CHAPTER-2

Device Design Methodology and Its Beam-Wave Interaction Analysis of Magnetically Insulated Line Oscillator (MILO)

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2.1. Introduction

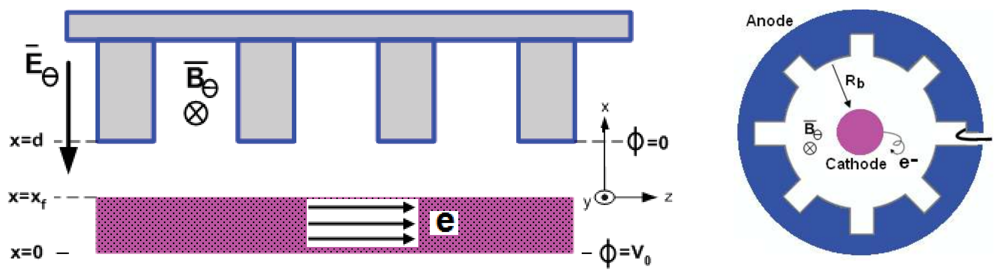
Beam-wave interaction behavior in a HPM source is understood based on the interaction between the slow or fast space charge waves and the RF wave in the interaction circuits. The slow space charge waves are referred as the waves with the phase velocities slower than the electron beam velocity, conversely the fast space charge waves are referred as the waves with phase velocities faster than the velocity of the beam. The high power microwave (HPM) generation requires self-magnetic insulation, which facilitates the requirement of external DC magnetic field. For efficient operation at higher power levels, it would be desirable to have oscillator that operates at lower impedance and also eliminate the problem of voltage matching. MILO is typically a reliable crossed field M-types slow-wave microwave tube oscillator producing pulse power of the order of giga-watts. This is a compact, stable and efficient HPM source. This inherent property makes this device compact and reliable for defense applications. MILO is similar to those of the relativistic magnetron however the relativistic magnetron deeds an external axial magnetic field. Therefore, it is necessary to understand the basic physics of self-magnetic insulation mechanism that takes place inside the MILO device.

This study accounts the relativistic electron flow, which significantly affect the microwave frequency and power level of the device. A number of independent research works have been reported in various literature as discussed in Chapter-1, for the analysis of magnetic insulation mechanism [Hull (1921), Lovelace and Edward Ott (1974), Davidson *et* al. (1984), Tsang and Davidson (1986), Lawconnell and Neri (1989)]. The non-relativistic single particle analysis of magnetron was explained by Hull, and the relativistic self-consistent Cartesian treatment had begun around 1970. Further, the expressions for critical magnetic field including the relativistic space-charge flow have also been reported [Bergeron and Poukey (1975), Creedon (1975)]. The analysis includes the assumption of the total energy and canonical momentum across flow is conserved, which is known as Brillouin flow theory. In 1979, Mendel explained canonical momentum and energy profiles for flows across the electron sheath for a variety of electron orbits. Creedon and Wang extended the laminar Brillouin flow theory in the cylindrical coordinates system. For confining electron flow in coaxial geometry, magnetically insulated flow mechanism has been defined considering three different processes. First process named as self- limited magnetically insulated flow, second process is load limited magnetically insulated flow and third process is constant flux limited magnetically insulated flow. MILO operates on the principle of load limited magnetically insulated flow. During the magnetic insulation between cathode to anode, electrons drift parallel to the cathode and an electron sheath is formed between the disc tip and the cathode surface. This electron sheath equilibrium is known as the relativistic Brillouin flow (RBF) in which all electrons have same constant Hamiltonian and Canonical momentum.

To understand the analytical fundamental and develop the design equations for MILO, the fundamentals of cross field devices, device design methodology, condition for relativistic Brillouin flow, magnetic insulation, beam-wave interaction mechanism and design flow chart are explained. The rest of the chapter is organized as follows. In Section 2.2 the fundamental and analytical theory of crossed field device is discussed. The analytical design equations for MILO including magnetic insulation, critical current, para-potential current are presented in Section 2.3.

2.2. Electron Motion in Crossed Field Devices

Electrons emitted from the cathode are accelerated towards the anode by the static electric field due to applied DC voltage and show a deflection due to the presence of the external magnetic field. The motion of negatively charged electron inside the RF circuits is studied using both classical and relativistic particle approach as follows.



(a) *(b)*

Fig. 2.1: Electron trajectories in crossed field *(a)* in the planer geometry *(b)* in the cylindrical geometry of the magnetron type [Cousin (2005)].

2.2.1. Classical Approach

Magnetic field is perpendicular to both electric field and electron motion, so for finding the trajectories of the electrons, one has to calculate force exerted by magnetic field and electric field on electron and that can be obtained, using expression (2.1). Typical tube 87electrodes. The electrons rotate due to static electric field $\vec{E}$ (which is created by a potential difference applied between these two electrodes) and a transverse magnetic field $\vec{B}$ perpendicular with the electric field. Electron velocity $\vec{v}$, thus subjected to the electric force $\overrightarrow{F_e} = -e\overrightarrow{E_0}$ and also with the magnetic force $\overrightarrow{F_m} = -e(\vec{v} \times \overrightarrow{B_0})$, and the projection of these forces in the Cartesian coordinate system (Fig. 2.1), is given as

$$m_0 \frac{d^2x}{dt^2} = eE_0 - eB_0 \frac{dz}{dt} \quad , \tag{2.1}$$

where, $B_0(dz/dt)$, is magnetic field component along z direction.

$$\Rightarrow m_0 \frac{d^2y}{dt^2} = 0 \quad . \tag{2.2}$$

There is no velocity component in y direction.

$$m_0 \frac{d^2z}{dt^2} = eB_0 \frac{dx}{dt} \quad , \tag{2.3}$$

where, m_0 = rest mass of electron, E_0 = electric field and B_0 = magnetic flux density.

Here $\dot{x}, \ddot{x}, \dot{y}, \ddot{y}, \dot{z}, \ddot{z}$, are the first and the second derivatives in Cartesian coordinates of space. The equations (2.1), (2.2), and (2.3) shows that electron motion is a planer motion since the initial condition for v_0 in y, is such that $\dot{y} = v_0 = 0$. Thus, electrons move in (x, z) plane. Now, integrating third equation and taking initial condition, $t = 0, \quad x = \dot{x}, \quad \dot{z}\{0\} = 0$ and one gets two results from the first three equations:

$$mv^2/r = qvB \quad , \tag{2.4}$$

$$\omega_c = v/r = qB/m_0 \quad , \tag{2.5}$$

where, ω_c is cyclotron angular frequency.

Now, integrating equation (2.3) and substituting the limit as followed, we get:

$$\int \frac{d^2z}{dt^2} = \frac{eB_0}{m_0} \int_{x_0}^{x} \frac{dx}{dt} \quad ,$$

$$\frac{dz}{dt} = \frac{eB_0}{m_0}(x - x_0) \quad , \tag{2.6}$$

Putting (2.6) in equation (2.1)

$$\frac{d^2x}{dt^2} = \frac{eE_0}{m_0} - \omega_c^{\,2}(x - x_0) \quad , \tag{2.7}$$

Equation (2.6) is converted as

$$dz/dt = \omega_c(x - x_0) \quad , \tag{2.8}$$

Equation (2.6) is second order differential equations, using (2.7) and (2.8) the second order coefficients can be calculated as follows. We put, $x = \sin\theta$, so $dx/dt = \cos\theta$ and $d^2x/dt^2 = -\sin\theta$, using these equation, one can obtain the value of sin θ, as:

$$\sin\theta = \frac{\omega_c^{\,2} x_0}{(\omega_c^{\,2} - 1)} + \frac{eE_0}{m_0(\omega_c^{\,2} - 1)} \quad , \tag{2.9}$$

After solving equation (2.9), we get

$$x\{t\} - x\{0\} = (E_0/\omega_c B_0) + R\sin(\omega_c t + \varphi) \quad , \tag{2.10}$$

Equation (2.8) is first order differential equation, after integrating and substituting the limit as calculated, we get

$$\int_0^t dz = \int \frac{eB_0}{m_0}(x - x_0)dt \ ,$$

$$\Rightarrow \qquad z(t) - z(0) = (eB_0/m_0)(x - x_0)t + c \ . \tag{2.11}$$

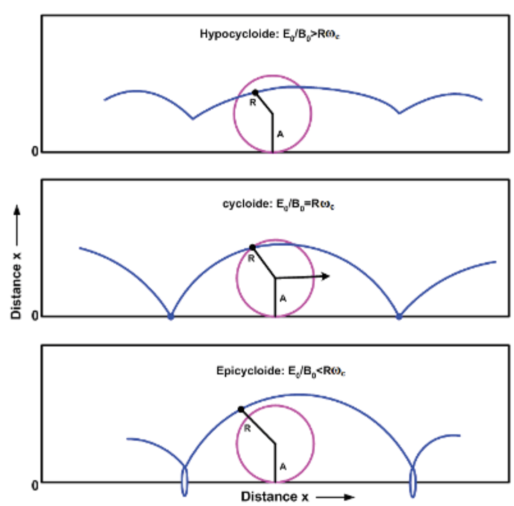


Fig. 2.2: Electron trajectories in all possible types of curvature [Slater (1950)].

Here, $z\{0\}$ is a constant of integration which fixes the initial condition of the electron flow in z direction, at time $t = 0$. The equations (2.10) and (2.11) thus obtained represents the electron trajectory in a (x, z) plane. This trajectory results from a combination between a rectilinear translatory uniform motion velocity ($v = E_0 / B_0$) and of a uniform circular motion (ω_c). The second term of (2.10) and (2.11) is the parametric representation of a circle of radius R in the (x, z) plane. Thus, the electron moved on a circular path of radius R with velocity v (Fig. 2.2), [Shevchik *et al.* (1966)]. The orbital electron velocity on this circle is $R\omega_c$. When velocity of electron orbital velocity $R\omega_c$ is equal to E_0 / B_0, then electron describes a cycloid (Fig. 2.2). The electron trajectory is a hypocycloid if $E_0/B_0 > R\omega_c$ and epicycloids if $E_0/B_0 > R\omega_c$ as shown in the Fig. 2.2.

2.2.2. Relativistic Approach

For the relativistic electrons in a crossed field tubes, the equation of motion has one more parameter, relativistic mass factor γ_0 and can be written as:

$$\frac{d\vec{p}}{dt} = \frac{d(\gamma_0 m_0 \vec{v})}{dt} = m_0 \vec{v}\frac{d\gamma}{dt} + m_0\gamma_0\frac{d\vec{v}}{dt} = -e[\vec{E}_0 + \vec{v} \times \vec{B}_0], \tag{2.12}$$

where p is the momentum the particle whose value is $m_0\vec{v}$, and that can be obtained as: $\vec{p}/t = \vec{F} = m\vec{v}/t$, and where γ_0 can be represented as

$$\gamma_0 = \frac{1}{\sqrt{1-(\vec{v}/c)^2}} = \frac{1}{\sqrt{1-((\vec{v}_x{}^2 + \vec{v}_y{}^2 + \vec{v}_z{}^2)/c^2)}} \quad , \tag{2.13}$$

v_x, v_y v_z are the electron velocity components in their respective coordinates axis. v can be written as

$$\vec{v} \cdot (d\vec{p}/dt) = -e(\vec{v} \cdot \vec{E}_0) = -e\vec{v} \times \vec{E}_0 + e\vec{v} \times \vec{v} \times \vec{B}_0 \quad , \tag{2.14}$$

Finally, we get the electron velocity

$$\vec{v}\cdot d\vec{p}/dt = -e\vec{v}\cdot\vec{E}_0 \qquad , \tag{2.15}$$

However, according to [Pierce (1949)], (2.15) expressed in term of kinetic energy

$$\vec{v}\cdot d\vec{p}/dt = \vec{v}\cdot\vec{F} = d\vec{E}_{cin}/dt \qquad , \tag{2.16}$$

where F is the resultant applied force at the electron. In this relation, $\vec{E}_{cin}$ is relativistic kinetic energy of the particles which can be expressed as

$$\vec{v}(m_0\cdot\vec{v}) = m_0\cdot\vec{v}^{\,2} = m_0c^2 = \vec{E}_{cin} \Rightarrow \vec{E}_{cin} = \gamma_0\, m_0c^2 \qquad , \tag{2.17}$$

So using (2.17), (2.15) can be rewritten as, $-e\vec{v}E_0 = m_0c^2(d\gamma_0/dt)$,

$$d\gamma_0/dt = -(e\vec{v}E_0/m_0c^2) \qquad , \tag{2.18}$$

Now, substituting (2.18) in (2.12), one obtains:

$$d\vec{p}/dt = m_0\vec{v}(e\vec{v}E_0/m_0c^2) + (m_0\gamma_0)(d\vec{v}/dt) \quad ,$$

which after simplification becomes

$$\Rightarrow \qquad (m_0\gamma_0)\frac{d\vec{v}}{dt} = \frac{d\vec{p}}{dt} + \frac{eE_0\vec{v}^{\,2}}{c^2} \qquad . \tag{2.19}$$

