CHAPTER 7

SUMMARY OF THE THESIS AND FUTURE SCOPE

7.1 Summary

Coupled theory of thermoelasticity takes into account of the fact that the changes in temperature in deformable body affect the state of strain and stress of the body and conversely any mechanical load as well as corresponding stress causes a temperature change in the body. Due to its vivid applications in the fields of machine structures, aircrafts, designing buildings, nuclear engineering, chemical engineering, acoustic engineering, micro- and nano-electromechanical resonator system and many more, the subject has become an integral subject of science. A significant progress in the subject has been made in last few decades with the advancement in technology and an extensive research work carried out both in mathematical and physical terms. The present thesis analyzes the important aspects of some recently developed generalized thermoelasticity theories using mathematical tools. The work under the thesis can be broadly divided into three parts on the basis of the generalized thermoelasticity theories to trace various features that characterize the modification in the classical thermoelasticity theory by solving some unsolved problems in the context of these new models. The important features arising out of the present work are therefore summarized in the following three parts:

I: Investigation on modified thermoelasticity based on exact heat conduction model with a single delay

Chapter 2 and **Chapter 3** are considered as the first part of the thesis work which investigates the thermoelasticity theory under Quintanilla's heat conduction law (Quintanilla (2011)) by studying three different problems. Quintanilla (2011) has introduced this model of heat conduction in order to reformulate the heat conduction law with three phase-lags and to establish a new model that is claimed to be more consistent as compared to three-phase-lag theory. **Subchapter 2.1** starts the discussion about the considered thermoelasticity theory for a homogeneous and anisotropic medium. In this theory, the Taylor's series expansion of this heat conduction law upto second-order in delay time parameter is considered. Some important theorems on this new thermoelasticity theory are established in this subchapter. Firstly, the uniqueness theorem for a mixed type boundary and initial value problem is proved using specific internal energy function which is very important step while working with the system of differential equations. Later, an alternative formulation of the mixed problem is provided using convolution which incorporates the initial conditions into the field equations. Using this formulation, the convolution type variational theorem is proved and a reciprocal relation is obtained for the model. These theorems play vital role in solving problems using finite element method and boundary element method. Subsequently, **Subchapter 2.2** deals with the Galerkin-type representation of solution for the system of equations of motion in the context of the same theory for isotropic medium. A theorem is established to obtain the same in terms of elementary functions. Further, a theorem stating the Galerkin-type solution of equations for steady state oscillations is established followed by the representation of general solution of the system of equations in the case of steady state oscillations. The general solution is acquired in terms of metaharmonic functions. Galerkin-type representation is further useful to obtain the fundamental solution of the considered problem.

The last study in the context of Quintanilla's thermoelasticity theory is articulated in **Chapter 3**. This chapter aims at describing the reflection of thermoelastic waves under considered generalized thermoelasticity theory to highlight some specific features regarding wave propagation predicted by this theory. The work is discussed for isotropic and homogeneous thermoelastic medium with variable elastic parameters. These parameters are considered to be temperature-dependent to extend the results to the elastic materials that experience change in mechanical properties with change in temperature. In order to observe the behavior of reflection in the present context, unified field equations for Quintanilla's theory along with three other existing theories, namely, classical theory (Biot (1956)), Lord-Shulman theory (LS) and Green-Naghdi theory (GN III) are considered. The three waves, i.e., elastic-mode longitudinal wave, thermal-mode wave and transverse wave are found among which only the first two are observed to be coupled. The amplitude ratio and phase velocity for various waves are obtained for incident longitudinal and incident transverse wave cases. The results are presented graphically for various angles of incident. The effects of variable material parameter is further observed for the present context. The comparative study in this chapter showcases that the reflection of thermoelastic waves is approximately of the same nature in case of Quintanilla and GN III theories whereas a notable difference in prediction by Quintanilla model is observed in comparison to the prediction by Biot model and LS model. These observations mark the significant effects of thermal conductivity rate, K^* and phase-lag, τ_q over delay time, τ. Further, the effects of empirical material constant, α^* on the magnitude of wave characteristics of reflected waves are prominently observed in the present investigation which highlights the effects of temperature-dependent elastic parameters. The analysis of phase velocity description clearly indicates that the Quintanilla model still holds the paradox of infinite speed of thermal wave propagation like GN III theory and Biot theory, on the other hand, LS theory accounts for the finite speed for both elastic longitudinal wave as well as of thermal wave and demonstrates the hyperbolic nature of the model.

