# PREFACE

Optimization is an approach that is used to characterizing, finding, and computing the maxima or minima of a function for a set of acceptable points or certain prespecified conditions. Its early stages were combined with ones of the differential calculus and mathematical analysis. The first idea of differential calculus and the rule for computing the maxima and minima was given by Fermat in 1638. Fermat introduced the optimality condition to obtain the extremum of a differentiable algebraic function $f$ as $f'(x) = 0$. The rule of Fermat remains valid for the differentiable function of several variables and differentiable functions defined on topological and Hilbert vector spaces.

Optimization is not just mathematical analysis. Many decision-making problems in management, engineering, economics, computer sciences, and statistics are formulated as mathematical programs to find the maximization or minimization of an objective function subject to constraints and conditions. Such programs often have special structures: convex, nonconvex, linear, nonlinear, quadratic, semidefinite, dynamic, integer, stochastic programming, etc. This was the source of more theory and efficient algorithms to find the solutions. For these types of problems, there are vast mathematical principles and optimization techniques to handle them.

Now-a-days, there are many optimization and real-world problems with imprecise or uncertain parameters that cannot be handled by only deterministic or probabilistic and linear or nonlinear models. To determine the imprecise or uncertain parameters in such problems, interval analysis was introduced by Moore [57]. Interval analysis is based on the representation of an uncertain variable by an interval, and it provides a natural way of incorporating the uncertainties of parameters. The importance of interval analysis from a theoretical and practical view point is explained by Moore

[57] in his book. Advances of interval analysis have been motivated by its wide application areas, such as control theory, dynamical theory, machine learning, artificial intelligence, etc.

Since its introduction almost 60 years ago, the subject has developed rapidly. It is serving as an impetus for research and rigor in numerical computations on machines. An element of interval analysis has a dual nature as both a number and a set of real numbers. Most of the algorithms for interval methods make use of this duality nature and combine set-theoretic operations such as set intersection with arithmetic operations. For the problem in which coefficients and initial data are acceptable as intervals values, the entire set of possible values of the solution will obtain in a single computation. The ability to compute with sets of values in interval arithmetic provides for some simple computational tests for existence, uniqueness, and convergence.

In this thesis, the author discusses some properties of interval analysis, smooth and nonsmooth analysis of interval-valued functions (IVFs), and optimality conditions for interval optimization problems (IOPs). The basic introduction of optimization, interval analysis with its origin, interval-valued functions, interval optimization problem with its origin are given at the starting of the first chapter. Interval arithmetic and some important properties of intervals are also explained. The definitions of continuity and convexity for IVFs along with their basic results, are explained in the last section of the first chapter.

The notions of directional, Gâteaux and Fréchet derivatives for IVFs are studied. Further, the conditions for the existence of these derivatives for IVFs are given. To observe the properties of these derivatives, the concepts of bounded, linear, monotonic, and Lipschitz continuity for IVFs are newly defined. It is also explained that the proposed derivatives are useful to check the convexity of IVFs. Further, it is observed that the efficient solutions of IOPs can find out with the help of these derivatives. The entire study on these derivatives for IVFs is supported by suitable illustrative examples.

The concepts of Clarke derivative, pseudoconvex and quasiconvex for IVFs are proposed. To describe the properties of Clarke derivative, the concepts of limit superior, limit inferior, and sublinear for IVFs are studied. Further, by using the derived concepts, the existence of Clarke derivative, the relation of Clarke derivative with directional derivative, the relation of convex with pseudoconvex, and relation of pseudoconvex with quasiconvex are shown for IVFs. With the help of the studied pseudoconvex, quasiconvex, and Lipschitz IVFs, we present a few results on characterizing efficient solutions to an interval optimization problem with upper Clarke and Fréchet differentiable IVF. The entire study on these concepts is supported by suitable illustrative examples.

Next, the notion of Hadamard semiderivative for IVFs is explained. In the presence of directonal derivative, continuity and Lipschitz continuity of IVFs, a necessary and sufficient condition for the existence of Hadamard semiderivative of IVFs are derived. The relation of Hadamard semiderivative with directional derivative and Gâteaux derivative for IVFs are shown. Further, the behavior of composition of two Hadamard semidifferentiable IVFs and maximum of Hadamard semidifferentiable IVFs are explained. Proposed semiderivative is observed to be useful to check the convexity of an IVF and also helpful to find out the efficient points of IOPs. For constraint IOPs, the Karush-Kuhn-Tucker sufficient condition to obtain efficient solutions are derived. The entire study on Hadamard semiderivative for IVFs is supported by suitable illustrative examples.

The Hadamard derivative for IVFs is studied subsequently. For an IVF, the relation of Hadamard derivative with Fréchet derivative and continuity are shown. Further, behavior of composition of two Hadamard differentiable IVFs and the maximum of Hadamard differentiable IVFs are explained. The proposed derivative is observed to be useful to check the convexity of an IVF and also helpful to characterize the efficient solutions of IOPs. For constraint IOPs, an extended Karush-Kuhn-Tucker necessary and sufficient condition by using the proposed derivative is derived. The entire study on Hadamard derivative for IVFs is supported by suitable illustrative examples.

Next, the notions of upper and lower Dini semiderivatives, upper and lower Hadamard semiderivatives for IVFs are studied. For an IVF, the relation of upper Dini semiderivative and upper Hadamard semiderivative with directional derivative, Clarke derivative, Hadamard semiderivative and continuity are shown. Proposed semiderivative is observed to be useful to characterize the efficient solutions of IOPs. The entire study on these semiderivatives for IVFs is supported by suitable illustrative examples.