Equation (2.19) has been expressed in terms of $(d\vec{v}/dt)$ as

$$\frac{d\vec{v}}{dt} = \frac{1}{\gamma_0 m_0}\left[-eE_0 + e\vec{v}\times B_0 + \frac{\vec{v}(\vec{v}eE_0)}{c^2}\right] \qquad , \tag{2.20}$$

Taking electron term as common from the whole equation

$$\frac{d\vec{v}}{dt} = \frac{e}{\gamma_0 m_0}\left[E_0\vec{v}\times B_0 - \frac{\vec{v}(\vec{v}E_0)}{c^2}\right] \qquad . \tag{2.21}$$

Using these fundamental equations of classical motion of particle in relativistic form can be obtained using (2.21) as

$$\frac{d\vec{v}_x}{dt} = \frac{d^2x}{dt^2} = \frac{eE_0}{\gamma_0 m_0} - \frac{eB_0}{\gamma_0 m_0}\frac{dz}{dt} - \frac{eE_0}{\gamma_0 m_0 c^2}\left(\frac{dx}{dt}\right)^2 \qquad , \tag{2.22}$$

$$\frac{d\vec{v}_y}{dt} = \frac{d^2 y}{dt^2} = -\frac{eE_0}{\gamma_0\, m_0 c^2}\frac{dx}{dt} \quad , \tag{2.23}$$

$$\frac{d\vec{v}_z}{dt} = \frac{d^2 z}{dt^2} = \frac{eB_0}{\gamma_0\, m_0}\frac{dx}{dt} - \frac{eE_0}{\gamma_0\, m_0}\frac{dz}{dt}\frac{dx}{dt} \quad . \tag{2.24}$$

It is possible to decouple the motion according to x and z position and it gives the results as that for the non relativistic motion of the electrons in a plane motion [Jackson (1999)].

2.3. Motion of Particle in MILO

In the distributed emission devices, the electrons are emitted over a large surface area. This is the specialty of cathode material which emits bunch of solid electron beam from the every part of surface due to application of a very high pulse. We can see from Fig. 2.3, there is a coaxial geometry in which the central part is cathode, the external outer surface and the anode forms a delay line. Electron beam is emitted from the entire length of cathode and terminates on the load which is also adjusted by the conditions of magnetic cutoff [Capua (1983)]. The static magnetic field is generated due to the flow electron beam current. The current in the structure flows towards anode structure via load, is resulted from superposition of the cathode current, and current due to magnetic cutoff.

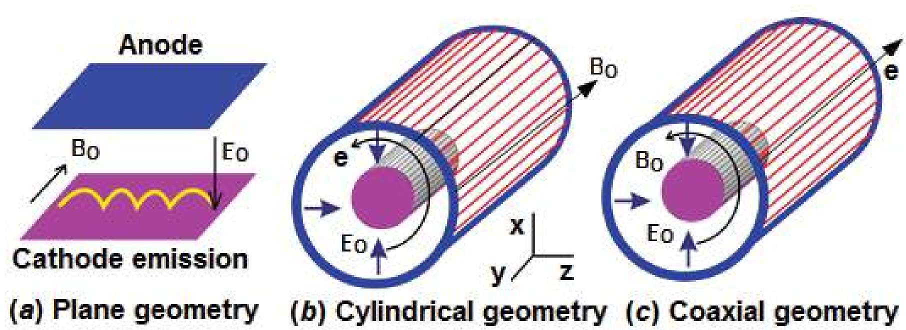


Fig. 2.3: Geometry of the crossed field type magnetron (a)Linear magnetron (b) Cylindrical magnetron (c) MILO .

This whole process is functioning under the same conditions, since the electrons get rotation due to the magnetic field, suitable for the beam and then interacts in the same manner synchronously with a longitudinal component of the delay line (coaxial disc-loaded structure). The difference created due to application of the high voltage. Here, the cathode surface which emits the electrons and in order to reach to the critical magnetic cutoff condition, several hundreds of kilovolts of the voltage is necessary, which can be calculated as on the line currents several tens of kilo amperes is required. With these currents, device work at a normal pulse mode. Fig. 2.4 show a schematic of the distributed emission occurs in the MILO device.

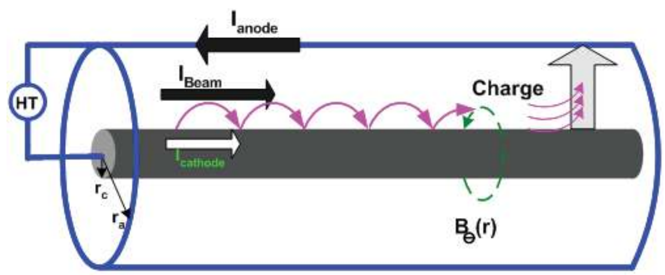


Fig. 2.4: Schematic of distributed emission in the MILO structure.

Magnetically insulated line oscillator (MILO) often uses coaxial disc-loaded RF structure with field emitted cathode, which has been discussed in details in the next chapters of this thesis. Here, we have taken cylindrical symmetry (Fig. 2.4) and used cylindrical coordinate system (z, r, φ). The electric and magnetic field in polar coordinates can be written as:

$$\vec{E}\{r\} = -E_r\{r\}\vec{u}_r \quad , \tag{2.25}$$

$$\vec{B}\{r\} = -B_\theta\{r\}\vec{u}_\theta \quad , \tag{2.26}$$

Here, $\vec{u}_r$ and $\vec{u}_\theta$ are the unit vectors in radial and azimuthal direction, respectively. The applied forces on the electrons have been calculated using the radial electric field and azimuthal magnetic field in form of radial, azimuthal and axial momentum form, as:

$$dp_r / dt = eE_r\{r\} - ev_z B_\theta\{r\} \quad , \tag{2.27}$$

$$dp_\theta / dt = 0 \quad , \tag{2.28}$$

$$dp_t / dt = ev_r\{r\} B_\theta\{r\} \quad , \tag{2.29}$$

Expressing momentum using relativistic factor $\gamma_0\{r\}$ and $\beta_z = v_z / c$, in (2.29), as:

$$m_0 c \frac{d}{dt}(\gamma_0\{r\}\beta_t\{r\}) = eB_\theta\{r\}\frac{dr}{dt} \quad , \tag{2.30}$$

Now, for finding the currents in the different parts of the structure, Ampere's current law has been used:

$$H\{r\} = (I\{r\} dl \sin\varphi / 4\pi r^2) \quad , \tag{2.31}$$

The general Ampere law equation is modified for the coaxial geometry as:

$$H_\phi\{r\} = (I / 4\pi r) \quad , \tag{2.32}$$

Using the well known relation of magnetic field intensity

$$\mu_0 H = B \Rightarrow H = B / \mu_0 \quad , \tag{2.33}$$

Substituting (2.33) in (2.32), we get

$$B = e\frac{I\mu_0}{4\pi r} \quad , \tag{2.34}$$

Now, substituting equation (2.34) in (2.30) yields,

$$d(m_0 c(\gamma\{r\}\beta_t\{r\})) = e\frac{I\mu_0}{4\pi r} dr \quad , \tag{2.35}$$

Boundary conditions applied on the cathode surface ($r = r_c, \gamma\{r_c\} = 1 \Rightarrow \beta_z\{r_c\} = 0$) and that at the anode, $r = r_a, \gamma\{r_a\} = \gamma_0 \Rightarrow v_r\{r_a\} = 0$, is necessary condition for critical magnetic cut-off of the structure. Magnetic cut-off condition imposes an additional condition on the cathode current, such as $I = I_{cr}$. Integrating equation (2.35) and using the above boundary condition we get:

$$m_0 d\gamma_0\{r\}\beta_z\{r\} = \frac{e\mu_0 I_{cr}}{4\pi}\int_{r_c}^{r_a}\frac{dr}{r} = \frac{e\mu_0 I_{cr}}{4\pi}\left[\ln r_a - \ln r_c\right] \quad , \tag{2.36}$$

Applying the condition, $\beta_z = \sqrt{\gamma_0^{\,2} - 1}/\gamma_0$ and $\varepsilon_0\mu_0 c^2 = 1$, in (2.36) yield;

$$I_{cr} = \frac{2\pi m_0 c\gamma_0\sqrt{\gamma_0^{\,2} - 1}}{\gamma_0(1/\varepsilon_0 c^2 e\ln(r_a/r_c))e\ln(r_a/r_c)} \quad , \tag{2.37}$$

Equation (2.37), can be rewritten as

$$I_{cr} = \frac{4\pi m_0 c^3\varepsilon_0\sqrt{\gamma_0^{\,2} - 1}}{e\ln(r_a/r_c)} \quad , \tag{2.38}$$

Substituting $I_0 = (4\pi m_0 c^3\varepsilon_0)/e$ in (2.38):

$$I_{cr} = \frac{I_0\sqrt{\gamma_0^{\,2} - 1}}{2\ln(r_a/r_c)} \quad . \tag{2.39}$$

Critical current I_{cr} is the minimum cathode current that is needed for magnetic cut-off to occur in the structure. A Hull criterion for magnetic insulation of the layer is used in equilibrium, that's why the electrons cannot cross the gap to reach toward the anode which is derived from very general theory, based on the conservation of energy and canonical momentum in the direction of the net electron drift.

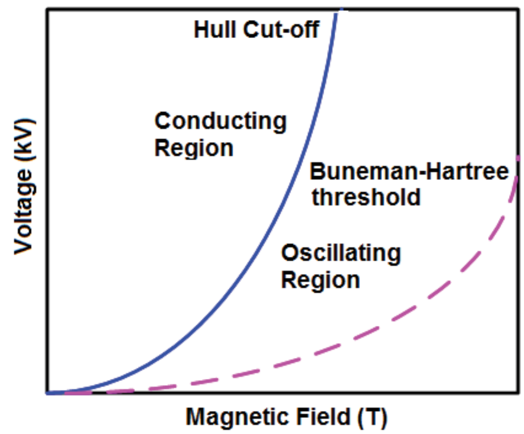


Fig. 2.5: Diagram showing the oscillation condition for the crossed field device.

Hull criteria has been applied due to the necessary conditions that if magnetic field is too weak, arcing occurs, and if magnetic field is too high then the oscillation is reached into the cutoff conditions. The space between Hull cutoff requirement and Buneman-Hartree threshold is the region of oscillation where efficient operation usually occurs just to the left side of the Buneman-Hartree threshold (Fig. 2.5).

Assuming that the electrons are emitted from the cathode and there is no system verification for the electrons along the direction of the drift, so that a time independent equilibrium is established. Now, Hull voltage as well as Hull magnetic field can be calculated using law of conservation of energy as follows:

$$\gamma_0 m_0 c^2 = m_0 c^2 + eV_H \quad ,$$

$$\Rightarrow \quad \gamma_0 E_{cin} = E_{cin} + eV_H \quad , \tag{2.40}$$

The critical magnetic field is calculated as:

$$(\gamma_0{}^2 - 1)mc^2 - eV = 0 \quad , \tag{2.41}$$

Using (2.41), one can derive the Hull magnetic field as:

$$B^* = \frac{mc}{ed_e}(\gamma_0{}^2 - 1)^{1/2} \quad , \tag{2.42}$$

For the beam-wave interaction condition, $B_z > B^*$, the critical magnetic field or Hull field, the electrons drift azimuthally within a bounded electron cloud called the Brillouin layer. As $B_z < B^*$ the electron cloud extends almost to the anode. As the magnetic field increases, the cloud is confined closer and closer to the cathode. Here, B_z is axial magnetic flux density. So, the cutoff magnetic field is,

$$B_c = \frac{\mu_0 I_{cr}}{2\pi r_c} = \frac{\mu_0 I_0 \sqrt{(\gamma_0)^2 - 1}}{2\pi r_c \ln(r_a/r_c)} \quad , \tag{2.43}$$

Using (2.40) and (2.41), equation (2.43) can be rewritten as:

$$B_c = \frac{\mu_0 I_0}{4\pi r_c \ln(r_a/r_c)} [((m_0 c^2 + eV_H)/m_0 c^2)^2 - 1]^{1/2} \quad , \tag{2.44}$$

which get simplified as:

$$B_c = \frac{\mu_0 I_0}{4\pi r_c \ln(r_a/r_c)} \left[\left(\frac{eV_H}{m_0 c^2} \right)^2 + \frac{2eV_H}{m_0 c^2} \right]^{1/2} \quad , \tag{2.45}$$

