Chapter 2

Remaining Useful Life Methodology

2.1 Introduction

The goal of prognostics, also known as system usable life prediction, is to forecast the RUL well before equipment fails. Failure of a machine is based on the current machine condition and previous operating characteristic. Diagnostics is generally defined as the process of finding and analyzing the relationship between the factors, with the goal of isolating flaws and determining the real reasons of failure. In order to achieve cleaner, greener production the paradigm transition from traditional to automated, and then to smart manufacturing, laid the foundation for a variety of ongoing maintenance. The Fig. 2.1 below shows a schematic of a continuously monitored system till it attains failure.

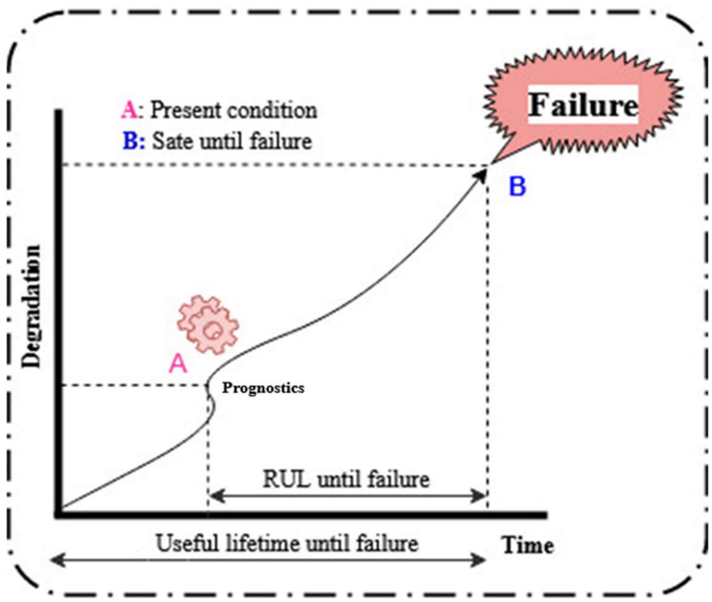

Figure 2.1 Schematic of RUL Prediction

In order to offer a promising reliability, there has been a wide variety of RUL estimation ideas for different domains.

Different terminologies that has been focused in relevance to RUL can be found as: residual life (Kaiser and Gebraeel 2009), time-to-failure (TTF) (Peng and Tseng 2009), RUL (Coble 2010), end-of-discharge (EOD) (Walker et al. 2015), Through-life Engineering Services (TES) (Okoh et al. 2014), state-of-health (SOH) (Wei et al. 2018), state-of-charge (SOC) (Qiu et al. 2020), and end-of-life (EOL) (Petrillo et al. 2020). The terminologies vary, while their commonality lies in their inference. The present study has adopted RUL as a terminology to discuss the results of prognosis.

The residual life (Chakraborty et al. 2009; Gebraeel et al. 2004, 2005; Kaiser and Gebraeel 2009; Kharoufeh and Cox 2005; Letot and Dehombreux 2012; Niknam et al. 2015; Yu et al. 2017; Zhou et al. 2011b) on the other hand provides a probabilistic estimate of the failure. It's the time between the end of economic life and the commencement of functional failure. The similarity follows during its mathematical calculation, which is the service time minus the effective time for intended purpose.

The TTF (Brown et al. 2009; Crk 2000; Kim et al. 2018; Li et al. 2018; Nakamura 2007; Yang et al. 2017) for a representative sample is a statistical approach to estimate the reliability of the product. The reliability analysis a.k.a., life data analysis, often conducted using a three parameter Weibull distribution, is the measure of a product life, measured in hours, miles, cycles (Liu et al. 2018). The parameters are the measure of how closely data fits to a distribution. Analysts have preferred lognormal, exponential, normal and Weibull probability distribution as a measure of lifetime distributions i.e., to investigate the period of successful operation of a particular product. The OEM groups prefer the term '*warranty*' in terms of the reliability of the product.

The useful life (Jiao et al. 2020; Li et al. 2020a; b; Xi et al. 2020; Yu et al. 2021) for the RUL of the system is the quantitative estimate of time left for the system until failure.

