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Converging shock wave in a dusty gas through nonstandard analysis

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Abstract A problem of propagation of strong plane and converging shock wave is studied in a mixture of a gas and small solid particles. It is assumed that the solid particles are continuously distributed in the gas. Jump conditions for plane and converging shock waves are derived using the nonstandard analysis. It is also assumed that the shock thickness occurs at infinitesimal interval and jump functions are smooth across this interval. The distribution of flow parameters across the shock wave are presented in terms of Heaviside functions and Dirac Delta measures.

1. Introduction

In recent years, the study of propagation of shock waves in a mixture of a gas and small solid particles have received considerable attention due to its application in space exploration, reentry capsules, nozzle flow, lunar ash flow, astrophysics space science and many other engineering problems. Many authors [1–7] studied the problems concerning the propagation and computation of flow field behind the strong shock wave in a mixture of a gas and small solid particles. Salas Manuel and Iollo [8] have derived the shock jump conditions in their primitive form using generalized functions. Hamad [9] studied the behavior of entropy across shock waves in dusty gases. Jena and Sharma [10] analyzed the self similar solution of shock wave in dusty gas. Baty et al. [11,12] have used the method of nonstandard analysis, developed by Robinson [13] to derive the jump conditions for converging shock wave in a perfect gas. Baty and Tucker [14] used the same method to derive the jump conditions for one-dimensional, diverging, magnetogasdynamics shock waves emerging on the surface of a star. Singh et al. [15,16] have studied the nonstandard analysis of converging shock wave in non ideal gas and nonideal magnetogasdynamics. In the above analysis it was assumed that the shock thickness occurs at an infinitesimal interval and jump functions in the gas parameters occur smoothly across this interval.

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In the present article, nonstandard analysis is used to study the propagation of strong shock wave in a one dimensional, unsteady, inviscid flow in a mixture of a gas and small solid particles. Here, it is assumed that the solid particles are continuously distributed in the medium. Predistributions of the Heaviside functions and Dirac Delta measures are used to model the microstructure the flow field across the shock wave.

The nonstandard jump functions used in this article are as follows: For standard jump function $\psi(y)$ with $[\psi] = \psi_1 - \psi_0$ at $y = 0$, a nonstandard jump function is defined below $\psi(y) = \psi_0 + [\psi]H(y)$,

where $H(y)$ is piecewise differentiable nonstandard Heaviside function. Distinct nonstandard jump functions of flow variables are derived for plane and converging shock waves. It is also assessed as to how (i) the ratio of specific heats of the gas, γ (ii) the ratio of density of the solid particles to that of initial density of the gas G and (iii) the mass concentration of the solid particles k_p in the mixture, affect the distribution of flow parameters across the shock wave.

2. Governing equations

The equations governing the motion of one dimensional, unsteady, compressible, inviscid, mixture of the gas and solid particles can be written in the following form [2]:

$$\frac{\partial \ln(v)}{\partial t} + u\frac{\partial \ln(v)}{\partial y} - \left(\frac{\partial u}{\partial y} + \frac{ju}{y}\right) = 0, \tag{2.1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial y} + v\frac{\partial p}{\partial y} = 0, \tag{2.2}$$

$$\frac{\partial e}{\partial t} + u\frac{\partial e}{\partial y} + vp\left(\frac{\partial u}{\partial y} + \frac{ju}{y}\right) = 0, \tag{2.3}$$

where u is the particles velocity, ρ is the density of the mixture, p is the pressure, e is the internal energy per unit mass of the mixture, $v(=1/\rho)$ is the specific volume; t is the time, y is the radial distance. The letter j takes value 0, 1 or 2 accordingly as the motion is planar, cylindrical or spherically symmetric, respectively. Here, it is assumed that the mixture of a gas and solid particles obey an equation of state of the form [10]

$$p = (1 - k_p)\frac{\rho RT}{1 - Z_p}, \tag{2.4}$$

where Z_p is the volume fraction of the solid particles, k_p is the mass fraction of solid particles, R is the gas constant and T is the absolute temperature. The relation between Z_p and k_p is given by

$$k_p = \frac{Z_p \rho_{sp}}{\rho}, \tag{2.5}$$

where ρ_{sp} is the density of the solid particles. The internal energy of the mixture is related to the internal energies of the two species, which may be written as

$$e = c_{vm}T = k_p c_{sp} + (1 - k_p)c_v, \tag{2.6}$$

where c is the specific heat of gas at constant volume, c_{sp} is the specific heat of solid particles and c_{vm} is the specific heat of the mixture at constant volume.