Substituting the value of I_0 in (2.45):

$$B_c = \frac{m_0}{e r_c \ln(r_a/r_c)} \left[\left(\frac{eV_H}{m_0 c^2} \right)^2 + \frac{2eV_H}{m_0 c^2} \right]^{1/2} \quad , \tag{2.46}$$

For finding the value of Hull voltage (V_H), we can simplify (2.46) as:

$$V_H = \frac{m_0 c^2}{e} \left[\left(1 + \left(\frac{eB_c D^*}{m_0 c} \right)^2 \right)^{1/2} - 1 \right] \quad , \tag{2.47}$$

where, $D^* = r_c \ln(r_a / r_c)$, (2.48)

Substituting (2.48) in (2.47), we get

$$V_H = \frac{m_0 c^2}{e} \left[\left(1 + \left(\frac{eB_c r_c \ln(r_a/r_c)}{m_0 c} \right)^2 \right)^{1/2} - 1 \right] \quad , \tag{2.49}$$

Putting the value of B_c from (2.43) and μ_0 from (2.33) in (2.49):

$$V_H = \frac{m_0 c^2}{e}\left[\left(1+\left(\frac{eI_c r_c \ln(r_a/r_c)}{2\pi m_0 c^3 r_c (\varepsilon_0)^2}\right)^2\right)^{1/2} - 1\right] \quad , \tag{2.50}$$

Finally, simplification of (2.50) yield

$$V_H = \frac{m_0 c^2}{e}\left[\left(1+\left(\frac{2I_c \ln(r_a/r_c)}{I_0}\right)^2\right)^{1/2} - 1\right] \quad , \tag{2.51}$$

As in magnetron, the electric field due to applied voltage is predominantly radial E_{r0}. The magnetic field is sum of contributions from the externally applied axial field, and the azimuthal components (created by axial current flow in the cathode and radial current flow from the spokes). Current flows axially into the magnetron to support both axial current flow which is at opposite end of the cathode to anode wall potential and radial current flow carried by the spokes in the anode-cathode gap. When flow become weak at the end of cathode get minimized, and also the azimuthal field from the axial current is relatively small, so the magnetic field is predominantly axial. Accordingly, to the left side of $v_d = \omega/k$, the drift velocity of an electron in this case is thus largely azimuthal, approximately given by E_{r0}/B_z, which decreases as B_z increases. Therefore, for a given voltage, maximum B_z above which the electrons will be too slow to achieve resonance with the phase velocity of the electromagnetic wave, supported by the anode structure. The relationship between the voltage and magnetic field at this threshold is called the Buneman-Hartree condition (Fig. 2.5) [Buneman (1950)]. For relativistic magnetron with an axial magnetic field, ignoring axial geometrical variations can be written as:

$$\frac{eV}{m_0 c^2} = \frac{eB_z \omega_n}{m_0 c^2 n} r_a d_e - 1 + \sqrt{1-\left(\frac{r_a \omega_n}{cn}\right)} \quad , \tag{2.52}$$

In equation (2.52), the value of d_e for coaxial geometry is given as

$$d_e = (r_a^{\,2} - r_c^{\,2}) / 2r_a \qquad , \tag{2.53}$$

Using the Buneman-Hartree condition for calculating Hull voltage as:

$$V_{BH} = B_{\max} D^* v_\varphi - (m_0 c^2 / e)[1 - (1 - (v_\varphi / c)^2)^{1/2}] \qquad , \tag{2.54}$$

In the conventional case, $v_\varphi << c$ and $\beta_\varphi = v_\varphi / c$, so (2.54) will become:

$$V_{BH} = B_{\max} r_c \ln(r_a / r_c) v_\varphi - (m_0 c^2 / 2e)(v_\varphi / c)^2 \quad , \tag{2.55}$$

$B_{\max}$ represents the magnetic field limits imposed by the cathode circulating current of beam radius r_c to allow synchronism condition in the structure. As from (2.52) and (2.53), one introduces the cathode current I_c and (2.55) becomes:

$$V_{BH} = (m_0 c^2 / e)[2(I_c / I_0) \ln(r_a / r_c) \beta_\varphi - (1 - (1 - \beta_\varphi^{\,2})^{1/2})] \ . \tag{2.56}$$

RF waves are being propagated in the medium which reflects on the diaphragms of the periodic structure of periodicity L (Fig. 2.4) and thus slowed down at the speed $v_\varphi < c$. With each reflection, the RF field is π out of phase of a cell of the waveguide to the following end and so for the periodic structure we can write:

$$\beta_\varphi = \omega L / \pi c = 2\pi c L / W \pi c = 2L / W \ . \tag{2.57}$$

In all the above equations V_H is Hull voltage, B_c is critical magnetic field, D^* is geometrical parameter characteristics of the coaxial line, V_{BH} is Buneman-Hartree voltage, v_φ is phase velocity, and β_ϕ is phase propagation constant. In (2.57), λ is wavelength at the resonant frequency of the periodic structure (for the π mode). However, each basic disc of depth h (Fig. 2.4) constitutes a quarter of wave resonator, as $\lambda \sim 4h$ [Slater (1950)], [Kleen (1958)], [Eastwood *et al*. (1998)] and [Mendel *et al*.

(1983)]. The relation (2.57) thus led to $\beta_{\varphi} \approx L/2h$ and defines the geometrical synchronism conditions of the delay line.

The threshold cases for the device described above show the operation of a crossed field tube with an annular beam created by the device geometry. In magnetic cutoff condition, the azimuthal magnetic field of the beam is not negligible and gets superimposed which is created by the passage of the cathode current. When total magnetic field (or the total current of the structure) exceeds the limiting magnetic field defined by the Buneman-Hartree condition, the system becomes on-cutoff and the synchronism conditions gets lost. The coaxial system in this case is a simple transmission line where the magnetic cutoff applied, know as magnetically insulated transmission line (MITL) [Mendel *et al*. (1983)], [Capua (1983)], no RF emission is then possible. Anode current breaks up at any moment (Fig. 2.4) [Lemke *et al*. (1997)] and the relation of current is given as:

$$I_{anode} = I_{cr} + I_{escape} \quad , \tag{2.58}$$

When the geometry of the line ensures synchronism between the electron drift velocity and the phase velocity of the RF waves, the energy exchanges takes place between RF waves and electron beam and then RF emission becomes possible. Thus, a fraction of the beam current in magnetic cutoff condition migrates towards the anode without passing by the load (Fig. 2.6) and characterizes a extra current known as escape (leakage) current I_{escape}, being accompanied by a local setting out of bunches of the electron beam, called ‘spokes’ current.

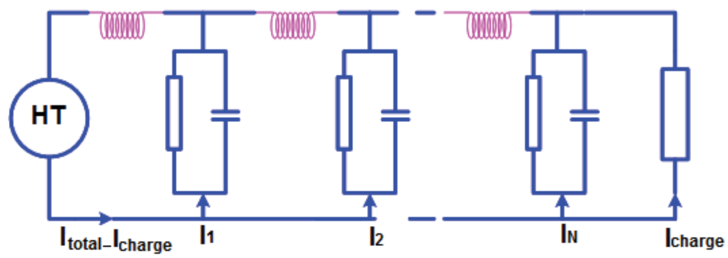


Fig. 2.6: Equivalent circuit for currents in MILO when leakage current I_{escape} of the partial currents I is the sum of all currents from I_1,… , I_N, characterizing the interaction mode of the beam with the RF field and I_{charge} is electric charging currents.

So from the equivalent circuits, the escape current is

$$I_{escape} = I_1 + I_2 + \cdots\cdots\cdots\cdots I_N = I_{anode} - I_{charge} \quad . \tag{2.59}$$

The electron bunches can be formed as the critical magnetic cutoff condition is reached, i.e., according to (2.59), as soon as, $I_{charge} = I_{cr}$. Thus, leakage current responsible for interaction in the structure and current flow through load to the anode directly using stub and form anode current which is necessary to evaluate and that corresponds to total current supported by the transmission line.

The drift electrons have a motion along equipotential surfaces. For this reason, total current of the structure is static and known as "parapotential current" [Humphries (1990)]. Such a current can be established, when static electric field responsible for emission of the cathode surface, such that magnetic field suitable for the beam to grip the electron flow [Creedon (1975)]. Now, for derivation of *parapotential current* using electric and magnetic field concept:

$$\vec{E} = -\vec{v} \times \vec{B} \quad , \tag{2.60}$$

Equation (2.60) is derived from Maxwell's equation and Faraday's law. Now taking divergence of both sides, we get:

$$\vec{\nabla} \cdot \vec{E} = -\vec{\nabla} \cdot (\vec{v} \times \vec{B}) \quad , \tag{2.61}$$

Using scalar product rule:

$$\vec{\nabla}\cdot\vec{E} = -\vec{B}\cdot(\vec{\nabla}\times\vec{v}) + \vec{v}\cdot(\vec{\nabla}\times\vec{B}) \quad , \tag{2.62}$$

where, $\vec{\nabla}\times\vec{B} = \vec{J} = \rho.\vec{v}$ which we get from Maxwell's equation, using this relation in (2.62), we get:

$$\vec{\nabla}\cdot\vec{E} = -\vec{B}\cdot(\vec{\nabla}\times\vec{v}) + \mu_0\rho\cdot\vec{v}^{\,2} \quad , \tag{2.63}$$

where, $\beta^2 = v^2/c^2$ and $c^2 = 1/\mu_0\varepsilon_0$, then (2.63) becomes:

$$\vec{\nabla}\cdot\vec{E} = -\vec{B}\cdot(\vec{\nabla}\times\vec{v}) + (\vec{\nabla}\cdot\vec{E})\beta^2 \quad , \tag{2.64}$$

From equation (2.64), we get

$$\vec{\nabla}\cdot\vec{E} = -\vec{B}\cdot(\vec{\nabla}\times\vec{v})/(1-\beta^2) \quad , \tag{2.65}$$

Using Poisson's equation in the cylindrical coordinate:

$$\vec{\nabla}\cdot\vec{E} = -\overrightarrow{\nabla^2}V \Rightarrow -\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) \quad , \tag{2.66}$$

In the same way, for the azimuthal coordinate φ independent components we get

$$\vec{\nabla}\times\vec{v} = -(\partial v_z/\partial r)\overrightarrow{u_\varphi} \quad , \tag{2.67}$$

Substituting (2.66) and (2.67) in (2.65):

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) = -\frac{1}{(1-\beta^2)}\frac{\partial v_z}{\partial r}B_\varphi \quad , \tag{2.68}$$

Using the relativistic mass factor, $\gamma_0\{r\} = 1/\sqrt{1-(\beta\{r\})^2}$, (2.67) and (2.68) can be expressed as follows:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) = -\gamma_0{}^2\frac{E_r}{v_z}\frac{\partial v_z}{\partial r} = \gamma_0{}^2\frac{\partial V}{\partial r}\frac{1}{v_z}\frac{\partial v_z}{\partial r} \quad , \tag{2.69}$$

where, $E_r = -\partial V/\partial r$, substituting in (2.69) gives:

$$\frac{\partial \beta_z}{\partial r} = \frac{\partial \beta_z}{\partial \gamma} \cdot \frac{\partial \gamma}{\partial r} = \frac{1}{\beta_z \gamma^3} \cdot \frac{\partial \gamma}{\partial r} \quad , \tag{2.70}$$

where , $\gamma_0\{r\} = (1/\sqrt{1-(\beta\{r\})^2}) \Rightarrow 1-(\beta\{r\})^2 = 1/(\gamma_0\{r\})^2$, (2.71)

Further, differentiating (2.71) with respect to γ_0 yields:

$$\frac{\partial \beta\{r\}}{\partial \gamma_0} = \frac{1}{\beta\{\gamma_0\{r\}\}(\gamma_0\{r\})^3} \quad , \tag{2.72}$$

Expressing (2.69) according to parameters of (2.70), it becomes:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) = \frac{\gamma_0^{\ 2}}{\beta_z}\frac{\partial \beta_z}{\partial r}\frac{\partial V}{\partial r} = \frac{1}{\beta_z^{\ 2}\gamma_0}\frac{\partial \gamma}{\partial r}\frac{\partial V}{\partial r} \quad , \tag{2.73}$$

Since, $(\beta\{r\})^2 = v_z^{\ 2}/c^2$, differentiating this equation with respect to r, and using (2.71) and (2.73), we get

$$(1/v_z)(\partial v_z/\partial r) = (1/\beta_z^{\ 2}\gamma_0^{\ 3})(\partial \gamma_0/\partial r) \ ,$$