The measure of how long it takes to reach a discharge threshold for a battery or a failure threshold for an aircraft engine is defined as the EOD/EOL (Barbieri et al. 2015; Daigle and Kulkarni 2016; Walker et al. 2015; Wang and Mamo 2019). It is a quantitative measure of the health of systems. The method relies on a state estimation approach. The methodology for EOL/EOD estimation remains the same for a linear and non-linear data. EOD being a direct function of battery characteristics, requires a very accurate mathematical model. The predicted values thereby incorporate the presence of bias, hence resulting in an over estimated EOD. The consideration of few more battery ageing parameters thereby assists in obtaining a higher order accurate model and therefore a more precise value of EOL (Daigle and Kulkarni 2016).

The SOH/SOC (Dong et al. 2014; He et al. 2011; Olivares et al. 2013; Orchard et al. 2014) is a qualitative measure of the system in comparison to its optimum circumstances. SOC presents a direct functional relationship with the charge characteristics, which again is a fairly accurate estimate for a battery life. Recent works with battery life estimation provides the measure of the health of system in terms of percentage. The fair estimate in terms of SOH (Lipu et al. 2018) for the system's performance again includes the functional relationship built with the correlated parameters during its optimal operation.

Through-life Engineering Services (TES) (Okoh et al. 2014) are a key in the manufacturing and servicing of complex engineering goods. In both the manufacturing and service industries, the idea of RUL has been made use to forecast the life-span of equipment with the goal of reducing catastrophic failure events. TES assists in better decision making on condition that support services are improved as a result of prognosis, when subjected to run-to-failure data. Considering the various terminologies used alongside for RUL, the definition has been defined keeping in mind the ISO-13381-1 standards (Tobon-Mejia et al. 2010).

2.2 Definition of RUL

The RUL of a system can be described as the useful amount of time left for the degraded system until its intended operation. It can be described in terms of a random variable that has a functional relationship with its present age and the operating conditions. The theoretical description along with the mathematical expressions has been summarized in the subsequent section.

2.3 Theory of RUL Estimation

Statistically derived RUL methodologies prefer non-trivial solutions that provides an estimate of uncertainty of the system besides the present status of health. A random variable, $\mathbf{X}_t$ denotes RUL of a system at any time, t having an operating history, $\mathbf{Y}_t$. In the absence of an operating history, Si et al. (2011) expresses the trivial solution for the conditional probability density, $Cp(\mathbf{X}_t|\mathbf{Y}_t)$ as:

$$C_p\left(x_t \mid Y_t\right) = p\left(\mathrm{x}_t\right) = \frac{f\left(t + x_t\right)}{S(t)} \tag{2.1}$$

where, S(t) is the survival function. The presence of operating history provides an additional information regarding the systems health. For a known distribution of the of $\mathbf{Y}_t$ it provides the life characteristics of the system, which often is scarce as it becomes cost intensive for any critical system. Therefore, the present study attempts to share some light into the past works relating to the use of degradation data rather than history of operating conditions. Condition monitored data unlike the vibration, pressure, and current data, etc consists an essential source of information. Si et al. (2011) elucidates the statistical approach for RUL estimation techniques into two broad categories, one which uses a directly and indirect observed information. The Fig. 2.2 depicts a data-driven (statistical) approach towards RUL estimation that has been implemented in the present study.

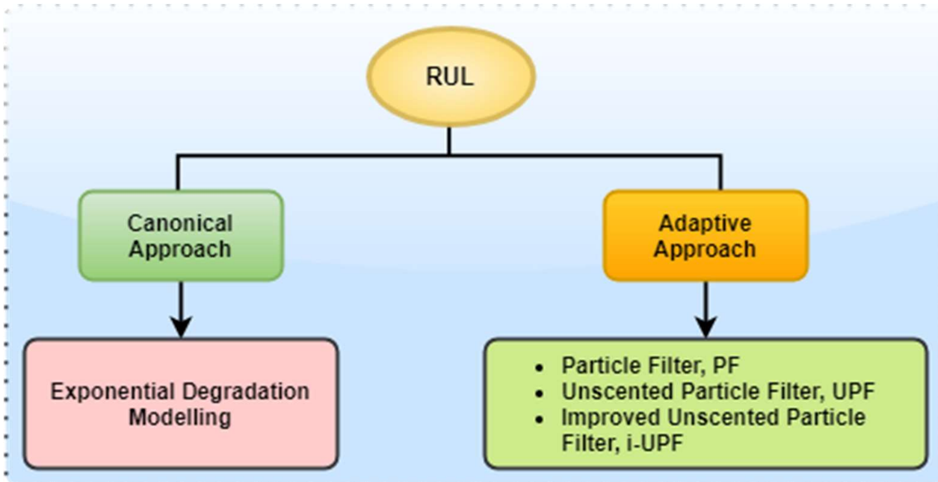

Figure 2.2 Data-driven remaining useful life estimation approaches in the present study

Statistically derived RUL uses probabilistic models to fit the measurement data without the intervention of first principles modelling. Table 2.1 summarizes the data-driven RUL approaches based on given objective type data.