The specific heat of the mixture at constant pressure is given by

$$c_{pm} = (1 - k_p)c_p + k_p c_{sp}, \tag{2.7}$$

where c_p is specific heat of the gas at constant pressure. The ratio of the specific heats of the whole mixture is given by

$$\Gamma = \frac{c_{pm}}{c_m} = \frac{\gamma + \delta\beta_{sp}}{1 + \delta\beta_{sp}}, \tag{2.8}$$

where

$$\beta_{sp} = \tfrac{c_{sp}}{c} \quad , \delta = \tfrac{k_p}{1 - k_p} \quad \text{and} \qquad \rho = (1 - Z_p)\rho_g + \rho_{sp}. \tag{2.9}$$

Relation between volume fraction of the solid particle Z_p and density of the mixture may be written as

$$Z_p = \frac{\rho Z_a}{\rho_a}, \tag{2.10}$$

where

$$Z_a = \frac{k_p}{G(1 - k_p) + k_p}, \tag{2.11}$$

where ρ_a is the initial density of the mixture, Z_a is the initial volume fraction of solid particles, $G = \frac{\rho_{sp}}{(\rho_g)_0}$ is the ratio of the density of the solid particles to the initial density of the gas. The particular case $G = 1$ corresponds to $Z_a = k_p$.

Internal energy of the mixture is given by

$$e = \frac{p(1 - Z_a)}{\rho(\Gamma - 1)}. \tag{2.12}$$

On using the relation (2.12) in Eq. (2.3) reduces to the following form

$$\frac{\partial p(1 - Z_a)v}{\partial t} + u\frac{\partial p(1 - Z_a)v}{\partial y} + pv(\Gamma - 1)\left(\frac{\partial u}{\partial t} + \frac{ju}{y}\right) = 0. \tag{2.13}$$

3. Jump conditions for normal shock

To derive the jump conditions for the strong shock waves, we assume the motion of normal shock wave propagating in the mixture. Also, on both sides of the shock wave the flow parameters v, u and p are assumed to be constant. Consequently, the shock wave does not accelerate and the shock speed U remains constant. Therefore, the characteristics for the corresponding shock will be straight lines in the $(y - t)$ space. Along the characteristic line the flow variables v, u and p may be assumed to have the following form across the shock wave [11]:

$$v(\eta) = v_l + [v]W(\eta), \tag{3.14}$$

$$u(\eta) = u_l + [u]V(\eta), \tag{3.15}$$

$$p(\eta) = p_l + [p]Z(\eta), \tag{3.16}$$

where W, V and Z are assumed to be predistributions of Heaviside function defined in [6], which belongs in fixed infinitesimal interval $(0, \varepsilon)$, ε being an arbitrary fixed infinitesimal, $\eta = y + Ut$ is a characteristic line, U is the shock velocity and $[\psi] = \psi_1 - \psi_0$, with subscripts and referring to the right and left conditions across the shock, respectively. On using Eqs. (3.14)–(3.16) in Eqs. (2.1)–(2.13) reduces in the following form:

$$u^*[v]W' - v[u]V' = 0, \tag{3.17}$$

$$u^*[u]V' + v[p]Z' = 0, \tag{3.18}$$

$$u^*(p[v]W' + v[p]Z') + (\Gamma - 1)(v - v_a Z_a)p[u]V' = 0, \tag{3.19}$$

where

$$u^* = u_l + U + [u]V. \tag{3.20}$$

The Eqs. (3.17), (3.18) and (3.19) may be expressed in the following matrix form:

$$\begin{bmatrix} [v]u^* & -[u]v & 0 \\ 0 & [u]u^* & p[v] \\ [v]pu^* & (\Gamma-1)(v-v_aZ_a)p[u] & pvu^* \end{bmatrix}\begin{bmatrix} W' \\ V' \\ Z' \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \tag{3.21}$$

The nontrivial family of ODE's may be found by specifying the nonstandard function and, if the determinant of the matrix of coefficients in Eq. (3.21) vanishes.