Again, using law of conservation of energy:

$$[\gamma_0(r)-1]m_0c^2 - eV(r) = \text{constant} \Rightarrow \frac{\partial V}{\partial r} = \frac{m_0c^2}{e}\frac{\partial \gamma_0}{\partial r} ,$$

$$m_0c^2\frac{\partial \gamma_0\{r\}}{\partial r} = 0 = e\frac{\partial V\{r\}}{\partial r} \quad , \tag{2.74}$$

Simplification (2.74) becomes,

$$\frac{\partial V\{r\}}{\partial r} = \frac{m_0c^2}{e}\frac{\partial \gamma_0}{\partial r} \quad , \tag{2.75}$$

Substituting (2.75) in (2.73) yields:

$$\frac{m_0c^2}{e}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \gamma_0}{\partial r}\right) = \frac{1}{\beta_z^{\ 2}\gamma}\frac{\partial \gamma_0}{\partial r}\frac{\partial \gamma_0}{\partial r}\frac{m_0c^2}{e} \quad , \tag{2.76}$$

So (2.76) further simplified as:

$$\frac{1}{r}\frac{\partial \gamma_0}{\partial r}\left(r\frac{\partial \gamma_0}{\partial r}\right) = \left(\frac{\partial \gamma_0}{\partial r}\right)^2\frac{1}{\beta_z^{\ 2}\gamma_0} \quad , \tag{2.77}$$

Substituting $\beta^2 = (\gamma_0{}^2 - 1/\gamma_0)$ in (2.77), we get,

$$\frac{1}{r}\frac{\partial\gamma_0}{\partial r}\left(r\frac{\partial\gamma_0}{\partial r}\right) = \left(\frac{\partial\gamma_0}{\partial r}\right)^2 \frac{\gamma_0}{\gamma_0{}^2 - 1} \quad , \tag{2.78}$$

Equation (2.78) can be rewritten as:

$$\frac{1}{r}\frac{\partial\gamma_0}{\partial r} + \frac{r}{r}\frac{\partial^2\gamma_0}{\partial r^2} = \frac{\gamma_0}{\gamma_0{}^2 - 1}\frac{\partial\gamma_0}{\partial r}\frac{\partial\gamma_0}{\partial r} \quad , \tag{2.79}$$

(2.79) is further simplified as:

$$\frac{1}{r} + \frac{\partial^2\gamma_0/\partial r^2}{\partial\gamma_0/\partial r} = \frac{\gamma_0}{\gamma_0{}^2 - 1}\frac{\partial\gamma}{\partial r} \quad , \tag{2.80}$$

Integrating (2.80), we get

$$\ln r + \ln(\partial\gamma_0/\partial r) = \ln(\gamma_0{}^2 - 1)^{1/2} + C \quad , \tag{2.81}$$

Converting (2.81) in exponential form yields

$$\partial\gamma_0(\gamma_0{}^2 - 1)^{1/2} = A\sqrt{(\gamma_0{}^2 - 1)} = \exp(C) \quad , \tag{2.82}$$

(2.82) can be rewritten as:

$$\partial\gamma_0/(\gamma_0{}^2 - 1)^{1/2} = A(\partial r/r) \quad , \tag{2.83}$$

Using integration relationship $\int \frac{1}{\left(x^2 - a^2\right)^{1/2}}dx = \ln\left(\frac{x}{a} + (\frac{x^2}{a^2} - 1)^{1/2}\right)$ and integrating (2.83), we get,

$$\ln(r) = (1/A)\ln(\gamma_0 + (\gamma_0{}^2 - 1)^{1/2}) + A_0 \quad , \tag{2.84}$$

To determine the integration constants A and A_0, using

$$r = r_c, \gamma\{r_c\} = 1 \Rightarrow \ln\{r_c\} = A_0 \quad ,$$

and $r = r_a, \gamma(r_a) = \gamma_0 \Rightarrow \ln(r_a) = (1/A)\ln(\gamma_0 + (\gamma_0{}^2 - 1)^{1/2}) + \ln r_c$, gives $$\frac{1}{A}=\frac{\ln(r_a/r_c)}{\ln(\gamma_0+(\gamma_0^{\,2}-1)^{1/2})} \quad , \tag{2.85}$$

and $$I_P=-(2\pi\, r\, B_\varphi\, /\, \mu_0)=B_\varphi\, 2\pi r c^2 \varepsilon_0 \quad , \tag{2.86}$$

(2.86) can be rewritten as

$$I_P=(2\pi m_0 c^3 \varepsilon_0\, /\, e)\gamma_0\, A \quad , \tag{2.87}$$

Now, alternate form of the parapotential current or the total anode expression can be given as:

$$I_P=\frac{I_0\,\gamma_0\,\ln(\gamma_0+(\gamma_0^{\,2}-1)^{1/2}}{2\ \ln(r_a/r_c)} \quad . \tag{2.88}$$

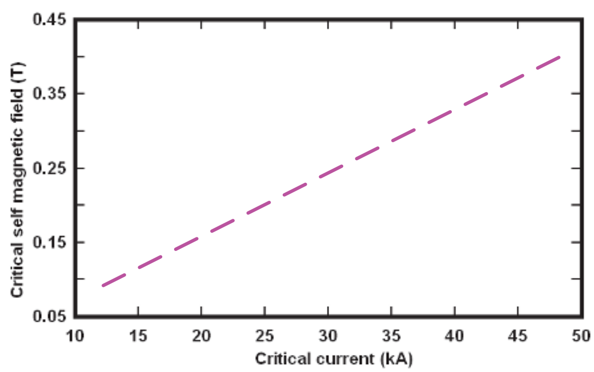


Fig .2.7: Critical current versus self-insulated critical magnetic field for r_c=25mm, r_a=40mm, L=13mm for various beam voltages.

For various beam voltages, the critical current (I_{cr}) varies linearly with critical self insulated critical magnetic field (B_c) for MILO device for the fixed value of r_c and r_a and L is shown in Fig.2.7.

2.4. Device Design Methodology of MILO

The following section comprising all equations pertaining to the design of conventional MILO has been explained.

2.4.1. Condition for Explosive Emission

As MILO is a high power device, high DC voltage is applied between cathode and anode, which causes explosive emission from the central cylindrical cathode surface due to expansion of cathode generated plasma, resulting in high beam current. Velvet cathode is often used for this device that acts as an explosive emitter. Materials implemented for explosive electron emission must have low electric field threshold for plasma initiation, emits uniformly, and has a low gap closure velocities.

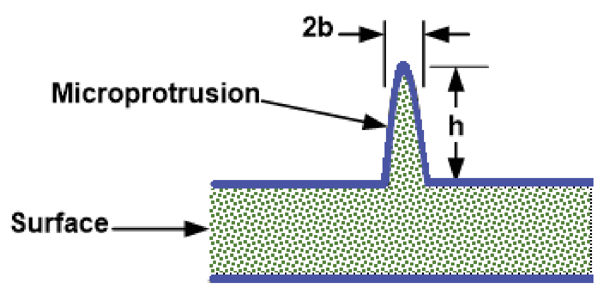


Fig. 2.8: Explosive emission from cathode surface [Miller (1998)].

With the application of a large voltage, the field lines converge on the tip of the microprotrusion, enhancing the localized fields, enabling electrons to tunnel through the decreased width of the potential barrier. The localized electric field at the tip can be expressed as:

$$E_{\text{local}} = f E \quad ,$$

In above expression, f is the field enhancement factor and E is the electric field in the absence of microprotrusion. The field enhancement factor can be written as [Miller (1998)]:

$$U_0 = \frac{\left(x^2-1\right)^{1.5}}{x\ln[x+(x^2-1)^{\frac{1}{2}}]-(x^2-1)^{\frac{1}{2}}} \quad , \qquad x=\frac{h}{b} \quad (x>1) \quad .$$

With the beginning of field emission, the flow of tunneling electrons increases the temperature of the microprotrusion due to Joule heating and eventually melts the whisker into a vapor state. On almost any cathode surface there exist microscopic surface protrusions (whiskers) which are typically on the order of 10^{-4} cm in height with a base radius of less than 10^{-5} cm and tip radius usually much smaller than the base radius. Whisker concentrations have ranged from 1 to 10^4 whiskers/cm^2 approximately.

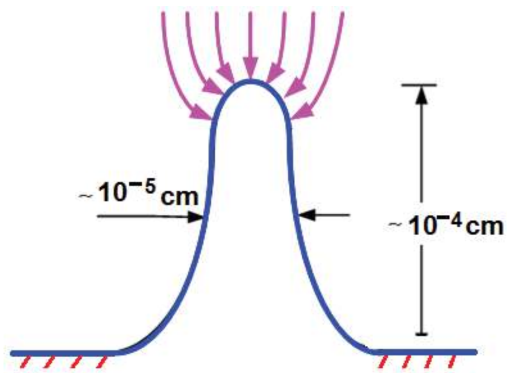


Fig. 2.9: Microscopic view of the local electric field enhancement at the tip of a whisker-like protrusion on the cathode surface [Miller (1998)].

Application of an electric field leads to the appearance of a large negative surface charge density on the top of the microprotrusions and consequently, to a significant enhancement (up to 10^4 times) of the local electric field. The enhanced electric field causes the intense electron field emission with current densities up to 10^6–10^7 A/cm^2 [Krasik *et al*. (2000)]. This leads to a fast increase in the temperature of the individual emitters due to Joule heating. When an electric current flows through a solid with finite conductivity, energy is converted to heat through resistive losses in the material. The heat is generated on the microscale, when the conduction electrons transfer energy to the conductor atoms through collisions. At this moment, the electron field emission transforms to thermionic electron emission. Further, because of the extremely high electron current density, explosion of the top of the micro-protrusions takes place. This explosion process causes the formation of a neutral cloud which is simultaneously ionized by electrons, leading to the formation of plasma. This plasma can provide an electron current density up to hundreds of MA/cm^2. However, the maximum electron current density is limited by the space-charge of the emitted electrons. These types of cathodes can be named ‘‘passive’’ cathodes because the plasma formation occurs under the application of the High Voltage (HV) pulse Therefore, it is extremely important that these cathodes provide uniform plasma simultaneously with the beginning of the accelerating pulse.

Maximum temperature of the micro-point before it becomes explosive:

The expression for the maximum value of T (at the tip of the emitter) is

$$T_{\max} = \frac{1}{2}\left(\frac{J^2 \rho}{k}\right) L^2 \quad ,$$

where, J= electron current density,

ρ = electrical resistivity,

k = thermal conductivity of material,

and L = length of micropoints.

Resistivity during explosive emission is calculated using expression for the plasma electrical conductivity and can be given by

$$\sigma_{dc} = \frac{n_e e^2}{m\nu} \quad ,$$

where, $\nu = T^{-3/2}$,

n_e = electron density,

T_e = electron temperature,

and ν = collision frequency which decreases with electron temperature.

For example, consider the case of tungsten, choosing intermediate values of ρ $=50$ X 10^{-6} Ω cm and k =0.25 cal/sec cm °C yields [Miller (1998)]:

$$T_{\max} = (2.5\times10^{-5})\mathrm{J}^2 L^2\ (^0\mathrm{C}) \quad .$$

where J is in A/cm^2 and L is in centimetres. For typical whisker lengths of 10^{-4} -10^{3} cm, the current density required to bring the tip of the whisker to the melting point (3400°C) is of the order of 10^{7}-10^{8} A/cm^2.

2.4.2. Condition for Critical Current

In high power device, high DC voltage is applied between cathode and anode, which causes explosive emission from the central cylindrical cathode surface, resulting in high relativistic current. The electron beam is initiated by explosive emission at the surface of a cylindrical velvet cathode. Due to self generated magnetic field, electrons in the beam can be insulated from the SWS [Cousin (2005)]. The self-azimuthal magnetic field generated due to the currents flowing in the system acts perpendicular to the electric fields to confine the electrons [Lawconnell and Neri (1989)]. When the currents (charging current, critical current, anode current) becomes appropriate, the self-magnetic field helps in confining electrons flow parallel to the cathode and greatly reduce the loss of electrons to the anode. When charging current becomes equal to the critical current, a magnetic cut-off condition establishes in the interaction (RF) periodic structure and sustains the EM fields present in the interaction region. Now, due to the presence of crossed electric (radial) and magnetic (azimuthal) fields in the device, emitted electrons drift toward the axial (z) direction of the structure, region between the cathode and the tip of metal coaxial discs with a velocity v_z. For the establishment of magnetic insulation condition, the electrons flow is generally relativistic and collisionless. Therefore, the relativistic Vlasov equation is used as a starting point for developing the physics of magnetic insulation. Writing z component of force equation in polar coordinate:

$$\frac{dp_z}{dt} = ev_r B_\theta \quad ,$$

Substituting relation for axial relativistic momentum and further rearranging, results in expression for critical current derived by [Dwivedi and Jain (2013)] and is written as:

$$I_{cr} = \frac{I_0\sqrt{\gamma_0^2 - 1}}{\ln(r_a / r_c)} \quad , \tag{2.89}$$

where $I_0 = \dfrac{4\pi m_0 c^3 \varepsilon_0}{e} = 17kA$ is the starting current.

