PREFACE

In this thesis the author has given a brief account of the historical development of wavelets (discrete as well as continuous) pointing out their connection with Fourier transform. He points out the application of wavelet theory to image processing, signal processing and generalized transform theory such as wavelet transforms of Schwartz distributions and Sobolev type of spaces useful in solving higher order differential equations and some other mathematical problems.

The thesis consists of six chapters as given below.

Chapter I

It is introductory and gives a brief account of the development of wavelet transform, basic properties of Fourier transform, Hankel transform, Bessel wavelet transform, Zemanian spaces and other spaces.

Chapter II

This chapter deals with the wavelet transform of Schwartz tempered distributions $f \in S'_F(\mathbb{R}^n)$ with respect to the wavelet kernel $\psi \in S(\mathbb{R}^n)$. It is assumed that the space $S'_F(\mathbb{R}^n) \subset S'(\mathbb{R}^n)$ which is obtained by filtering all non-zero constants belonging to the space $S'(\mathbb{R}^n)$. The wavelet $\psi \in S(\mathbb{R}^n)$ is so chosen that none of its moments of order m, $m \geq 1$, is zero. Here $m = (m_1, m_2, \ldots, m_n)$ and each of $m_i \geq 1$.
i.e.

$$\int_{\mathbb{R}^n} \psi(x) x^m dx = \int_{\mathbb{R}^n} \psi(x) x_1^{m_1} x_2^{m_2} \ldots x_n^{m_n} dx_1 dx_2 \ldots dx_n \neq 0.$$

Chapter III

Continuous Bessel wavelet transform is extended to distributions in $H'_\mu(\mathbb{R}^+)$. It is proved that

$$\langle B'_\psi T,\, \phi\rangle = \langle T,\, B_\psi \phi\rangle, \qquad \phi \in H_\mu(\mathbb{R}^+)$$

and

$$T \in (H^1_\mu)'(\mathbb{R}^+ \times \mathbb{R}^+).$$

Chapter IV

In Chapter IV, the characterization of the Besov and Triebel-Lizorkin type spaces are discussed by using the multiplier theorem.

Chapter V

Using the theory of Hankel transform in the space $H'_\mu(\mathbb{R}^+)$ the inversion formula for the continuous Bessel wavelet transform of distributions is proved. Using the inversion formula for the continuous Bessel wavelet transform of distributions, the Calderón reproducing formula is proved. The continuous Bessel wavelet transform of distributions is used to solve certain heat equation problems.

Chapter VI

In this chapter the author has proved the inversion formula for the Bessel wavelet transform of elements in β'_μ- space with respect to the Bessel wavelet kernel whose even order moments weighted by the factor $x^{\mu+\frac{1}{2}}$ are non-zero; the author has used here the properties of Hankel transform.