The condition

$$\begin{vmatrix} [v]u^* & -[u]v & 0 \\ 0 & [u]u^* & p[v] \\ [v]pu^* & (\Gamma-1)(v-v_aZ_a)p[u] & pvu^* \end{vmatrix} = 0, \tag{3.22}$$

is satisfied if

$$u^* = \sqrt{p(\Gamma v - (\Gamma-1)v_aZ_a)}. \tag{3.23}$$

Eq. (3.22) in conjunction with (3.23) show that a nontrivial solution exists defining the ODE's specifying the nonstandard Heaviside function for isentropic flow. Since shock propagation is not an isentropic process, Eq. (3.22) may be replaced by following underdetermined system:

$$\begin{bmatrix} [v]u^* & -[u]v & 0 \\ 0 & [u]u^* & p[v] \end{bmatrix}\begin{bmatrix} W' \\ V' \\ Z' \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \tag{3.24}$$

together with the entropy defined in a dusty gas [9]

$$s_m = c_p\left[\ln\frac{T}{T_0} - \frac{(\Gamma-1)(1-k_p)}{\Gamma}\frac{p}{p_0}\right], \tag{3.25}$$

where T_0 is the initial temperature, p_0 is the initial pressure of the mixture. Nonstandard Heaviside functions W, V and Z on the interval $(0, \varepsilon)$ are determined satisfying the boundary conditions

$$\begin{bmatrix} W(0) \\ V(0) \\ Z(0) \end{bmatrix} \approx \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \text{and} \begin{bmatrix} W(\varepsilon) \\ V(\varepsilon) \\ Z(\varepsilon) \end{bmatrix} \approx \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}. \tag{3.26}$$

Integration of matrix (3.24) subject to boundary conditions (3.26) gives the relationship between the Heaviside functions. To determine the solution of nonstandard Heaviside functions W' and V' Eq. (3.24) is rewritten in terms of nonsingular matrix and an unknown function depending on Z'.

$$\begin{bmatrix} [v]u^* & -[u]v \\ 0 & [u]u^* \end{bmatrix}\begin{bmatrix} W' \\ V' \end{bmatrix} = \begin{bmatrix} f_1(Z') \\ f_2(Z') \end{bmatrix}, \tag{3.27}$$

where

$$\begin{bmatrix} f_1(Z') \\ f_2(Z') \end{bmatrix} = \begin{bmatrix} 0 \\ -[p]vZ' \end{bmatrix}, \tag{3.28}$$

Eqs. (3.27) and (3.28) then gives

$$\begin{bmatrix} W' \\ V' \end{bmatrix} = \frac{1}{[u][v]u^{*2}}\begin{bmatrix} [u]u^* & [u]v \\ 0 & [v]u^* \end{bmatrix}\begin{bmatrix} 0 \\ -[p]vZ' \end{bmatrix}, \tag{3.29}$$

which may be written as

$$W' = -\frac{[p]vZ'}{[v]u^{*2}}, \tag{3.30}$$

$$V' = -\frac{[p]vZ'}{[u]u^*}. \tag{3.31}$$

Combining Eqs. (3.30) and (3.31) yields

$$V' = \frac{u^*[v]W'}{[u]v}. \tag{3.32}$$

Integrating Eq. (3.32) and applying the boundary conditions (3.26) shows that

$$V \approx W, \tag{3.33}$$

with

$$\frac{v_l[u]}{[v]u_l^*} = 1,$$

where

$$u_l^* = u_l + U. \tag{3.34}$$

Also, combining (3.33) and (3.34) with Eq. (3.31) and then integrating the resulting Eq. (3.32) with boundary conditions (3.26) yields

$$V \approx Z, \tag{3.35}$$

$$\frac{[u]^2}{[v][p]} = -1. \tag{3.36}$$

By combining (3.33) and (3.35) we get

$$W \approx V \approx Z, \tag{3.37}$$

which shows that the microstructure for the Heaviside functions for the flow parameters v, u and p, jump conditions are coincident across an inviscid shock in dusty gas for an arbitrary infinitesimal interval $(0, \varepsilon)$.