Critical current (I_{cr}) is the minimum cathode current, needed to generate the self-magnetic field to create the insulation between the tip of disc and cathode. Expression for cut-off magnetic field can be written as:

$$B_c = \frac{\mu_0 I_{cr}}{2\pi r_c} = \frac{\mu_0 I_0 \sqrt{(\gamma_0)^2 - 1}}{2\pi r_c \ln\left(r_a / r_c\right)} \quad . \tag{2.90}$$

Critical magnetic field is minimum magnetic field required for magnetic insulation to occur. Conditions of magnetic cut-off and interaction in the device are controlled by the load or critical current. Further, the load current is decided by the space between collector and cathode. The space between the collector and cathode is optimized in order to allow a maximum current, in the slow wave structure, for better interaction. Condition for magnetic insulation is being described in next section.

2.4.3. Condition for Parapotential or Anode Current

Anode current should be more than the minimum critical current required by the interaction structure to create sustained self-magnetic insulation. When the anode current is less than the critical current, all the electrons emitted by the cathode reaching the anode causes electrical breakdown between anode-cathode gap and no insulation occurs between tip of discs and cathode. When the anode current is more than critical current, all the electrons are grazed from the anode surface towards cathode. When charging current become equal to critical current, electrons are confined between tip of discs and cathode due to self-azimuthal magnetic field. This is the basic principle for producing very high-power microwave pulses in a HPM device MILO. The flow of electrons between cathode and tip of the metal disc can be described using relation

$$\frac{dp}{dt} = -e(E + v \times B) \ , \tag{2.91}$$

Writing Maxwell's law of electrostatic and magnetostatic as:

$$\nabla.\mathrm{E} = \frac{\rho}{\varepsilon_0}, \qquad \nabla \times B = \mu_0 J \tag{2.92}$$

Poisson equation can be written as:

$$\nabla^2 V = -\frac{\rho}{\varepsilon_0} \quad ,$$

For the relativistic Brillouin flow pattern, the electron trajectories are straight line along equipotential surfaces, so that $dp/dt = 0$ thus, Equation (2.91) can be written as,

$$E = -v \times B$$

Taking divergence on both sides,

$$\nabla.E = \frac{-B.(\nabla \times v)}{(1-\beta^2)} \quad ,$$

Using cylindrical coordinates in above equation and after rearranging, above equation can be written as [Dwivedi and Jain (2013)]:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V\{r\}}{\partial r}\right) = \gamma^2 \frac{\partial V\{r\}}{\partial r}\left(\frac{1}{v_z}\frac{\partial v_z}{\partial r}\right) \quad , \tag{2.93}$$

Using law of conservation of energy, $(\gamma\{r\}-1)mc^2 = eV\{r\}$, one can write:

$$\frac{\partial V\{r\}}{\partial r} = \frac{mc^2}{e}\frac{\partial \gamma\{r\}}{\partial r} \quad , \tag{2.94}$$

Substituting Equation (2.94) in (2.93), one gets differential equation that descried parapotential flow, same as derived by [Lemke (1989)] as:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \gamma}{\partial r}\right) = \frac{\gamma}{\gamma^2 - 1}\left(\frac{\partial \gamma}{\partial r}\right)^2 \quad , \tag{2.95}$$

Above equation describe differential equation describing magnetic insulation and on solving this we get [Creedon (1975)]:

$$\ln(r) = C_1 \ln\left[\gamma + (\gamma^2 - 1)^{\frac{1}{2}}\right] + C_2 \qquad r_c \le r \le r_m \quad , \tag{2.96}$$

where γ_m is the value of γ at the insulated flow pattern, then the boundary condition for Equation (2.96) can be written in terms of r. At $r = r_c$, $\gamma(r_c) = 1$ and at $r = r_m$, $\gamma = \gamma_m$. Satisfying these boundary conditions in Equation (2.96), $C_2 = \ln(r_c)$ and

$$C_1 = \frac{\ln(r_m) - \ln(r_c)}{\ln(\gamma_m + (\gamma_m{}^2 - 1))} \quad , \tag{2.97}$$

To relate potential V_m at r_m to anode potential V_0 at r_0, it is necessary to solve Laplace's equation in vacuum region between r_m and r_0. Set $\rho = 0$ results in $\nabla^2 V = 0$ then $\nabla.E = 0$ and making transformation from V to γ and from r to ln(r) given by $\gamma = 1 + \frac{eV}{m_0 c^2}$ and $\frac{d(\ln(r))}{dr} = \frac{1}{r}$, then results in $\frac{d^2\gamma}{d(\ln(r))^2} = 0$ with solution ,

$$\gamma = \frac{\gamma_0\left(\ln(r) - \ln(r_m)\right) - \gamma_m(\ln(r) - \ln(r_0))}{\ln(r_0) - \ln(r_m)} \quad , \qquad \ln(r_m) \le \ln(r) \le \ln(r_0) \ , \tag{2.98}$$

where, $\ln(r_0)$ and γ_0 are the value of ln(r) and γ at the anode.

Differentiating Equation (2.96) and Equation (2.98) setting $\gamma = \gamma_m$ and equating the results gives:

$$C_1 = \frac{\ln(r_0) - \ln(r_m)}{\gamma_0 - \gamma_m}\left(\gamma_m^2 - 1\right)^{\frac{1}{2}} \quad , \tag{2.99}$$

$\ln(r_m)$ can be expressed as a function of γ_0, γ_m and $\ln(r_c)$ by equating expression for C_1 given by Equations (2.97) and (2.99) and then calculating value of $\ln(r_m)$ and then substituting in Equation (2.99), we get:

$$C_1 = [\ln(r_0) - \ln(r_c)] / \ln[\gamma_m + (\gamma_m^2 - 1)]^{1/2} + (\gamma_0 - \gamma_m)(\gamma_m^2 - 1)^{-1/2} \quad , \tag{2.100}$$

Relation between the azimuthal magnetic field and current flowing is written as:

$$B_\theta = \frac{\mu_0 I\{r\}}{2\pi r}, \quad \text{also } E_r = v_z B_\theta \quad ,$$

On substituting above values, $I\{r\} = \dfrac{E_r(2\pi r)}{\mu_0 v_z}$, making transformation from v to γ and v_z by $-\beta c$ gives,

$$I\{r\} = I_\alpha \frac{1}{\beta} \frac{d\gamma}{d(\ln(r))} \quad , \tag{2.101}$$

where, $$I_\alpha = \frac{2\pi}{\mu_0 c} \frac{m_0 c^2}{e} = 8500A \quad ,$$

Differentiating Equation (2.96), we get

$$\frac{d\gamma}{d(\ln(r))} = \frac{\left(\gamma^2 - 1\right)^{1/2}}{C_1} = \frac{\beta\gamma}{C_1} \quad , \tag{2.102}$$

Substituting Equation (2.102) in Equation (2.101), to get:

$$I\{r\} = I_\alpha \frac{\gamma}{C_1} \quad , \tag{2.103}$$

Total current flowing is given by putting $\gamma = \gamma_m$ in above equation and combine with Equation (2.100),

$$I_0 = I_\alpha g \gamma_m \left[\ln(\gamma_m + (\gamma_m^2 - 1)^{1/2} + \frac{\gamma_0 - \gamma_m}{\left(\gamma_m^2 - 1\right)^{\frac{1}{2}}} \right] \quad , \tag{2.104}$$

where, $$g = \frac{1}{\ln(r_a / r_c)} \quad ,$$

Substituting $\gamma = 1$ in Equation (2.103), it can be seen that part of total current must flow on the cathode surface or in the cathode plasma. Thus, charging current is given by Equation (2.104):

$$I_c = \frac{I_0}{\gamma_m} \quad . \tag{2.105}$$

Part of the total current I_0 that flows as an electron beam in the diode is $I_e = I_0 - I_c$. Combining Equations (2.104) and (2.105), it can be shown that this beam current is maximum when $\gamma_m = \gamma_0$. Corresponding current is known as saturated parapotential current and is calculated by setting $\gamma_m = \gamma_0$ in Equation (2.104) to get:

$$I_p = I_\alpha g \gamma_0 \ln \left[\gamma_0 + (\gamma_0^2 - 1)^{\frac{1}{2}} \right] \quad . \tag{2.106}$$

From Equations (2.104) and (2.105), it can be shown that (for a fixed value of γ_0 and g), I_c is minimum when $I_0 = I_p$.

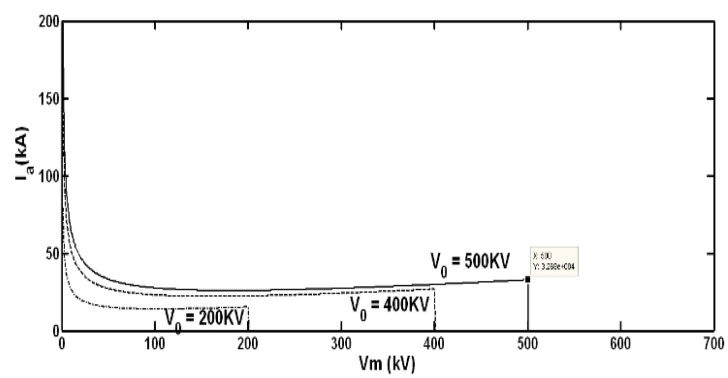


Fig. 2.10: Variation of parapotential current I_p versus V_m (maximum developed potential). Here $V_m = V_0$ and $I_0 = I_p$.

2.4.4. Cathode Design

Static electric field in vacuum centered coaxial line is used to calculate the device operating voltage. Gauss theorem gives static electric field (Fig. 2.4):

$$\overrightarrow{E(r)} = -\frac{Q}{2\pi L_{SE}\varepsilon_0 r}\overrightarrow{u_r} \quad , \tag{2.107}$$

Structure potential V_0 ($E = -\nabla V$) related to charge Q on the surface of drivers,

$$\int_{-V_0}^{0} dV = -\int_{r_c}^{r_a} E_r dr \Rightarrow V_0 = \frac{Q}{2\pi L_{SE}\varepsilon_0}\int_{r_c}^{r_a}\frac{q}{r}dr = \frac{Q}{2\pi L_{SE}\varepsilon_0}\ln\left(\frac{r_a}{r_c}\right) \tag{2.108}$$

By expressing the charge Q for cylindrical capacitor formed according to voltage in the structure, as:

$$Q = C(V_a - V) = CV_0 \Rightarrow 2\pi L_{SE}\varepsilon_0 / \ln(r_a/r_c) \quad , \tag{2.109}$$

Static electric field is related to (2.109) for MILO structure as:

$$E_r = \frac{V_0}{r\ln(r_a / r_c)} \quad , \tag{2.110}$$

Potential is applied at the internal electrodes of the coaxial structure, and anode is kept at the ground potential. Electric field, (2.110), is maximum on the cathode surface, decrease as $1/r$ towards the anode and follows impedance variations of the coaxial line.

The length of the cathode, L_{SE} is arrived from the perveance of the device;

$\mu = I_C / V_o^{3/2}$, given as [Parker *et al.* (1974)],

$$L_{SE} = 6.82\times10^4 \mu r_i \left(\ln\left(r_i / r_c\right)\right)^2 \quad . \tag{2.111}$$

where, μ is the diode perveance that depends on geometrical parameters, I_c is the cathode current and V_o is the beam voltage or cathode voltage.

2.5. Condition of Relativistic Brillouin Flow (RBF) and Magnetic Insulation

Davidson and Tsang (1986) describe the equilibrium properties of relativistic electron flow in planar diode configuration. This section describes the equilibrium properties of relativistic electron flow along the insulated sheath for cylindrical diode configuration of MILO. During MILO operation, when its charging current becomes equal to the critical current, a magnetic cutoff condition establishes in the RF periodic structure and sustains the EM fields present in the interaction region. This section mainly explains the condition for magnetic insulation in MILO. This condition is being derived here taking into account formation of plasma on the cathode surface due to explosive emission that creates high density electron beam. Application of high voltage DC pulse between cathode and anode causes electrons emission from the cylindrical cathode surface. The electrons are field emitted from the cold cathode by the process termed as explosive electron emission. This type of emission results in the formation of non-neutral plasma sheath which is a good source of electron emitter [Hilsabeck and O'Neil (2001)]. Electro-magneto dynamics equation and momentum conservation lead to complete description of the magnetically insulated flow. During analysis it has been assumed that equilibrium is represented by relativistic Laminar flow in crossed electric and magnetic fields, with space charge limited emission.

