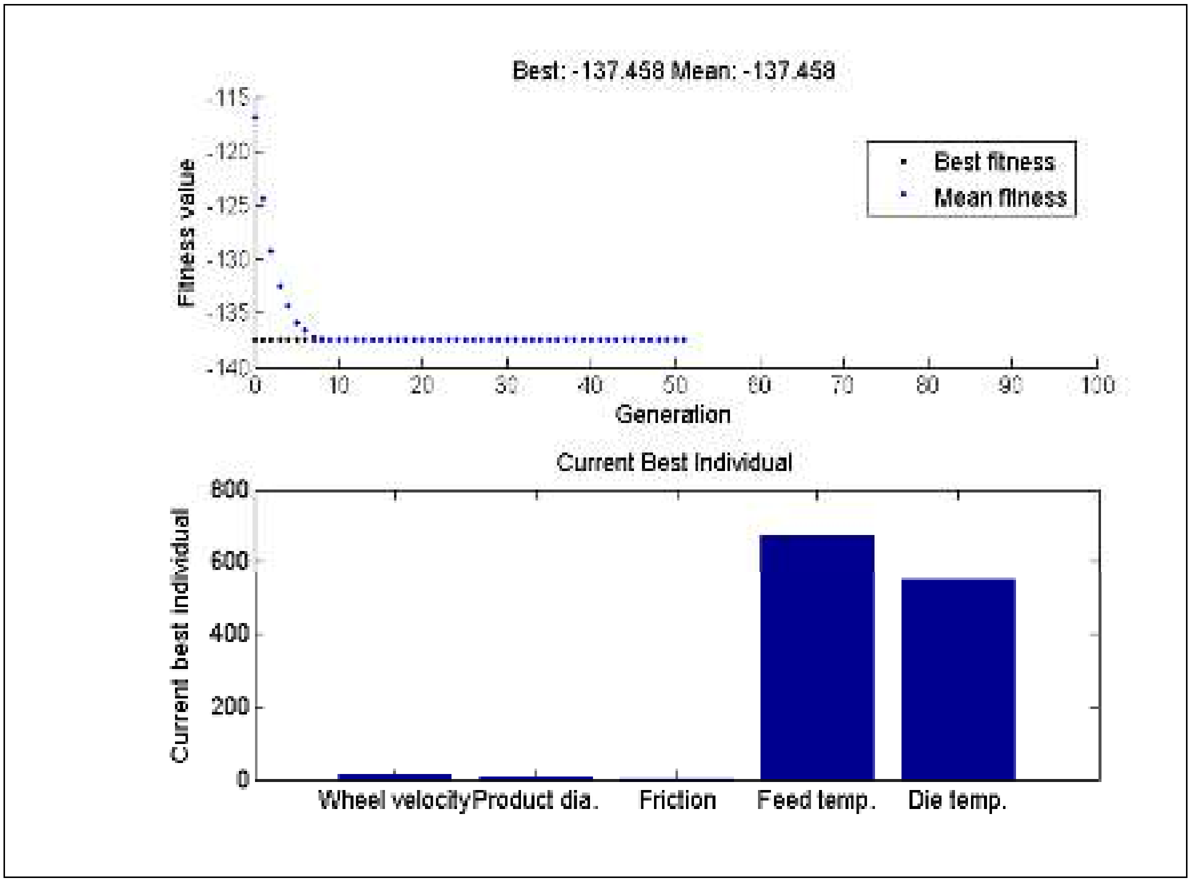

Figure 6.55: Best fitness plot showing the progressive performance (for Load required) of GA over generations till the achievement of optimum solution (upper plot).Variables (in lower plot) showing the level of wheel velocity, product diameter, friction condition, feedstock temperature and die temperature.

The validation of the optimal solutions has been carried out by independent experiments using the same conditions. The experimental data of each generated trial have shown that the optimum load is achieved as 136.4 kN.

Figure 6.55 showed the best fitness plot achieved during the iterations of GA over generations describes the gradual convergence of results towards the optimal solution for load required for extrusion as 136.4 kN .The optimum value of input process parameters for optimum value of load obtained has been 13 RPM as wheel velocity, 5 mm as product diameter, 1.95 as friction condition, 671°C as feedstock temperature and 548 °C as die temperature.

6.7 Comparison of RSM and ANN

Table 6.40: Results of comparison of R^2 value for RSM and ANN

Modeling Technique	**R^2 value**
RSM	0.9149
ANN	0.972

The neural network prediction has been analyzed by calculating the coefficient of R^2 using the experimental and predicted data as shown in Table 6.40. The R^2 (entire data including training and testing data) has been found to be 0.972 indicating that the used network is significant and the obtained data is more accurate. The value of correlation coefficient close to unity represents the accurate predictions of result. Therefore it can be concluded that ANN provides accurate result as compared to RSM.