Using Eq. (3.33) in (3.34) gives the following expression for shock speed

$$U = \frac{1}{[v]}[v_l[u] - u_l[v]]. \tag{3.38}$$

Eq. (3.38) is shock speed determined from equation of motion in conservative form.

4. Jump conditions for converging shock

To have an analysis similar to the normal shock analysis discussed earlier, the governing equations of motion are considered along characteristic curves in $(y-t)$ space. We now introduce the non dimensional variables π, g and so that the flow variables are written in terms of these new variables in the following form [12]:

$$\rho(y,t) = \rho_0 g(\eta), u(y,t) = \dot{X}_s(t)\upsilon(\eta), \quad \text{and} \quad p(y,t) = \rho_0\dot{X}_s^2(t)\pi(\eta), \tag{4.39}$$

where

$$\eta = \frac{y}{\dot{X}_s(t)} \quad \text{and} \quad \dot{X}_s(t) = A(-t)^\alpha. \tag{4.40}$$

Here $X_s(t)$ is the location of the shock front and ρ_0 is the initial density in front of the shock wave. A and α are constants. The functions of Eqs. (4.39) and (4.40) are defined on $1 \leqslant \eta < \infty$, $t < 0$. It may be noted here that the converging shock fronts are located at $\eta = 1$ along the characteristic curve, nonstandard jump functions across the shock front can be written in terms of non dimensional variables π, g and in the following form

$$g(\eta) = g_0 + [g]L(\eta), \tag{4.41}$$
$$v(\eta) = v_0 + [v]K(\eta), \tag{4.42}$$
$$\pi(\eta) = \pi_0 + [\pi]N(\eta), \tag{4.43}$$

where $L(\eta)$, $K(\eta)$ and $N(\eta)$ are assumed to be differentiable on $(1, 1 + \varepsilon)$ and belong $L_{loc}(R)$ space of locally integrable functions. Also, each Heaviside predistribution function is assumed to have its jump contained on the same interval $(1, 1 + \varepsilon)$. The boundary conditions for the nonstandard Heaviside functions at the end point of the interval $(1, 1 + \varepsilon)$ are given as

$$\begin{bmatrix} L(1) \\ K(1) \\ N(1) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} L(1+\varepsilon) \\ K(1+\varepsilon) \\ N(1+\varepsilon) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}. \tag{4.44}$$

In the strong shock limit, the density ρ_1, velocity u_1, and pressure p_1 behind the converging shock front are assumed to have the following form [2]

$$\rho_1 = \frac{\Gamma+1}{\Gamma-1+2Z_a}\rho_0, \quad u_1 = \frac{2(1-Z_a)}{\Gamma+1}\dot{X}_s \quad \text{and} \quad p_1 = \frac{2(1-Z_a)}{\Gamma+1}\rho_0\dot{X}_s^2. \tag{4.45}$$

Using the above relations we have the nonstandard jump function on the boundary as [2].

$$g_0 = 1 \quad \text{and} \quad [g] = \frac{\Gamma+1}{\Gamma-1+2Z_a} - 1, \tag{4.46}$$
$$v_0 = 0 \quad \text{and} \quad [v] = \frac{2(1-Z_a)}{\Gamma+1}, \tag{4.47}$$
$$\pi_0 = 0 \quad \text{and} \quad [\pi] = \frac{2(1-Z_a)}{\Gamma+1}. \tag{4.48}$$