The equilibrium configuration is shown in Figure 2.5. In the present section, conditions for the RBF flow are studied, considering the ($E \times B$) drift of a relativistic electron beam in the region between the cathode and anode. Electro-magneto dynamics equation and momentum conservation lead to complete description of the magnetically insulated flow.

A radial electric field E_r gets established due to applied DC potential between explosive emissive cathode and anode. Gauss's Law in cylindrical coordinate is written as [Levy (1965)]:

$$\frac{1}{r}(\frac{\partial}{\partial r}(rE)) = \frac{en_e\{r\}}{\varepsilon_0} \quad ,$$

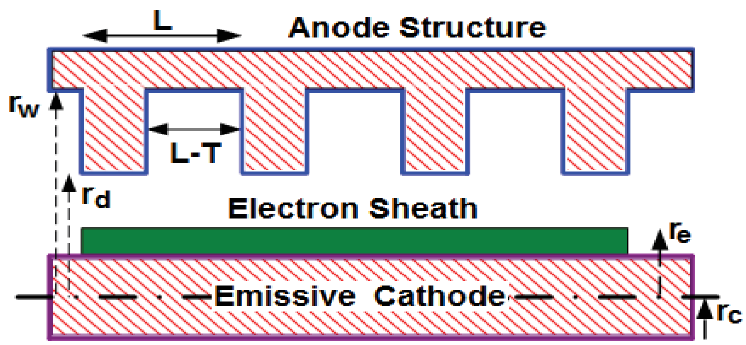


Fig.2.11: Schematic of RF interaction structure.

where $n_e\{r\}$ is the electron density in the radial direction. Considering property of electric field at cylindrical charged surface and space charge limited emission ($E_r = 0$ $(r=r_c)$), above Equation can be written as:

$$\frac{\partial E_r}{\partial r} = \frac{en_e\{r\}}{\varepsilon_0} \quad , \qquad (2.112)$$

Differential form of Ampere's law in cylindrical coordinates is written as,

$$\frac{1}{r}(\frac{\partial}{\partial r}(rB_\theta)) = \mu_0 n_0 e v_z \quad ,$$

Initially during electron emission, no azimuthal component is present, $B_\theta / r = 0$ in above expression,

$$\frac{\partial B_\theta}{\partial r} = \mu_0 e n_0 v_z \quad ,$$

Substituting $J = \rho v$ and $\mu_0 = 1/(\varepsilon_0 c^2)$ in the above equation, we get:

$$\frac{\partial B_\theta}{\partial r} = en_e\{r\}v_z / (\varepsilon_0 c^2) \quad , \tag{2.113}$$

This self magnetic field redirects electrons back towards the cathode due to $(v \times B)$ force. Thus, electrons gets drift in z direction with the velocity v_z. Under the influence of electromagnetic forces, the momentum equation can be written as

$$\dot{p}_r = eE_r\{r\} - ev_z B_\theta\{r\} \quad , \tag{2.114}$$

Electrons present on the plasma sheath which results due to explosive emission from the cathode surface evolve or rotate at a velocity $\dot{\theta}\{r\}$ [Tsang and Davison (1985)]. Relativistic electron beam (charging current) results in the self azimuthal magnetic field at which magnetic insulation takes place as a specific condition. For magnetic insulation condition to occur, the centrifugal force must be balanced by the Lorentz force such that,

$$\frac{e}{\gamma}[E_r + r\dot{\theta}B_\theta] = mr\dot{\theta}^2 \quad ,$$

Introducing relativistic cyclotron frequency, $\omega_c = eB_\theta / \gamma m_e$ and relativistic plasma frequency, $\omega_p = ne^2 / \gamma m_e \varepsilon_0$, the angular speed satisfied quadratic equation:

$$\dot{\theta}^2 + \omega_c \dot{\theta} + qE_r / m_e r = 0 \quad ,$$

Here, $q = \omega_p^2 / \omega_c^2$, represents parameter that limits instability along the magnetically insulated sheath. Existence of this condition results in drift of the each electron along z direction at drift velocity $v_z = E_r / B_\theta$. Thus, insulated sheath follows condition of RBF equilibrium.

For the electrons streaming parallel to the z-axis, radial momentum will be zero and drift velocity of the relativistic electron beam can written as $v_z = E_r / B_\theta$ [Dwivedi and Jain (2013)]. Substituting value of v_z and $n_e\{r\}$ from Equation (2.112) into Equation (2.113), then

$$c^2 \frac{\partial B_\theta^2\{r\}}{\partial r} = \frac{\partial E_r^2\{r\}}{\partial r} \quad , \tag{2.115(a)}$$

Integrating above Equation, within limits of the space charge electric field and the self magnetic field B_θ generated by the beam, one gets:

$$B_\theta^2\{r\} - \frac{E_r^2\{r\}}{c^2} = \text{constant} = w \quad , \tag{2.115(b)}$$

Under RBF condition, the canonical momentum and total energy across the electron sheath become zero [Lemke (1989)]. From above equation (2.115(b)), $c^2 B_\theta^2\{r\} - E_r^2\{r\}$ represents equilibrium condition establishes during the magnetic insulation and is equal to a constant quantity, imposing the boundary condition $r_c \le r \le r_e$. Thus, the drift velocity will modify the relativistic mass factor γ along the electron sheath:

$$\gamma\{r\} = \frac{1}{\sqrt{1 - v_z^2 / c^2}} = \frac{1}{\sqrt{1 - E_r^2\{r\} / c^2 B_\theta^2\{r\}}} \quad , \tag{2.116}$$

Substituting Equation (2.115(b)) in Equation (2.22),

$$B_\theta\{r\} / \gamma\{r\} = \sqrt{w} = \text{constant} \quad , \tag{2.117}$$

Equations (2.115) and (2.117) define the stable equilibrium condition for the relativistic electron flow (RBF) in crossed electric and magnetic field. Multiplying both side of Equation (2.115(a)) by r^2, Equation can be rewritten as,

$$[r^2 (E^2 - (cB)^2)]' = 0 \quad ,$$

Integrating above Equation from cathode radius r_c to some arbitrary r_e (radius of insulated electron sheath) [Lawconnell and Neri (1990)],

$$r_e^2 [E_e^2 - (cB_m)^2] - r_c^2 [E_c^2 - (cB_c)^2] = 0 \quad , \tag{2.118}$$

Considering space charge limited emission condition (E_c = 0) and writing above equation in terms of anode and cathode currents, considering $B_m = \frac{\mu_0 I_a}{2\pi r_e}$ and can be written as [Lawconnell and Neri (1990)],

$$E_e = -\frac{c\mu_0}{2\pi r_e}(I_a^2 - I_c^2)^{\frac{1}{2}} \quad , \tag{2.119}$$

E_e represents electric field developed at the electron sheath. In above equation, B_m represents magnetic field along the electron sheath. During RBF, total energy of the electrons remains conserved along the electron sheath [Lemke (1989)],

$$(\gamma\{r\} - 1)\, m_0 c^2 - e\phi_0\{r\} = \text{constant} \quad , \tag{2.120}$$

Now, imposing the boundary condition, at the cathode surface, γ $(r=r_c)$ = 1 and $\phi_0(\mathrm{r} = r_c) = -V_0$, above Equation can be written, after solving constant value, after rearranging,

$$(\gamma\{r\} - 1)\, m_0 c^2 - e\phi_0\{r\} = eV_0 \quad , \tag{2.121}$$

Total energy of electrons is expressed using Equation (2.121). Differentiating the above expression with respect to r :

$$\frac{\partial \gamma}{\partial r} = \frac{e}{m_0 c^2}\frac{\partial \phi_0\{r\}}{\partial r} \quad ,$$

Substituting $\partial\gamma/\partial r$ from (2.116) and $\partial\phi_0/\partial r = -E_r$ in above equation:

$$\gamma^3\{r\}\frac{E_r\{r\}}{B_\theta\{r\}}\frac{\partial}{\partial r}\left[\frac{E_r\{r\}}{B_\theta\{r\}}\right] = -\frac{e}{m}E_r\{r\} \quad ,$$

Expanding the above using equations (2.113) and rearranging,

$$\left[\frac{eB_\theta\{r\}}{m_0\gamma\{r\}}\right]^2 = \frac{e^2 n_e\{r\}}{m_0\varepsilon_0\gamma\{r\}} \Rightarrow \omega_c = \omega_p \quad , \tag{2.122}$$

In above Equation (2.122), ω_c and ω_p represent relativistic cyclotron and plasma frequencies, respectively [Davidson and Tsang (1984)]. Equation (2.122) represents condition of relativistic Brillouin flow ($r_c \le r \le r_e$) for achieving magnetic insulation condition due to explosive emission process. It is under the action of above defined electromagnetic fields that magnetic cut off is reached inside the structure. Imposing condition (2.117) into expression (2.122), then rearranging, results

$$n_e\{r\} / \gamma\{r\} = (\varepsilon_0/\mathrm{m}_0)w = \text{constant} \quad , \qquad \text{for } r_c \le r \le r_e \qquad (2.123)$$

To ensure the formation of magnetically insulated sheath and fulfilling the RBF flow condition, the ratio of electron density to azimuthal magnetic field must also be equal to a constant and can be explained using Equations (2.117) and (2.123):

$$n_e\{r\} / B_\theta\{r\} = \text{constant} = (\varepsilon_0 / m_0)\sqrt{w} \quad . \qquad (2.124)$$

Now, differentiating (2.113) with respect to radius r and then substituting in (2.112) using property (2.124), yields [Tsang *et al.* (1985), Davidson *et al.* (1984)]:

$$\frac{\partial^2 B_\theta\{r\}}{\partial r^2} - s^2 B_\theta\{r\} = 0 \quad , \qquad (2.125)$$

where,

$$s^2 = \frac{\omega_p^4\{r_e\}}{\omega_c^2\{r_e\}\, c^2} = \frac{\omega_D^2\{r_e\}}{c^2}; \quad \omega_D\{\mathrm{r}\} = \frac{\omega_p^2\{r\}}{\omega_c\{r\}} \quad . \qquad (2.126)$$

Here, ω_D represents Diocotron frequency and s represents electromagnetic field strength along the magnetically insulated sheath. Solution of Equation (2.125):

$$B_\theta\{r\} = M\cosh(sr) + N\sinh(sr) \ ,$$

where, M and N are the integration constants and can be computed on applying boundary conditions at $r = r_c$, the above expression becomes [Tsang and Davidson (1986)]:

$$B_\theta\{r\} = B_c \frac{\cosh(sr)}{\cosh(sr_e)} \quad , \qquad r_c \le r \le r_e \qquad (2.127)$$

and $$B_\theta\{r\} = B_c \quad . \quad r_e \le r \le r_d \tag{2.128}$$

In this regard, using equilibrium properties, above magnetic field expressions (2.127) and (2.128) emphasizes the self-azimuthal magnetic field in the two structure regions. B_c represents the critical self magnetic field at the anode due to which electrons just graze the anode surface. During RBF, expression for electron density can also be calculated using equations (2.124) and (2.127), as:

$$n_e\{r\} = n_e^0 \frac{\cosh(sr)}{\cosh(sr_e)} \quad , \tag{2.129}$$

where n_e^0 is the electron density at boundary $r = r_e$. Considering Equations (2.117) and (2.125), the expression for the relativistic factor can be written as,

$$\gamma = \cosh(sr) \quad , \tag{2.130(a)}$$

Substituting Equation (2.129) in (2.112) after integrating results in expression for electric field and can be written as:

$$E_r(r) = -\frac{en_e^0}{\varepsilon_0 s}\frac{\sinh(sr)}{\cosh(sr_e)} = -cB_c\frac{\sinh(sr)}{\cosh(sr_e)} \quad . \tag{2.130(b)}$$

Velocity for drift electrons is given by: $v_z(r) = c\tanh(sr)$.

Considering RBF or insulation condition (2.122), expression for the self azimuthal magnetic field can be written as:

$$B_\theta\{r\} = [n_e\{r\}m_e / \varepsilon_0]^{1/2} \quad .$$

Substituting above expression in Equation (2.127) and substituting value of critical magnetic field $B_c = \mu_0 I_{cr} / 2\pi r_c$, further recasting results in expression for critical current:

$$I_{cr} = \left[\frac{n_e\{r\}m_e}{\varepsilon_0}\right]^{1/2}\left[\frac{\cosh(sr_e)}{\cosh(sr)}\right]2\pi r_c \varepsilon_0 c^2) \quad . \tag{2.131}$$

Expression (2.131) represents expression for critical current considering explosive emission from the cathode surface.