II: Study on Modified Green-Lindsay (MGL) thermoelasticity

The second part of the thesis includes **Chapter 4** and **Chapter 5**, which emphasizes on the features of recently developed generalized thermoelasticity theory by Yu et al. (2018). This theory is referred to as modified Green-Lindsay theory (MGL). With the help of extended thermodynamics, Yu et al. (2018) have established this thermoelastic model by including the strain-rate and temperature-rate terms in the constitutive relations. This model is also an attempt to remove the discontinuity in the displacement field under temperature rate-dependent thermoelasticity theory (Green and Lindsay (1972)) as reported by several researchers. This theory is also referred to as the strain and temperature rate-dependent thermoelasticity theory. **Chapter 4** derives the representation of Galerkin-type solutions in the context of this recently proposed model in terms of elementary functions. The representation theorem of the Galerkin-type solution of the system of equations for steady oscillation is also proved. In accordance with this theorem, a theorem is established which expresses the general solution of the system of homogeneous equations of steady oscillation in terms of metaharmonic functions.

Chapter 5 deals with a detailed investigation on the propagation of plane harmonic waves in the context of the same MGL theory for an unbounded isotropic homogeneous elastic medium. In order to investigate the effects of strain-rate term, the unified governing equations related to three thermoelastic models namely, classical coupled thermoelastic model, the GL model and MGL have been considered. The dispersion relation for propagation of longitudinal plane waves in the contexts of all three models has been derived in a unified way, which is further solved by computational tool for a particular problem. Two different modes (predominantly elastic-mode and predominantly thermal-mode) of longitudinal plane wave are identified to propagate through the medium. The transverse wave is observed to be unaffected due to thermal field. Hence, special attention is paid on the propagation of longitudinal wave propagation. The behavior of different wave components such as phase velocity, specific loss and penetration depth of longitudinal wave is examined through graphical representation to investigate the effects of temperature-rate and strain-rate terms on the plane wave propagation. It is found that similar to GL model, phase velocity for thermal-mode wave in case of MGL thermoelastic model attains a finite value of 10. On the other hand, as angular frequency, $\omega \to \infty$, phase velocity for elastic-mode wave under MGL model tends to infinity after attaining a local minima at , $\omega \approx 2.601$. This nature of elastic-mode wave is different from the nature observed in GL and classical thermoelasticity where it propagates with finite speed. The infinite phase velocity of elastic-mode wave is also detected in case of thermo-viscoelasticity theory (Roychoudhuri and Mukhopadhyay (2000)), therefore, it can be stated that MGL thermoelasticity theory behaves similar to thermo-viscoelasticity theory. Specific loss for thermal-mode wave starts with the value 4π and steadily decreases to zero in case of MGL theory, which is similar to the pattern in GL theory. However, unlike GL and classical thermoelasticity theory, specific loss due to elastic-mode wave starts with zero and gradually attains the limiting value of 4π in case of MGL theory. Further, similar to specific loss, the nature of penetration depth of thermal-mode wave in MGL theory is same as that in GL theory. On contrary, the penetration depth for elastic-mode wave under MGL model starts with infinite value and gradually decreases to zero, which is different from the predictions by other two models. Moreover, the significant effect of relaxation time parameters, τ_0 and τ_1 on various wave characteristics are observed in case of MGL model. It has been noted that τ_0 has more effect on behavior of thermal-mode wave, whereas τ_1 affects elastic-mode wave more in comparison to τ_0. As a concluding remark, it can be stated that inclusion of only temperature-rate term (GL Theory) in the constitutive relations removes the drawback of infinite speed behavior of thermal-mode wave predicted by classical theory whereas the additional strain rate term along with temperature rate term has a significant influence on the propagation of elastic-mode wave in comparison to that of thermal-mode wave. This new thermoelasticity theory is thus investigated to suffer from the drawback of infinite speed behavior like classical theory.

III: Investigation on dual-phase-lag thermoelasticity

Chapter 6 illustrates the last part of the thesis where various aspects of dual-phase-lag thermoelasticity theory are discussed by solving three different problems. The first problem in **Subchapter 6.1** discusses the thermoelastic interactions in a half-space in the context of the theory of dual-phase-lag thermoelasticity due to stochastic conditions applied at the boundary. An isotropic elastic homogeneous medium with traction-free boundary subjected to two types of thermal distributions is considered to deal with a one-dimensional problem. Further, in order to make the problem more realistic, white noise is added to the boundary conditions. Solutions of different field variables are derived analytically under both deterministic and stochastic conditions using properties of Laplace transform and stochastic process, and short-time approximations. The problem has been further illustrated with numerical results carried out for copper material to highlight the variations of physical fields in both the cases. Five thousand sample paths and their solutions are analyzed to mark the difference between distributions of deterministic type field variables and their corresponding stochastic distributions. It has been observed that the domain of influence for all the field variables is finite in both the cases, i.e., the field variables vanish after finite distance at any time in case of deterministic distributions as well as of stochastic distributions. This observation is also confirmed from analytical results where expressions for field variables consist of terms of two coupled waves both propagating with finite wave speed. This further implies that the noise at the boundary creates the disturbance in the propagation without changing the affected area. Significant difference in stochastic and deterministic distribution is noticed in numerical results for stress and displacement fields near the boundary of the half space whereas for temperature field, both results show negligible difference.