On substituting Eq. (4.39) in mass and momentum Eqs. (2.1) and (2.2) we get the following system of equations written in the matrix form as

$$\begin{bmatrix} [v] & 0 \\ 0 & [\pi] \end{bmatrix} \begin{bmatrix} K' \\ N' \end{bmatrix} = \begin{bmatrix} F_1(L) \\ F_2(L) \end{bmatrix}, \tag{4.49}$$

where

$$F_1(L) = (\eta -)\frac{g'}{g} - \frac{j}{\eta},$$
$$F_2(L) = \frac{g(1-\alpha)}{\alpha} + (-\eta)'.$$

Using Eqs. (4.41) and (4.40) in (4.49) and simplifying we get

$$\frac{d}{d\eta} + \left(\frac{j}{\eta} + \frac{g'}{g}\right) = \eta\frac{g'}{g}, \tag{4.50}$$

where is function of g and η. Integrating (4.50) from 1 to η in the interval $(1, 1 + \varepsilon)$ yields

$$v(\eta) = \frac{1}{\eta^j g(\eta)}\left[\eta^{j+1}g(\eta) - 1 - (j+1)\int_1^\eta \zeta^j g(\zeta)d\zeta\right]. \tag{4.51}$$

Eq. (4.51) satisfies the boundary condition (4.47) and g is monotonically increasing on the interval $(1, 1 + \varepsilon)$. Since the value of the integral (4.51) is very small therefore, Eq. (4.51) reduces approximately

$$v(\eta) \approx \left[\eta - \frac{1}{\eta^j g(\eta)}\right]. \tag{4.52}$$

and

$$v(1+\varepsilon) \approx \frac{2(1-Z_a)}{\Gamma+1}. \tag{4.53}$$

Second equation in (4.49) reduces to a first order linear ODE for π in terms of functions g, and η which may be written as

$$\frac{d\pi}{d\eta} = g\left(\frac{(1-\alpha)}{\alpha} - (-\eta)'\right). \tag{4.54}$$

Integrating Eq. (4.54) on the interval $(1, 1 + \varepsilon)$ yields

$$\pi(\eta) = \frac{1-\alpha}{\alpha}\int_1^\eta g(\zeta)(\zeta)d\zeta - \int_1^\eta g(\zeta)((\zeta) - \zeta)'(\zeta). \tag{4.55}$$

Since the value of second integral of (4.55) is very small and g is monotonically increasing function, Eq. (4.55) may be written approximately as

$$\pi(\eta) \approx \int_1^\eta \zeta^{-j\prime}(\zeta)d\zeta. \tag{4.56}$$

Integrating Eq. (4.56) gives

$$\pi(\eta) \approx \eta^{-j}(\eta) - (1) + j\int_1^\eta \zeta^{-j-1}(\zeta)d\zeta. \tag{4.57}$$

Evaluating Eq. (4.57) at $\eta = 1$ and $\eta = 1 + \varepsilon$ gives

$$\pi(1) \approx 0 \quad \text{and} \quad \pi(1+\varepsilon) \approx \frac{2(1-Z_a)}{\Gamma+1}. \tag{4.58}$$

The Eqs. (4.52) and (4.57) are approximately same, therefore, we deduce that

$$\pi(\eta) \approx (\eta). \tag{4.59}$$

The nonstandard functions π, v and g belong in $L_{loc}(R)$, and the solution of nonstandard functions π, v and g may be determined from the boundary value problem defined in equations (4.46)–(4.49).