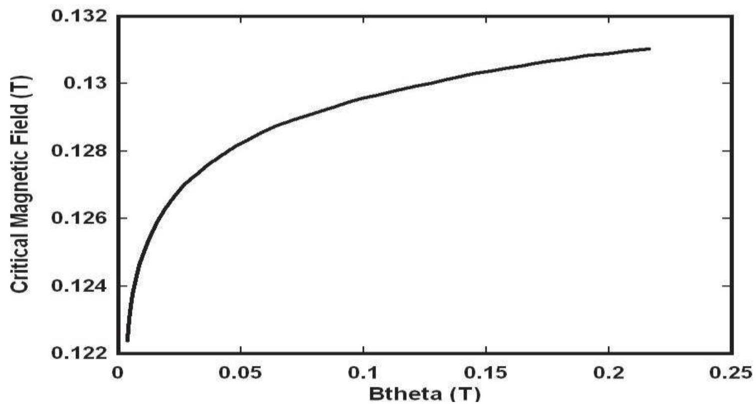


Fig. 2.12: Variation of self-azimuthal magnetic field with critical magnetic field.

In above figure, B_θ is azimuthal self magnetic field at the anode during magnetic insulation due to which electrons just graze the anode surface. It is inferred from figure, at about when critical magnetic field become equal to B_{theta} (0.13T), magnetic insulation occurs and electrons drift in axial direction.

2.5.1. Hull cut-off and Buneman-Hartree Condition

Hull cut-off represents the magnetic insulation condition, the minimum magnetic field required to prevent the emitted electrons from reaching the anode and in MILO this condition occurs when charging current becomes equal to critical current. Substituting expression for relativistic factor from Equation (2.130) in (2.121), resulting potential of the structure is defined as:

$$[\cosh\{sr\}-1] = \frac{e}{m_0 c^2}(V_0 + \phi_0\{r\}) \quad , \quad 0 \le r \le r_e \quad . \tag{2.132}$$

Differentiating Equation (2.132) with respect to r,

$$\frac{\partial \phi_0\{r\}}{\partial r} = \frac{s m_0 c^2}{e}\sinh\{sr\} \quad . \tag{2.133}$$

At the electron sheath, potential can be expressed as,

$$\phi_0\{r\} - \phi_0\{r = r_e\} = -E_r\{r_e\}(r - r_e) \quad . \tag{2.134}$$

Substituting Equation (2.133) in (2.134) , substituting $E_r = d\phi / dr$

$$\phi_0\{r\} - \phi_0\{r = r_e\} = -\frac{m_0 c^2}{e} s(r - r_e)\sinh\{sr_e\} \quad . \tag{2.135}$$

Differentiating above with respect to *r*,

$$\frac{\partial \phi_0(r)}{\partial r} = \frac{s m_0 c^2}{e} \sinh(sr) \quad .$$

With respect to above defined potential, radial electric field at insulated sheath due to space charge effect [Lemke (1989)]:

$$E_r = -\frac{s m_0 c^2}{e} \sinh(sr) \quad .$$

Considering continuity of electric field between different regions, potential in the region $r_e \le r \le r_d$, can be obtained using Equations (2.132) and (2.135),

$$\frac{e}{m_0 c^2}(V_0 + \phi_0\{r\}) = [\cosh\{sr_e\} - 1] + s(r - r_e)\sinh\{sr_e\} \quad , \tag{2.136}$$

It is desirable to couple B_c to the initial magnetic field strength $B_\theta(r)$ developed between anode and cathode prior to the formation of Brillouin layer. This can be done by taking into account conservation of magnetic flux theorem, dividing structure into two regions, i) $r_c \le r \le r_e$ and ii) $r_e \le r \le r_d$,

$$\int_{r_c}^{r_d} B_\theta dr = \int_{r_c}^{r_e} B_c \frac{\cosh\{sr\}}{\cosh\{sr_e\}} dr + \int_{r_e}^{r_d} B_c dr \quad . \tag{2.137}$$

Solving above integral for these two regions, as shown in Figure 2.5, and substituting $B_c = B_0 \cosh\{sr_e\}$ Equation (2.137) can be rewritten as,

$$\frac{eB_0}{m_0 cs}[\sinh\{sr_e\} + s(r_d - r_e)\cosh\{sr_e\}] = \frac{e}{m_0 c} B_\theta d \quad . \tag{2.138}$$

In above equation, B_0 represents magnetic field developed due to the flow of parapotential or anode current. Drift electrons have motion along equipotential surface, thus total current is known as parapotential or anode current which is a function of the structure geometry and applied potential.

$$I_p = \frac{I_0}{2\ln(r_a / r_c)} \gamma_0 \ln[\gamma_0 + (\gamma_0^2 - 1)^{1/2}] \quad .$$

During RBF flow, $\omega_p = \omega_c$ and $\gamma\ (r = r_c) = 1$, $s = \omega_p / c$, Equation (2.138) can be written as,

$$\frac{e B_\theta r_d}{m_0 c} = \sinh\{s r_e\} + s(r_d - r_e)\cosh\{s r_e\} \quad . \tag{2.139}$$

Considering, r_e tends toward r_d , one obtains, potential in region $r_e \le r \le r_d$,

$$\left(\frac{e B_\theta r_d}{m_0 c}\right)^2 = \cosh^2\{s r_d\} - 1 \quad . \tag{2.140}$$

When $r_e = r_d$ and $\phi_0\{r_d\} = 0$, Equation (2.37) can be written as,

$$\frac{eV_0}{m_0 c^2} = \cosh\{s r_d\} - 1 \quad . \tag{2.141}$$

Substituting Equation (2.140) in (2.141), potential of the structure can be expressed as:

$$\frac{eV_H}{m_0 c^2} = \left[1 + \left(\frac{e B_\theta r_d}{m_0 c}\right)^2\right]^{1/2} - 1 \quad . \tag{2.142}$$

Above expression represents cut-off condition for MILO. Potential corresponding to this case is called Hull cut-off voltage. Equation (2.142) linked potential of the structure to the resultant total self magnetic field and defines Hull cut off voltage at specific condition of the magnetic insulation. When magnetic field is above cut off condition, there is an empty space between tip of the discs and electron sheath, where Equation (2.136) satisfies for the potential. At $r = r_d$, $\varphi(r_d) = 0$, Equation (2.136) can be rewritten as:

$$\frac{eV_0}{m_0 c^2} = [\cosh\{s r_e\} - 1] + s(r_d - r_e)\sinh\{s r_e\} \quad . \tag{2.143}$$

Combining equation (2.48) with equation (2.44), one obtains:

$$\frac{eV_0}{m_0c^2} = [\cosh\{sr_e\} - 1] + \frac{eB_\theta r_d}{m_0c^2} c\tanh\{sr_e\} - \frac{[\cosh^2\{sr_e\} - 1]}{\cosh\{sr_e\}} \quad . \tag{2.144}$$

Since $\gamma_0 = \cosh\{sr_e\}$, we can easily verify that $v_z = c\tanh\{sr_e\}$. Also require that $v_z = \beta_p c$ at the threshold voltage V_{BH}, we get the expression for the new potential when synchronism between the phase velocity of desired mode of the structure and the velocity of the drift electrons exist, on rearranging this equation:

$$\frac{eV_{BH}}{m_0c^2} = \frac{eB_\theta r_d}{m_0c^2} v_\phi - \left[1 - \left(1 - \frac{v_\phi^2}{c^2}\right)^{1/2}\right] \quad . \tag{2.145}$$

Above expression defines the limiting condition at which the beam interacts with the RF interaction structure. When the EM waves in the RF periodic structure are sufficient slowed down to allow synchronism with velocity of the drift electrons:

$$\frac{e}{m_0c^2} V_{BH} \approx \frac{e}{m_0c^2} B_\theta d v_\phi \quad . \tag{2.146}$$

Expression defines the limiting condition for which the beam interacts with the anode structure and same as expressed by [Lau *et al.* (1987)]. When the electromagnetic waves in the anode structure called periodic structure are sufficiently slowed down to allow synchronism with the velocity of the drift electrons.

Thus potential grows in a linear manner with the magnetic field in the structure. Condition given in Equation (2.146) represents Buneman-Hartree condition. For steady Brillouin flow, inequality $V < V_H$ should exist to assure magnetic insulation of electron flow from contact with anode at $r = r_e$. Condition $V > V_{BH}$ should sustain for interaction of outer electrons with the wave field for specified voltage. To identify the region of operation and oscillation condition, expressions for Hull cut-off voltage as well as Buneman-Hartree condition for beam wave synchronism are derived above. Buneman-Hartree voltage is the voltage at which oscillations should start provided at the same time magnetic field is sufficiently large so that undistorted space charge does not extend to the anode. For voltages less than Buneman-Hartree threshold, the MILO will be non-resonant and it will not radiate microwave. The region of operation can be emphasized between the two conditions. Efficient operation generally occurs near the B-H threshold. The two conditions are proportional and their ratio is a function of only the voltage and phase velocity of the resonant wave.

From Fig. 2.13, it could be concluded that for resonant condition in MILO, voltage must be greater than Buneman-Hartree threshold (V_{BH}) and less than that of Hull cut- off voltage (V_H). During RBF flow, drift electrons have motion along equipotential surface, thus total current is known as parapotential or anode current. Existence of various operating currents that are responsible for magnetic insulation and beam wave interaction mechanism at specified voltage can be determined from above Fig. 2.14.

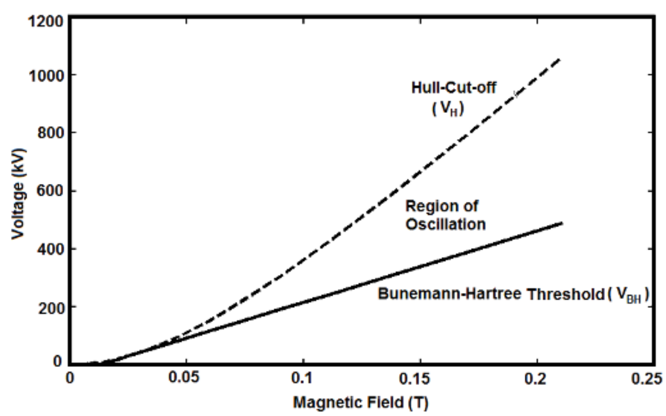


Fig. 2.13: Graph showing region of oscillation in MILO.

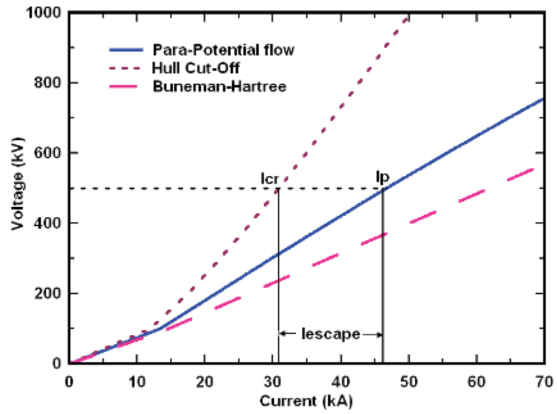


Fig. 2.14: Operating voltage versus current

2.6. Beam-Wave Interaction Mechanism in MILO

Beam-wave interaction process will be described in details in Chapter-3. Here, only the fundamental procedure that is occurred during interaction, the necessary steps, magnetic cutoff, Diocotron instability and mode of oscillation are discussed.

2.6.1. Magnetic Cut-off

When the charging current reaches to the critical current value, electrons do not reach to anode structure. It is said that, it is isolated by magnetic field suitable for beam forming and the phenomenon is named as magnetic cutoff.

When, $I_{charge} = I_{cr}$ electron do not reaches toward the anode structure.

To study the effect of length of covering of collector on cathode L_{col} compared with the minimum load length L_{cmin} where beam do not insulates under the collector. For that, we calculate relativistic charging current in a general form. Using Poisson's equation, in cylindrical coordinate, we can write:

$$\nabla V = -\frac{\rho\{r\}}{\varepsilon_0} = \frac{j\{r\}}{v\{r\}\varepsilon_0} = \frac{I_{charge}}{2\pi\varepsilon_0 r L_c v\{r\}} \Rightarrow \frac{d}{dr}\left(r\frac{dV\{r\}}{dr}\right) = \frac{I_{charge}}{2\pi\varepsilon_0 r L_c v\{r\}} \quad , \qquad (2.147)$$

where $j\{r\}$ is current density under the collector and I_{charge} is associated charging current, $v\{r\}$ is radial velocity of modulated electrons and L_c is length of the load.