To maximize Ultimate Tensile Strength and the Hardness of the Aluminum extruded product at some specific combination of extrusion wheel velocity and product diameter or extrusion ratio, the optimum values of the input process parameters in Continuous Extrusion process for extrusion wheel velocity and product diameter are 6.57(RPM) and 6.33(mm) respectively. For Continuous Extrusion process of Pure Aluminum feedstock with optimum parametric combination, Ultimate Tensile Strength can be achieved as high as 106.45 MPa and the Hardness can be achieved as high as HV 34.6.

To maximize the Yield Strength and the % Elongation of the Aluminum extruded product at some specific combination of extrusion wheel velocity and product diameter or extrusion ratio. The optimum values of the input process parameters in Continuous Extrusion process for extrusion wheel velocity and product diameter are 6.06(RPM) and 6.18(mm) respectively. For Continuous Extrusion process of Pure Aluminum feedstock with optimum parametric combination, Yield Strength can be achieved as high as 70.94 MPa and the % Elongation can be achieved as high as 46.45.

To maximize the Ultimate Tensile Strength of the Copper extruded product at some specific combination of extrusion wheel velocity and product diameter or extrusion ratio. The optimum values of the input process parameters in Continuous Extrusion process for extrusion wheel velocity and product diameter are 10(RPM) and 5.72(mm) respectively. For Continuous Extrusion process of Pure Copper feedstock with optimum parametric combination, Ultimate Tensile Strength can be achieved as high as 250.5 MPa.

To maximize the Hardness of the Copper extruded product at some specific combination of extrusion wheel velocity and product diameter or extrusion ratio. The optimum values of the input process parameters in Continuous Extrusion process for extrusion wheel velocity and product diameter or extrusion ratio are 10(RPM) and 5(mm) respectively. For Continuous Extrusion process of Pure Copper feedstock with optimum parametric combination, Hardness can be achieved as high as 95.9 HV. To maximize the Yield Strength of the Copper extruded product at some specific combination of extrusion wheel velocity and product diameter or extrusion ratio. The optimum values of the input process parameters in Continuous Extrusion process for extrusion wheel velocity and product diameter are 6.36(RPM) and 6.87(mm) respectively. For Continuous Extrusion process of Pure Copper feedstock with optimum parametric combination, Yield Strength can be achieved as high as 59 MPa. To maximize the % Elongation of the Copper extruded product at some specific combination of extrusion wheel velocity and product diameter or extrusion ratio. The optimum values of the input process parameters in Continuous Extrusion process for extrusion wheel velocity and product diameter are 6.28(RPM) and 6.84(mm) respectively. For Continuous Extrusion process of Pure Copper feedstock with optimum parametric combination, % Elongation can be achieved as high as 59.46.

Numerical modeling and optimization of process parameters in Continuous Extrusion process of Aluminum alloy has also been done through RSM, ANN and ANN-GA in sections 6.4, 6.5 and 6.6 respectively. ANN-GA has been found the best optimization tool among RSM, ANN and ANN-GA. The accuracy of ANN-GA approach is better than ANN and accuracy of ANN is found to be better than RSM. ANN can be used as an efficient tool in predicting composite properties.

Table 6.41: RSM results for Aluminum and Copper feedstock

Material	Optimization technique used	Optimum Parametric Combination		Optimum Result
		Wheel Velocity (RPM)	Product Diameter (mm)	
Aluminum	RSM	6.57	6.33	UTS(Max) = 106.45 MPa
		6.57	6.33	Hardness(Max) = 34.6
		6.06	6.18	YS(Max) = 70.94 MPa
		6.06	6.18	% Elongation(Max) = 46.45
Copper	RSM	10	5.72	UTS(Max) = 250.5 MPa
		10	5	Hardness(Max) = 95.9 HV
		6.36	6.87	YS(Max) = 59 MPa
		6.28	6.84	% Elongation(Max) = 59.46

The optimum value of input process parameters for optimum value of load obtained in numerical modeling and optimization process of Continuous Extrusion for Aluminum feedstock has been found as 13 RPM as wheel velocity, 5 mm as product diameter, 1.95 as friction condition, 671°C as feedstock temperature and 548 °C as die temperature using ANN-GA technique and optimum value of load achieved is 136.4 kN.

The mechanical properties of Aluminum extrusions shown in Table 6.41 are greatly affected by the presence of Mg-Si particles. For the mechanical properties, it was generally observed that deformation speed did not itself have a dominant effect, and could simply be regarded as a tool for affecting maximum extrusion temperature [Zhao et al. (2013)].As the extrusion wheel velocity increases, the maximum extrusion temperature increases which leads to increase of mechanical properties such as tensile strength and hardness. But increase of extrusion temperature beyond a limit results in the presence of coarse grain particles which results in decrease of mechanical properties such as UTS, YS and hardness.