5. Example

Assume that for the microstructure of inviscid converging shock wave the density jump function $g(\eta) \in {}^*L_{loc}(R)$ defined in (4.41) and satisfying the Rankine Hugoniot condition (4.46) is given by

$$g(\eta) = 1 + [g]\left(\frac{\eta-1}{\varepsilon}\right)^n, \tag{5.50}$$

on the interval $(1, 1 + \varepsilon)$, for $n \geqslant 1$. Using Eq. (5.50) in (4.51) we get the following relation for (η)

$$v(\eta) = \eta - \frac{1}{\eta^j g(\eta)} - \frac{j+1}{\eta^j g(\eta)}\int_1^\eta \left[\zeta^j(1 + [g]\left(\frac{\zeta-1}{\varepsilon}\right)^n d\zeta\right]. \tag{5.51}$$

For $n = 1$, integrating (5.51) and applying boundary condition (4.47) yields

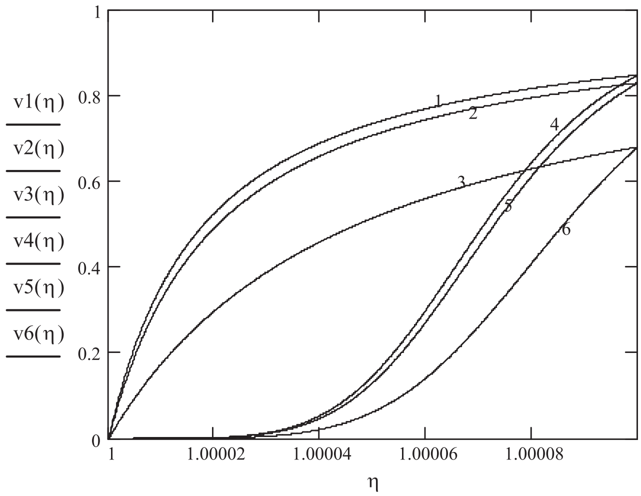

Figure 1 The velocity profiles: (1) $v1 - G = 1$, $n = 1$; (2) $v2 - G = 10$, $n = 1$; (3) $v3 - G = 100$, $n = 1$; (4) $v4 - G = 1$, $n = 5$; (5) $v5 - G = 10$, $n = 5$; (6) $v6 - G = 100$, $n = 5$ with fixed value of $k_p = 0.2$, $\gamma = 1.4$ and $\beta = 0.5$.

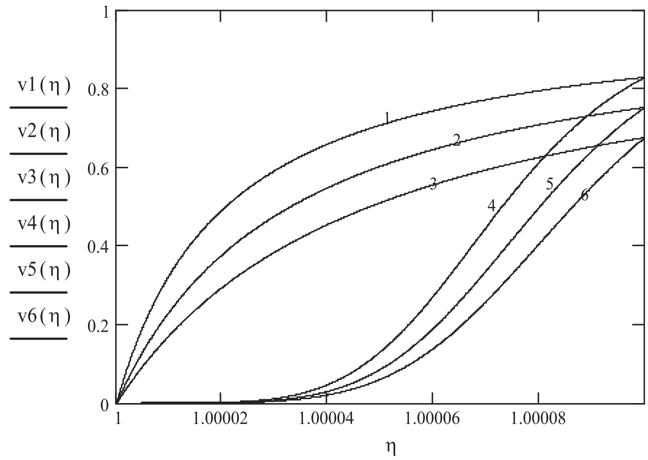

Figure 3 The velocity profiles: (1) $v1 - \gamma = 1.4, n = 1$; (2) $v2 - \gamma = 1.67$, $n = 1$; (3) $v3 - \gamma = 2.0$, $n = 1$; (4) $v4 - \gamma = 1.4$, $n = 5$; (5) $v5 - \gamma = 1.67$, $n = 5$; (6) $v6 - \gamma = 2.0$, $n = 5$ with fixed value of $k_p = 0.2$, $G = 10$ and $\beta = 0.5$.