Current density in terms of charge density and velocity can be written as:

$$j = \rho\{r\}\, v\{r\} \qquad , \tag{2.148}$$

Using law of conservation of energy, one can write,

$$eV\{r\} = m_0 c^2 (\gamma_0\{r\} - 1) \qquad , \tag{2.149}$$

Differentiating (2.149) we get:

$$e\frac{d}{dr}\left(\frac{dV\{r\}}{dr}\right) + 0 = m_0 c^2 \frac{d}{dr}\left(\frac{d\gamma_0\{r\}}{dr}\right) \qquad , \tag{2.150}$$

Substituting (2.147) in (2.150), we get:

$$\frac{e}{m_0 c^2}\left(\frac{I_{charge}}{2\pi\varepsilon_0 r L_c v\{r\}}\right) = \frac{d}{dr}\left(\frac{d\gamma_0\{r\}}{dr}\right) \qquad , \tag{2.151}$$

Now, $\gamma_0 = (1/\sqrt{1-(v/c)^2}) \Rightarrow v = c\sqrt{(\gamma_0^2 - 1/\gamma_0^2)} \qquad ,$ (2.152)

Substituting (2.152) in (2.147), we get:

$$\frac{d}{dr}\left(r\frac{d\gamma_0\{r\}}{dr}\right) = \frac{e}{m_0 c^2}\frac{I_{charge}}{2\pi\varepsilon_0 L_c}\left(\frac{\gamma_0(r\}}{c\sqrt{\gamma_0\{r\}^2 - 1}}\right) \qquad , \tag{2.153}$$

Rearranging equation (2.153) as:

$$\frac{d}{dr}\left(r\frac{d\gamma_0\{r\}}{dr}\right) = \frac{e}{m_0 c^3}\frac{I_{charge}}{2\pi\varepsilon_0 L_c}\left(\frac{\gamma_0\{r\}}{\sqrt{\gamma_0\{r\}^2 - 1}}\right) \qquad , \tag{2.154}$$

Substituting $\qquad k = \dfrac{e}{m_0 c^3}\dfrac{I_{charge}}{2\pi\varepsilon_0 L_c} = \dfrac{2I_{charge}}{I_0 L_c} \quad ,$

and $\qquad y = \ln\left(\dfrac{r}{r_c}\right) \Rightarrow \dfrac{dy}{dr} = \dfrac{1}{r} \qquad .$ (2.155)

we can rewrite (2.154) as:

$$\frac{d}{dr}\left(r\frac{d\gamma_0\{r\}}{dr}\right) = \frac{d}{dy}\frac{dy}{dr}\left(r_c\frac{d\gamma_0}{dr}\frac{dy}{dr}\right) \Rightarrow \frac{d}{dy}\left(\frac{1}{r_c}\right)\left(r_c\frac{d\gamma_0}{dr}\frac{1}{r_c}\right) = \frac{1}{r_c}\frac{d^2\gamma_0}{dy^2}, \tag{2.156}$$

also, $$\frac{1}{r_c}\frac{d^2\gamma_0}{dy^2} = k\frac{\gamma_0}{\sqrt{\gamma_0{}^2 - 1}} r_c \quad . \tag{2.157}$$

Multiplying (2.157) by $2d\gamma_0 / dy$ yields:

$$\frac{d}{d\gamma_0}\left[\left(\frac{d\gamma_0}{dy}\right)^2\right] = 2k\frac{\gamma_0}{\sqrt{\gamma_0{}^2 - 1}} r_c \frac{d\gamma_0}{dy} \quad . \tag{2.158}$$

Integrating (2.158) we get:

$$\left(\frac{d\gamma_0}{dy}\right)^2 = 2kr_c\sqrt{\gamma^2 - 1} + D \quad . \tag{2.159}$$

Imposing initial conditions on (2.159) as $\gamma_0(r = r_c) = 1$and field become null at the metal, cathode surface, $(d\gamma_0 / dy)_{r=r_c} = (e / m_0c^2)(dV / dy)_{r=r_c} = 0$ that is to say D = 0, one can write:

$$\left(\frac{d\gamma_0}{dy}\right) = \sqrt{2kr_c}\,(\gamma_0{}^2 - 1)^{1/4} \quad . \tag{2.160}$$

Integrating (2.160), we get:

$$\int_1^{\gamma_2}\frac{d\gamma}{(\gamma_9{}^2 - 1)^{1/4}} = \sqrt{2kr_c}\int_0^{\ln\frac{r_a}{r_c}} dy = \sqrt{2kr_c}\,\ln(r_a / r_c) \quad . \tag{2.161}$$

The first term of (2.161) is an elliptic integral of first type. We can assume $G(\gamma_0) = \int_1^{\gamma_2}\frac{d\gamma_0}{(\gamma_0{}^2 - 1)^{1/4}}$ so the relativistic charging current becomes:

$$I_{charge} = \frac{I_0}{4}\left(\frac{L_c}{r_c}\right)\frac{[G\{\gamma_0\}]^2}{[\ln(r_a/r_c)]^2} \quad . \tag{2.162}$$

If we open the series of $(\gamma_0^2 - 1)^{-1/4}$, and then integrating the series yields:

$$G(\gamma_0) \approx 2(\gamma_0{}^{1/2} - 0.847) \quad . \tag{2.163}$$

Now (2.163) can be expressed as:

$$I_{charge} = I_0 (L_c / r_c) \frac{[\sqrt{\gamma_0} - 0.847]^2}{[\ln(r_a / r_c)]^2} \quad . \tag{2.164}$$

However, in the SWS magnetic cutoff is possible, if the charging current is equal to critical current for the device. As discussed earlier for $I_{\text{charge}} = I_{\text{cr}}$, a minimum length of load in this case is defined by [Lemke *et al*. 1997)],

$$L_{c,\min} = 2 r_c \ln(r_a / r_c) \frac{\sqrt{\gamma_0^{\ 2} - 1}}{[G\{\gamma_0\}]^2} \quad . \tag{2.165}$$

The expression given (in a different form) by Calico [Calico (1995)],

$$L_{c,\min} = 1.6 \frac{d^2}{r_c \ln(r_a / r_c)} \left(\frac{\sqrt{\gamma_0} + 1}{\gamma_0 - 1} \right) \quad . \tag{2.166}$$

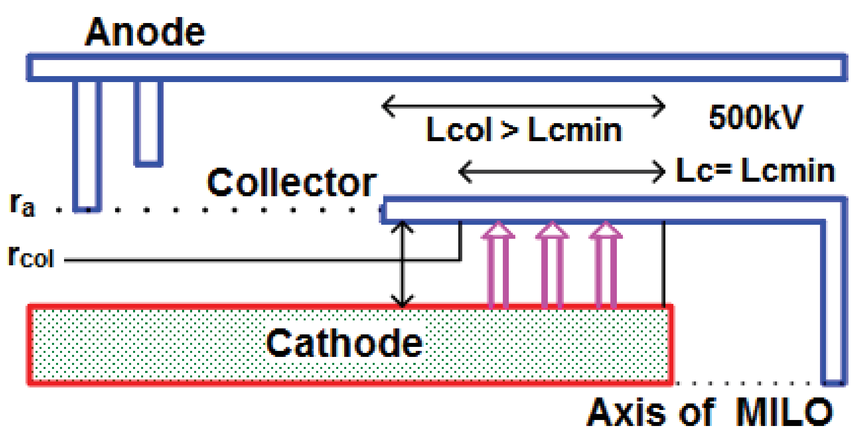


Fig. 2.15: Schematic of charging current under collector, with nominal voltage of 500kV, L_c=L_{cmin}. Length of covering of the collector on cathode is designed such that $L_{\text{col}} > L_{\text{c,min}}$.

where, *d* is inter-electrode spacing as shown in Fig.2.15. However, parameter d fixed the magnetic cutoff conditions of upstream under the SWS and takes a precise value [Cousin (2005)], fixed by the geometry of the diode and voltage of operation to ensure the synchronism conditions. For MILO operation, necessary condition is length of collector is higher than the length of collector over cathode surface [Cousin (2005)].

The collector–cathode spacing fixes the diode operating condition. The recovery length of collector on cathode makes it possible to lengthen the duration of microwave pulse.

2.6.2. Beam Instability (Diocotron Effect):

There are three stages from generation of beam to energy extraction, which are pre-oscillation phase, linear regime, and the non-linear regime. In pre phase –oscillation phase, electric field is generated due to high applied voltage, which causes high emission current in the structure due to which high magnetic field is developed in the device. In the linear regime, a magnetic cutoff occurred; the transient state allows establishment of an RF wave in the structure and oscillations starts in the periodic resonator in a stable *TM* mode of operation. Now, when beam-wave interaction starts, then under Diocotron instability [Shevchik *et al.* (1966)], [Lau (1987)], [Kervalishvili *et al.* (2002)], [Chen (2002)], spokes form due to radial component of electric field and energy transfer occurs in the structure due to axial component of electric field. In the last phase, i. e., non-linear regime, energy extraction phenomenon takes place.

2.6.3. Mode of Oscillation

Under Diocotron instability, few electrons of the beam can reach the anode. Whereas few electrons form shells which remain in magnetic cutoff, and other electrons migrate towards the anode in bunch form that is called as spokes, characterizing a leakage current resulting from the interaction in SWS. They form a periodic package of electrons (bunching-spokes) and the mode of oscillation of MILO is established and beam-wave interaction occurs when electron drift velocity is equal to phase velocity of RF wave which can be defined as:

$$v_e = \frac{E_0}{B_0} = \frac{V_0}{2\ln(r_a/r_c)} \cdot \frac{2\pi r}{\mu_0 I_t} = \frac{2\pi\varepsilon_0 c^2}{\ln(r_a/r_c)} \frac{V_0}{I_t} \approx 0.3c \quad , \tag{2.167}$$

$$v_\varphi = L/2hc \approx 0.3c$$

$$= \frac{\omega}{\beta_0} = \frac{2\pi f}{(\pi/L)} = 2Lf \qquad . \qquad (2.168)$$

where, I_t is total current of coaxial structure compared to the parapotential current.

Stable mode on which the interaction takes place is a mode or the RF field is out of phase 180° of one disc of the periodic structure to the following next. This mode is commonly called as π-mode. It then becomes possible to connect relation (2.168) at the oscillation frequency of MILO in π mode to (2.167). Here ω is the angular frequency and β_0 the phase propagation constant of wave.

Thus, after interaction there is no energy transfer via radial component of electric field and RF power extracted from MILO is given as

$$P = \iiint_{\tau} (\overrightarrow{E_z} \cdot \overrightarrow{J_z}) d\tau \qquad . \qquad (2.169)$$

Here, E_z is axial magnetic field in z-direction and J_z is current density for the coaxial disk loaded structure and can be represented as

$$J_z\{r\} = -e \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dp_z dp_\theta dp_r v_b f_0\{r, p\} \qquad , \qquad (2.170)$$

where, $f_0\{r, p\}$ is the distribution function (radius, momentum), representing charge in space-charge equilibrium, calculated using relativistic Brillouin flow (RBF) and dp_r, dp_θ, dp_z are momentum of charge particles. v_b is the velocity of electrons.

2.7. Design Flow Chart for S-Band MILO

The following steps are required to design all parameters of MILO using analytical equations.

Inputs: Frequency and Beam voltage

For given voltage, calculate relativistic constant γ_o and wavelength.

↓

For given cathode radius (r_c), calculated anode radius (r_i) using r_i/r_c=1.6.

↓

Calculate critical current (I_{cr}) ,parapotential current (I_p) and Spoke current(I_{escape}).

↓

Calculate the depth of cavity (d) using $\lambda/4$ and period (s) using $s=v_\varphi/2f$.

↓

Calculate the length of beam dump covering cathode (L_{cmin}) and cathode length (L_{SE}).

↓

Calculate the anode outer radius (r_o) using $r_i + d$ and choose radius of choke vane less than r_i .

↓

Using trial and run method, decide the radius of extractor vane radius (r_{extr}) and thickness of vanes (w).

↓

Calculate output RF power and power conversion efficiency using Lemke's equation.

↓

Device length is decided by period, choke cavity, SWS and extractor cavity, length of collector over the cathode and remaining part of collector.

2.8. Conclusion

Magnetically insulated line oscillator (MILO) is an attractive HPM source for defence applications due its simple and compact structure as well as no DC magnetic field requirements, although its simulation and experimental results are reported in the literature; its theoretical analysis is limited. In this chapter, a fundamental analytical approach of crossed field devices based on classical approach and relativistic approach is described. Mechanism of self-magnetic insulation has been discussed in detail considering process of explosive emission from the surface of cathode which is related to generation of plasma frequency in HPM sources. To understand the analytical fundamental and development of device, the device design equations using critical current, anode current and cathode design have also been described. Considering relativistic Brillouin flow condition, expressions describing the behaviour of self magnetic field have been explained in disc-loaded coaxial waveguide. These expressions have provided basic insight for in-depth understanding the operation of MILO device. Mode of oscillation and Beam instability in MILO due to Diocotron principle has been explained. Further, a flow chart for the design of conventional MILO has been described. The concepts developed in this chapter will be used in the subsequent chapters for the further study and exploration of MILO.