Table 2.1 Data-driven prognostic models for RUL estimation

Data Type	Uses historical / time-to-failure data	Uses lifetime / probability data	Uses known threshold
Model Type	Similarity model development	Survival model development	Degradation model development

By means of CBM, finding the optimal critical intervals ahead of failure has been practiced using random coefficients growth model with a known prior for the model coefficients (Wang 2000). Using random coefficients model, we have a model $y(t) = \theta + \varepsilon$ by random coefficients. The relationship between the condition measurement, $\boldsymbol{y}(t)$ and the TTF maintaining the degradation reaches at maximum threshold, $\boldsymbol{x}_{\text{threshold}}$ has been modeled by Wang (2000). Mathematically the said statement can be written as:

$$P(Y \mid \theta) = P\left(y(t) \le \mathrm{Y} \mid \theta\right) \qquad (2.2)$$

$$
\begin{aligned}
&= P\left(\theta + \varepsilon \le Y\right) \\
&= P\left(\varepsilon \le Y - \theta\right) \\
&= \phi \, \frac{Y - \theta}{\sigma}
\end{aligned} \tag{2.3}
$$

for, ϕ is a standard normal distribution and integrating Eq. (2.2) yields the joint pdf. The distribution of TTF is when Θ meets $\boldsymbol{x}_{\text{threshold}}$. For a random variable, T

$$
\begin{aligned}
&P\left(T \le t \mid \theta\right) = F_T\left(t \mid \theta\right) \\
&F_T = \begin{cases} \dfrac{1 \;\; if\; x_{threshold} \le t}{0 \;\; if\; x_{threshold} > t} \end{cases}
\end{aligned} \tag{2.4}
$$

with $\mathbf{g}(\Theta)$ the joint probability distribution and **R** the sample space for Θ, the distribution of TTF can be expressed as:

$$
F_T(t) = \int_R g(\theta) F_T(t \mid \theta)\; d\theta \tag{2.5}
$$

The extensive usage of the model extends to statistical quality control applications in industry. The random coefficients model was however earlier introduced by Lu and Meeker (1993). It was then concluded that the model does not yield a closed form solution for multiple paths. Besides, a point estimate was insufficient to approach towards any reasonable decision. CBM very often refers to finding the probability density function of RUL rather than a point estimate. The RUL pdf is helpful in characterizing the associated uncertainty and thereby assists in decision making. The application of Bayesian methods upon condition monitored accelerated testing data from bearings were further utilized to obtain a closed form solution of the residual life (Gebraeel et al. 2005). In the process, exponential degradation models were fitted to the bearing data and the stochastic model parameters were updated using Bayesian methodology. The use of random coefficients approach extended to a partially degraded bearing which was fitted to a linear and an exponential model. Gebraeel et al. (2009) presented a mathematical methodology for converting failure time distributions into a form that could also be utilized to estimate the stochastic parameters of degradation models. The collective use of Bayesian updating and expectation maximization in estimating the RUL by means of degradation data was seen in the works of Fan et al. (2013). Exponential model had been a preferred choice for data fitting while Bayesian was used to update the model parameters.

The similar components under different working conditions generally exhibit an unique functional form. The stochastic degradation signal, $S(t_i)$ describing the path having coefficients, $\boldsymbol{\Phi}$ as a the deterministic parameter and the stochastic coefficient, β bears the relationship: $S(t_i) = \gamma(t_i \mid \beta, \phi) + \varepsilon(t_i)$ $\mathrm{i} = 1,2,3...,$. The β follows a known prior distribution and the independent and identically distributed error to model transients for a non-linear study is found to be normally distributed $\mathbf{N}(0, \sigma^2)$. Therefore, for a known failure threshold, *x_threshold* at time '*t*', the expected RUL (Gebraeel et al. 2009) can be expressed as:

$$RUL_T = P(T \le t) = P\left[\gamma(t \mid \beta, \phi) + \varepsilon(t) \ge x_{threshold}\right] \tag{2.6}$$