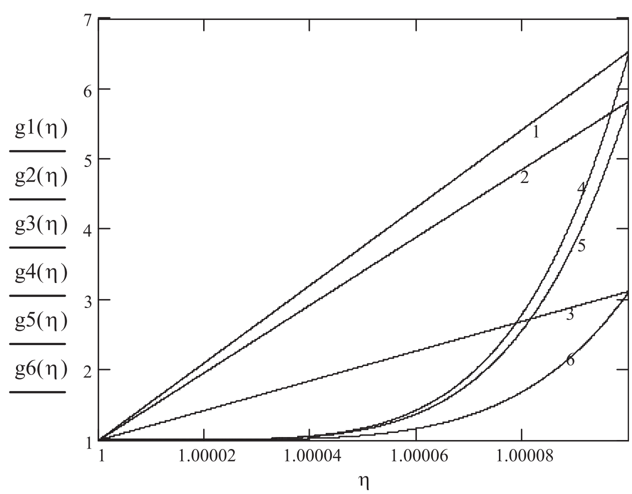

Figure 2 The density profiles: (1) $g1 - G = 1$, $n = 1$; (2) $g2 - G = 10$, $n = 1$; (3) $g3 - G = 100$, $n = 1$; (4) $g4 - G = 1$, $n = 5$; (5) $g5 - G = 10$, $n = 5$; (6) $g6 - G = 100$, $n = 5$ with fixed value of $k_p = 0.2$, $\gamma = 1.4$ and $\beta = 0.5$.

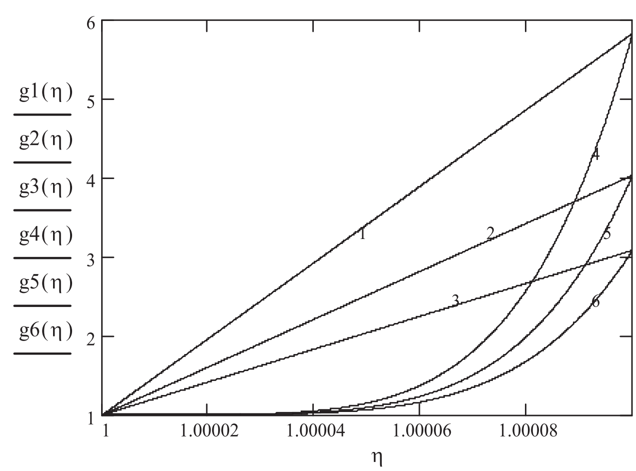

Figure 4 The density profiles: (1) $g1 - \gamma = 1.4, n = 1$; (2) $g2 - \gamma = 1.67$, $n = 1$; (3) $g3 - \gamma = 2.0$, $n = 1$; (4) $g4 - \gamma = 1.4$, $n = 5$; (5) $g5 - \gamma = 1.67$, $n = 5$; (6) $g6 - \gamma = 2.0$, $n = 5$ with fixed value of $k_p = 0.2$, $G = 10$ and $\beta = 0.5$.

$$v(\eta) = \eta - \frac{1}{\eta^j g(\eta)}\left[1 - (j+1)\left(\left(1 - \frac{[g]}{\varepsilon}\right)\frac{1}{j+1}(\eta^{j+1} - 1) + \frac{[g]}{\varepsilon(j+2)}(\eta^{j+2} - 1)\right)\right], \tag{5.52}$$

satisfying the following boundary conditions

$$v(1) \approx 0 \quad \text{and} \quad (1 + \varepsilon) \approx \frac{2(1 - Z_a)}{\Gamma + 1}. \tag{5.53}$$

6. Results and discussion

The density and velocity jump functions specified by Eqs. (5.50) and (5.52) are computed for various values of k_p, mass concentration of the solid particles in the mixture G, ratio of solid to that of initial density γ, specific heat ratio. The profiles for density and velocity distribution are plotted in Figs. 1–6 with small fixed infinitesimal values of ε for converging shock wave with cylindrical symmetry ($j = 1$) which are used to analyze the qualitative behavior of nonstandard shock wave microstructure.

In the numerical results, the typical values of the non-dimensional parameters taken are $\gamma = 1.4$, 1.67 and 2.0, $G = 1$, 10 and 100, $k_p = 0.1$, 0.2 and 0.6, $n = 1$ and 5, $\beta = 0.5$.

