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DECLARATION BY THE CANDIDATE

I, ***Jay Singh Maurya***, certify that the work embodied in this thesis is my own bonafide work and carried out by me under the supervision of ***Prof. Santosh Kumar Upadhyay*** from ***July, 2017*** to ***May, 2022*** at the ***Department of Mathematical Sciences, Indian Institute of Technology (Banaras Hindu University), Varanasi***. The matter embodied in this thesis has not been submitted for the award of any other degree/diploma. I declare that I have faithfully acknowledged and given credits to the research workers wherever their works have been cited in my work in this thesis. I further declare that I have not willfully copied any other's work, paragraphs, text, data, results, *etc.*, reported in journals, books, magazines, reports dissertations, theses, *etc.*, or available at websites and have not included them in this thesis and have not cited as my own work.

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List of Symbols and Abbreviations

$\mathbb{N}$	Set of natural numbers
$\mathbb{N}_0$	Set of non-negative integers
$\mathbb{R}^+$	Open interval $(0, \infty)$
$\mathbb{R}$	Set of real numbers
$\mathbb{R}^n$	Usual Euclidean space of dimension n
$\mathbb{C}$	Set of complex numbers
$E(x)$ or $[x]$	Integer part of x
$\|x\|$	Norm of x
$D_x \equiv \frac{\partial}{\partial x}$	Partial derivative with respect to variable x
a.e.	Almost everywhere
R.H.S.	Right hand side
L.H.S.	Left hand side