Further works (Ompusunggu 2012) employed logistic regression to model the degradation data, $\boldsymbol{g}$ obtained from wet friction clutches, in which, the logarithm of the odds-of-success were used as the predicted variable, $\boldsymbol{v}$ expressed as: $v = \log\left(g/(1-g)\right)$. Adaptive RUL methodology describes the dependability of the weighted mean slope to a sufficient amount of data denoting failure threshold, *x_threshold*. With the linear combination of features, the RUL approach heads the following four steps:

1. First time instant to start prediction
2. Model development from data
3. Predicting the trajectory of the predicted variable, $\boldsymbol{v}$
4. RUL estimation

For $\boldsymbol{v} = \{\boldsymbol{v_1}, \boldsymbol{v_2}, \ldots \boldsymbol{v_N}\}$ and $t = \{t_1, t_2, \ldots t_N\}$ represents the logarithm of the odds-of-success and the corresponding time sequence. The weighted mean slope, s_w is computed as:

$$\begin{aligned} s_w &= \sum_{n=2}^{N} w_n s_n \\ w_n &= \frac{n}{\sum_{n=2}^{N} n} \end{aligned} \tag{2.7}$$

$$s_n = \frac{v_n - v_{n-1}}{t_n - t_{n-1}}, \quad n = 1, 2, \ldots N \tag{2.8}$$

$\boldsymbol{w}_n$ denotes the weighting factor and $\boldsymbol{s}_n$ the local slope. Then $\sigma_w = \sqrt{\sum_{n=2}^{N} w_n \left(s_n - s_w\right)^2}$. Assuming linearity within the three condition monitored variables, the trend is linearly modeled. For the known failure threshold, the expected RUL at any arbitrary instant, N = t is given as:

$$RUL = \frac{x_{threshold} - v_N}{s_w} \quad for\ v_N = v_t \tag{2.9}$$

The Eq. (2.9) deals with clutch health prognostics incorporating the logistic model. Since the logistic model ranges between [0, 1], only two specific states are defined.

Recently published works propose RUL$_i$ as a measure of time from the current instant, t_i to the failure point, *x_threshold*. Also discussion in the general guidelines as per ISO 13381-1, RUL is the time left before the rotating machinery cross the failure time noted as an inferior limit of the variable:

$$RUL_i = \inf\left(t : x\left(t + t_i\right) \geq x_{threshold}\right) \tag{2.10}$$

The health state at, $t+t_k$ is $\mathbf{x}(t+t_k)$ for $t > 0$.

The methodology for RUL estimation was further employed while SOH estimation for Li-ion battery involving the filtering technique that uses a state-space approach. Degradation data is initially fitted to produce an empirical model. The model coefficients a.k.a., states were defined with their initial values: $x_t = \left[p1_t; p2_t; p3_t; p4_t\right]$, where p1, p2, p3, p4 are coefficients of a cubic polynomial model with process noise, $\boldsymbol{u}_t$ for a given time, t is given as:

$$\text{f(t)}_t = p1_t * \text{t}^3 + p2_t * \text{t}^2 + p3_t * \text{t} + p4_t + u_t \quad u_t \sim N(0, \sigma_u) \tag{2.11}$$

The filtering is a sequential Monte-Carlo approach that uses a set of particles to represent the density of the states and noise. The predicted posterior density function at time instant, $t+\Delta$t is approximated using the uniformly distributed particle set and can be expressed as:

$$\text{f}(t)_{t+\Delta t} = \sum_{j=1}^{M} p1_t^j * (\text{t} + \Delta\,\text{t})^3 + p2_t^j * (\text{t} + \Delta\,\text{t})^2 + p3_t^j * (\text{t} + \Delta\,\text{t}) + p4_t^j \tag{2.12}$$

The estimated posterior makes use of the normalized weights of the particles, w_t and the Dirac delta function, δ to generate the predicted pdf which can be evaluated using the expression

$$p(\mathrm{f(t)}_{t+\Delta t} \mid \mathrm{f(t)}_{0:t}) = \sum_{j=1}^{M} w_t^{(\mathrm{j})} \delta\left(\mathrm{f(t)}_{t+\Delta t} - \mathrm{f(t)}_{t+\Delta t}^{(\mathrm{j})}\right) \tag{2.13}$$

For a degradation dataset, the RUL$_t$ pdf at cycle t is expressed as

$$p(\mathrm{RUL}_t \mid \mathrm{RUL}_{0:t}) = \sum_{j=1}^{M} w_t^{(\mathrm{j})} \delta\left(RUL_t - RUL_t^{(\mathrm{j})}\right) \tag{2.14}$$

All the above cases for estimating the RUL makes use of a point and an interval estimate. Published research works have been highlighted the uncertainty with the point estimation of RUL and it is suggested to measure the RUL in terms of probability distribution function. The pdf will also provide the two bound of uncertainty for the prognostics measure. The next section will elaborate few of the recent works for RUL estimation along with their accuracy indices.