Figs. 1 and 2 show that an increase in the value of G causes to decreases the velocity and density in the region of microstructure. Also, an increase in specific heat ratio and mass concentration of solid particles k_p causes to decrease the velocity and density distribution in the region which can be observed from Figs. 3–6. Further, for $n = 1$ the velocity profiles across

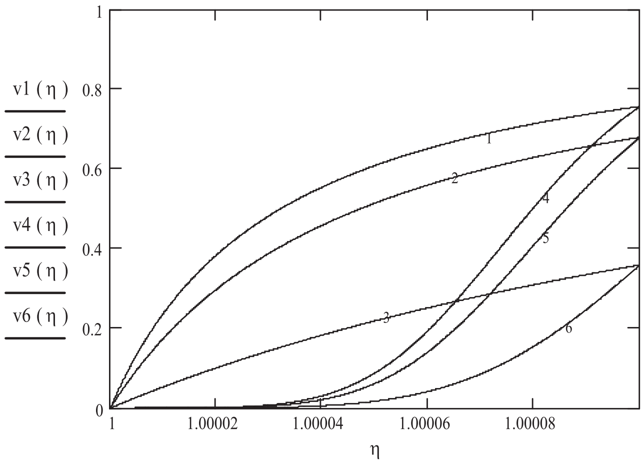

Figure 5 The velocity profiles: (1) $v1 - k_p = 0.1$, $n = 1$; (2) $v2 - k_p = 0.2$, $n = 1$; (3) $v3 - k_p = 0.6$, $n = 1$; (4) $v4 - k_p = 0.1$, $n = 5$; (5) $v5 - k_p = 0.2$, $n = 5$; (6) $v6 - k_p = 0.6$, $n = 5$ with fixed value of $\gamma = 1.4$, $G = 10$ and $\beta = 0.5$.

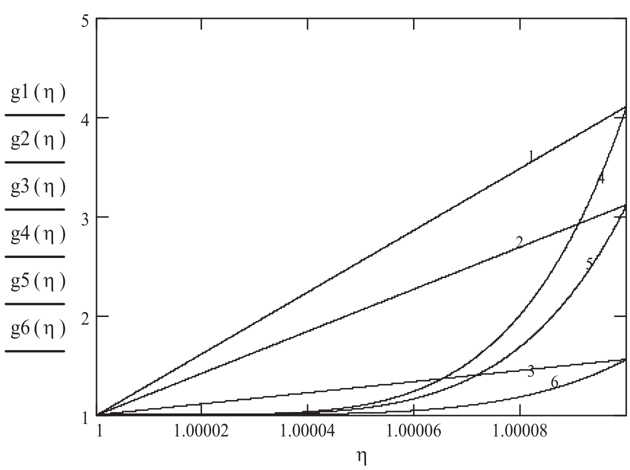

Figure 6 The density profiles: (1) $g1 - k_p = 0.1$, $n = 1$; (2) $g2 - k_p = 0.2$, $n = 1$; (3) $g3 - k_p = 0.6$, $n = 1$; (4) $g4 - k_p = 0.1$, $n = 5$; (5) $g5 - k_p = 0.2$, $n = 5$; (6) $g6 - k_p = 0.6$, $n = 5$ with fixed value of $\gamma = 1.4$, $G = 10$ and $\beta = 0.5$.

the shock wave microstructure have reverse trend as compared to the case $n = 5$. For $n = 1$ the density increases linearly with increasing η where as for $n = 5$ the profiles show a polynomial growth in density distribution with increasing η as expected.

7. Conclusions

In this paper, we used the nonstandard analysis to derive the shock wave jump functions for one-dimensional strong converging shock wave in the mixture of a gas and small solid particles. Here, it is assumed that the solid particles are continuously distributed in the gas. The flow field across the shock wave is modeled in terms of Heaviside functions. It is observed that the predistributions of the Heaviside functions for density, velocity and pressure jump conditions are coincident across an inviscid shock wave in a dusty gas. Also, it is found that the effect of increasing value of k_p, mass concentration of solid particles in the mixture γ, the specific heat ratio and G, the ratio of the solid particles to that of initial density is to decrease the velocity and density across the shock wave. Further, the effect of the values of $n > 1$ on the density and velocity profiles is to completely reverse the trend of profiles with respect to the case $n = 1$.

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