Contents

List of Figures xvii

List of Tables xix

Symbols xxi

Preface xxiii

1 Introduction 1
- 1.1 Singularly perturbed problems . . . 1
- 1.2 Numerical solutions of singularly perturbed differential equations . . . 3
- 1.3 Mesh equidistribution . . . 6
- 1.4 Literature review . . . 8
  - 1.4.1 Singularly perturbed degenerated convection-diffusion problems 9
  - 1.4.2 Singularly perturbed parabolic reaction-diffusion problems . . . 9
  - 1.4.3 Singularly perturbed time delayed parabolic reaction-diffusion problems . . . 11
  - 1.4.4 Nonlinear singularly perturbed Volterra integro-differential equation . . . 12
- 1.5 Outline of the thesis . . . 13

2 A robust adaptive numerical method for singularly perturbed degenerate parabolic convection-diffusion problems 17
- 2.1 The time semidiscretization . . . 19
- 2.2 Spatial mesh generation and discretization . . . 21
  - 2.2.1 Layer-adaptive equidistribution mesh . . . 21
  - 2.2.2 The fully discrete scheme . . . 24
- 2.3 Error analysis . . . 26
  - 2.3.1 Error analysis of the regular component . . . 29
  - 2.3.2 Error analysis of the singular component . . . 30
- 2.4 Numerical experiments . . . 36

2.5 Conclusions 43

3 A robust adaptive numerical method for singularly perturbed parabolic reaction-diffusion problems with Robin boundary conditions 45
3.1 Discretization and adaptive mesh generation 47
3.1.1 The discretization strategy 47
3.1.2 Layer-adaptive equidistribution mesh 48
3.2 A stationary problem 51
3.3 Error analysis 54
3.4 Numerical experiments 58
3.5 Conclusions 66

4 A robust adaptive numerical method for singularly perturbed delay parabolic problems with Robin boundary conditions 67
4.1 Properties of the continuous problem 69
4.2 Discretization and mesh generation 73
4.2.1 The discrete problem 73
4.2.2 Mesh equidistribution 75
4.3 Error analysis 79
4.4 Numerical experiments 89
4.5 Conclusions 98

5 A high order robust adaptive numerical method for singularly perturbed parabolic reaction-diffusion problems 101
5.1 The time semidiscretization 103
5.2 Mesh equidistribution 106
5.3 The spatial discretization and error analysis 109
5.3.1 The discretization strategy 109
5.3.2 Stability of scheme (5.19) 110
5.3.3 Error analysis 112
5.4 The total discretization scheme 118
5.5 Numerical results 120
5.5.1 Numerical experiments 122
5.6 Conclusions 134

6 A robust adaptive numerical method for nonlinear singularly perturbed Volterra integro-differential equations 135
6.1 Stability of the continuous problem 136
6.2 The discretization and its stability 137
6.3 Error analysis 141
6.4 Numerical experiments 145
6.5 Conclusions 152

Bibliography **153**

List of Publications **173**