2.4 Past RUL Works

Any RUL estimation model not only forecasts RUL but also make available a confidence bounds on the prediction. This section aims to emphasize RUL works with different shortcomings (likewise, the sparse and unlabeled data, computational complexity and efficiency) and of the approaches while majorly focusing on the statistical data-driven RUL estimation approach. Research contributions from the past inclined towards the three stage development of a prognostic (Atamuradov et al. 2018; Jardine et al. 2006; Laddada et al. 2017; Rezvanizaniani et al. 2014) method:

1. CI Selection
2. Model Development, and
3. RUL Prediction method.

The previous chapter introduces the past CBM works and its role in predictive maintenance. The construction of CI for different data types and their modeling approaches under two categories was further classified: degradation approach and adaptive filtering. Both the data-driven approaches require best-fitted data for further training the model. In most actual applications, no single CI's are sensitive to a components failure mode. Sequential sampling techniques handle non-linear dynamic systems and non-Gaussian noises in a PF based prognostics approach (He et al. 2012). The inability of the present day diagnostics system to detect the failure of system is the scope of the present study.

To account economic flow analysis due to interacting components, the use of system level prognostics were proposed in the works (Tamssaouet et al. 2019) using inoperability input-output model (IIM) algorithm. The challenge was to cultivate a comprehensive understanding and development of the system to derive physics based or an economic based models. Limitations were made to overcome by employing data driven models. However, the direct RUL method was aimed to train the AI models and mapping the RUL data of the target system. The direct RUL approach made use of huge amount of run-to-failure data while performance measurement using a single health indicator led to successful achievements in RUL prognosis for systems (Kim et al. 2021). The present work made use of the single prognostic indicator to determine the RUL in a system-level approach.

Four categories were provided based on RUL prediction methodology (Lei et al. 2018), namely:

1. Statistical approaches
2. Physics based approaches
3. Hybrid approaches
4. AI approaches

The RUL prediction results were presented as a conditional PDF in statistical models, to construct data-driven models based on empirical knowledge and given observations. RUL prediction models are constructed by using a probabilistic strategy to fit given observations into random coefficient models or stochastic process models. Random variances are commonly added into model parameters to account for external factors such as spectral, component, and measurement variability. As a result, statistical model-based approaches are helpful in explaining the degrading process' uncertainty and its impact on RUL prediction. The use of normally distributed random coefficients model for estimating RUL pdf involved Monte-Carlo simulation (Lu and Meeker 1993), exponential model with random error terms (Gebraeel et al. 2005), prior beliefs into random coefficients (Coble 2010) etc. Results disregarded the effects of temporal variability and the methodology suffered restricted applications due to the assumption of Gaussianity.

Addressing cost minimization and sustainable operational management for a scares and unlabeled data, Cox proportional hazards deep learning (Chen et al. 2020a) provided improved benefits in regards to predictive maintenance. Pre-processed maintenance data had been used to train the long short term memory (LSTM) model that had been applied for predicting TBF. The hazards model makes use of both event and condition monitored data (Heng et al. 2009). Achieving dual data types becomes a challenge besides, the covariate function need to be modeled using another function which makes it computationally exhaustive.

Research expending Weiner's model saw massive emphasis involving linear and exponential models were seen for RUL estimation of a rotating machinery (Gebraeel et al. 2005). The process presented the notion of Brownian motion introducing the drift term. Their use expanded to finding failure probability of structure (Mishra and Vanli 2016) besides contributing in generation of an optimal operational control policy for systems working in an uncertain condition (Usynin 2007).