However, analytical as well as numerical results establish that the mean of stochastic solutions along different sample paths coincides with the deterministic solution for each field variable.

Subchapter 6.2 tackles the natural stress-heat flux problem in the context of dual-phase-lag thermoelasticity theory. The primary motive of this subchapter is to establish the domain of influence theorem when an isotropic and homogeneous material is considered for mixed initial and boundary value problem in terms of stress and heat-flux in a three dimensional Euclidean space. Mathematically, it has been shown that there exists a bounded domain outside which the disturbance caused by any thermoelastic loading vanishes in the case when the phase-lag parameters satisfy the inequality, $2\tau_\theta > \tau_q$. The hyperbolicity of DPL model for natural stress-heat flux problem is therefore demonstrated by the proposed theorem. An upper bound to the speed of stress-heat-flux disturbance is also obtained. It is investigated that the finite speed of thermoelastic wave propagation is significantly affected by two phase-lags, τ_q and τ_θ, and other material parameters. Furthermore, it has been established that the results for LS and classical thermoelasticity theories can be derived as special cases from the outcomes of the present theorem.

The last problem demonstrated in the **Subchapter 6.3** is an attempt to examine the non-local dual-phase-lag heat conduction model (DPL model) and to propose a modified thermoelastic model based on this non-local heat conduction law given by Tzou and Guo (2010). A thermal shock is applied at the traction free boundary of a one-dimensional elastic half-space to trace the thermoelastic disturbances under this non-local model. This problem is solved using Laplace transformation and the final results for field variables are obtained numerically using Stehfest method (Stehfest (1970)) for inversion of Laplace transforms. The predictions are analyzed via graphical results to mark the effects of the non-local length parameter, λ_q, which is the characteristic of this non-local heat conduction model. A prominent emphasis of this parameter is observed on all the physical fields, i.e., displacement, temperature and stress fields. The observations for this model are presented along with the outcomes of LS and DPL model to notice the difference in results caused due to non-local parameter. It is highlighted that as the value of λ_q increases, the disagreement of this non-local model with LS and DPL model increases. It is also observed that λ_q has significant effect on the domain of influence of field variables. Further, the impacts of phase-lag parameters, τ_q and τ_θ are examined in presence of λ_q and it is detected that the effect of τ_q is more prominent as compared to that of τ_θ. It is believed that this analysis might help in understanding the effects of non-local heat conduction in mutual interactions due to thermomechanical loading in a medium. Further study in this direction will result in developing the most realistic model of thermoelasticity.

7.2 Future Scope

As mentioned earlier, thermoelasticity theory has applications in wide range of fields in science and technology. However, there has been continuous progress in the technology which requires gradual modifications in the theory of thermoelasticity as well. Through the detailed mathematical study of various problems in the present thesis it can be judged that the removal of paradox of infinite speed of wave propagation may not always occur as it is observed in the first two generalized thermoelasticity theories. The predictions by modified models vary significantly on the way of alteration and can prominently affect the nature of both thermal and mechanical distributions. Therefore, further mathematical as well as experimental examinations are necessary to mark the significance of applying these models in studying real world problems. Incorporation of non-local phenomenon in the theory is one of the ways to model the thermomechanical aspects at nanoscale technology. This inclusion can be done in either time domain or spatial domain or both. One such type of modification has been given by Tzou and Guo (2010) based on which a thermoelasticity theory is presented in the present thesis. Good results have been observed in the same which further motivate to carry out similar work in other theories to overcome the paradox of infinite speed propagation. Also, more analytical work is required to highlight the basic features of the model and relation of this model to that of thermomass theory. Fractional order thermoelasticity theory is another field that has drawn attention in recent times as fractional calculus is becoming popular to model complex real-time scenarios. Therefore, analyzing problem on fractional thermoelasticity holds scope both in terms of application and theoretical development and it is worth pursuing research in that direction. Moreover, stochastic enhancement of thermoelastic theory needs more attention as it aids in predicting the results close to practical outcomes. With the great expectation for future endeavors in the areas mentioned before, the thesis is concluded.