RUL estimate for a Markov failure time process with a hybrid model of PHM and Markov property for the covariate development was studied by Banjevic and Jardine (2006) as a particular instance. One of the earliest moves taken by researchers was to employ Kalman filters to track changes in characteristics during prognostics (Swanson 2001). T3-Aluminum alloy specimens were used to simulate the residual lifetime distribution of a single-unit system exposed to Markovian deterioration. The methodology addressed failure based reliability in the presence of scares data (Kharoufeh and Cox 2005). Bridging the gap between physics based models and data-driven approach, first order HMM for non-linear and non-Gaussian systems had been proposed for estimating RUL for control systems. The introduction to HMM assisted in CBM, PHM, remanufacturing alongside reliability for the systems (Hu et al. 2019). The estimation of unobservable health states using observable sensor signals for drill bits of CNC drilling machine saw the use of HMMs applications (Camci and Chinnam 2010).

Extrapolation (Butler et al. 2012) to the possibly identified problematic behavior for a bearing wind turbine in case of a PF based RUL estimate has been described for attaining probabilistic results. The RUL is then effectively described as a probability distribution that narrows as the failure point approaches, resulting in an RUL estimate and a confidence measure. Estimation of model parameters (Pang et al. 2018) that accounts to accurate RUL results for the fitted degradation model has been found using a modified simulation and extrapolation (SIMEX) method. the process considered multiple sources of variability for a real world example.

An et al. (2013) conducted a review of CBM research combining the Gaussian process and neural networks for a data-driven and physics-based approach. The most extensively utilized statistical methods for RUL prediction are regression and Bayesian techniques (Angadi et al. 2020; Yi and Song 2018).The Bayesian approaches helped in incorporating the effects of temporal variability while regression helped in attaining extrapolation benefits. Ao and Qiao (2010) examine the evaluation of the RUL using logistic regression mixed with an autoregressive moving average model. The application of support vector machines for forecasting engine dependability was demonstrated by Hong and Pai (2006). Silva and Pederiva (2013) used artificial intelligence to examine failure detection in induction motors.

ANN, SVM, GPR's are some of the most used AI approaches today in industries. DC motor fault analysis using coiflet wavelet transformed acoustic data have been performed by *k*-NN classifier approach (Glowacz 2015). The deep-NN alongside the adaptive neuro fuzzy inference systems (Elattar et al. 2016) and e-SVR (Loutas et al. 2013)have been employed for rotating machinery bearing's RUL estimation. However, deep learning based RUL prediction framework were seen to be established using deep autoencoder and DNN (Ren et al. 2018) for bearing prognosis. Recurrent neural network (RNN) based autoencoders (Yu et al. 2021) were devised for classification of the data ahead of machine prognosis.

Prognostic accuracy metrics were developed keeping in mind the constraints of ambiguous and inconsistent interpretations. Furthermore, the metrics must be capable of incorporating probabilistic uncertainty as well as a full visual perspective that can be employed in the design of the prognostic system. With the significantly increased attention in the prognostics community regarding the evaluation of the standard metrics, Saxena et al. (2008) introduced an offline evaluation of prognostic performance. Prognostic horizon, alpha-lambda performance, relative accuracy and convergence were the four proposed metrics. The four metrics provide a schematic for their selection to alleviate some of these issues: does the algorithm predict within desired accuracy and in advance, consistency in accuracy within the desired levels, quantification of accuracy, and computational easiness. RUL metrics likewise MAPE, RMSE, standard deviation of error, absolute prediction error, MSE besides individual RMSE values for the Li-ion battery training and validation datasets were found in a comprehensive work by Lipu et al. (2018). Pdf width and absolute error metrics was found in the works employing NN based PF-bat algorithm (Wu et al. 2019). Consistency in the prognostics error index (*e*_PR) were however addressed by the authors in the estimation of motor RUL (Banerjee et al. 2021).

While, the Type I collects degradation data from similar/dissimilar machines that demonstrate a similar tendency for a run to failure machine. Type II uses a proportional hazards model to estimate RUL using component failure probability estimates. Type III prognostics employ a degradation measure (also known as a known failure threshold) to assess the rise and fall of RUL estimates. The resulting approach would enable its extended support in futuristic high-level decision-making such as device replacement, reusability, schedule maintenance operation, and inventory management. This leads us to tackle the present day challenge in getting introduced to the concepts of Industry 4.0 via cyber physical systems. An approach has also been taken from the authors end to make use of RUL as an effective cognitive tool in the domain of manufacturing. Keeping in mind the challenges and the present scope of the work, the subsequent chapter 3 will elaborate the experimental setup required for data generation and data driven modeling.