# PREFACE

The thesis consists of six chapters. In Chapter 1, review of wavelets, Legendre scaling basis functions, Abel integral equation, operational matrix of differential and operational matrix of integration and various terms related with fractional calculus is presented. In chapters 2-6, stable algorithms for numerical evaluation of generalized Abel integral equations and systems are evolved by use of different orthonormal wavelets and polynomial bases.

The fractional order singular volterra integro-differential equations which are typical for the theory of Brownian motion based on the fractional Langevin equation and Basset force. Such a differential equation also appears in the theory of classical Brownian motion based on the fractional Langevin equation which was introduced by Kubo (1966). Singular integro-differential equation has many applications in several fields like magnetic field induction in dielectric media (Tarasov (2009)), anomalous diffusion (Sokolov and Klafter (2005)), micro and nano-technology (Grzybowski *et al.* (2005)), velocity fluctuation of a hard-core Brownian particle (Widom (1971)), transfer of dust and fog particles in atmosphere (Aartijk and Clercx (2010)). It is worth noting that analytical evaluation of singular integro-differential equation of fractional order is difficult so the numerical methods are important. Chapter 2 of the thesis is an effort in this direction. This chapter deals with construction of operational matrices for the fractional integration and differentiation. These newly constructed operational matrices have been used to propose an algorithm based on the Legendre scaling functions for numerical evaluation of singular volterra integro-differential equations of fractional order as well as for integer order. Further the proposed algorithm is followed by an error analysis of the constructed algorithm.

The chapter 3 deals with study of linear as well as nonlinear system of generalized Abel integral equations. Abel's integral equation (Abel (1826)) occurs in many branches of science and technology, such as plasma diagnostics and flame studies, where the most common problem of deduction of radial distributions of some important physical quantity from measurement of line-of-sight projected values is encountered. For a cylindrically symmetric, optically thin plasma source, the relation between radial distribution of the emission coefficient and the intensity measured from outside of the radial source is described by Abel transform. The challenging task of reconstruction of emission coefficient from its projection is known as Abel's inversion. The Abel integral equation is given by

$$I(y)=2\int_{y}^{1}\frac{\varepsilon(r)r}{\sqrt{r^{2}-y^{2}}}dr, \qquad 0\le y\le 1 \tag{1}$$

where $\varepsilon(r)$ and $I(y)$ represent, respectively the emissivity and measured intensity, as measured from outside the source (Griem (1963)).

The analytical inversion formula for Eq. (1) is given as (Tricomi(1975)):

$$\varepsilon(r)=-\frac{1}{\pi}\int_{r}^{1}\frac{1}{\sqrt{y^{2}-r^{2}}}\frac{dI(y)}{d(y)}dy, \qquad 0\le r\le 1\,. \tag{2}$$

Singh *et al*. (2009), constructed an operational matrix of integration based on orthonormal Bernstein polynomials and used it to propose a stable algorithm to invert the following form of Abel integral equation

$$I(y)=2\int_{0}^{y}\frac{\varepsilon(r)r}{\sqrt{y^{2}-r^{2}}}dr, \qquad 0\le y\le 1\,. \tag{3}$$

In the year 2010, Singh *et al*. (2010) constructed yet another operational matrix of integration based on orthonormal Bernstein polynomials and used it to propose an algorithm to invert the Abel integral equation (1). This motivated us to look for a stable algorithm which can be used for numerical inversion of the generalized Abel integral equation obtained by joining the two integrals (1) and (3). The operational matrices constructed in chapter 2 are used to propose an algorithm for numerical solution of linear as well nonlinear system of generalized Abel integral equations in chapter 3. This chapter also includes few numerical experiments to show the effectiveness of the proposed algorithms. The convergence criterion for approximate solutions under certain mild conditions is also dealt in the chapter.

In Chapter 4, a numerical wavelet and polynomial method for the solution of a class of system of singular volterra integro-differential equations is proposed. In this chapter, we have constructed the operational and almost operational matrices of integration based on wavelet and orthogonal polynomials to reduce the main problem into linear system of algebraic equations which are computationally most simple and having an advantage of low cost of setting the algebraic equations without using artificial smoothing factors. This chapter also deals with the error analysis of the proposed algorithm together with some numerical examples.

In Chapter 5, we deal with initial and boundary value problem for non-homogeneous fractional order partial differential equations of the kind

$$\frac{\partial^{\alpha} y(x,t)}{\partial x^{\alpha}}+\frac{\partial^{\beta} y(x,t)}{\partial x^{\beta}}=g(x,t),\ \ 0<\alpha,\ \beta\le 2,$$

with initial conditions $y(x,0)=a(x), D_t y(x,0)=c(x)$ and boundary conditions $y(0,t)=b(t), D_x y(0,t)=d(t)$, where, $g(x,t)$ is known continuous function. In this chapter we have constructed operational matrices using Legendre scaling functions as basis. These operational matrices are used for the evaluations of approximate solutions of partial differential equations of the above kind. The algorithm is also accompanied with error and stability analysis and some numerical experiments. The results so obtained are compared with those which are obtained by known methods. The Chapter 5 is based on our paper publised in *Ain Shams Engineering Journal*. http://dx.doi.org/10.1016/j.asej.2016.03.013 2090-4479.

The chapter 6 presents an approximate method for solving nonlinear system of fractional differential equations defined in terms of Caputo derivative. The method used in the chapter Bernstein's operational matrix of fractional differentiation (BOMFD), which is applied to solve nonlinear system of fractional differential equations and fractional stiff system of nonlinear differential equations.

The operational matrix method combined with the typical tau method reduces the fractional systems into system of nonlinear algebraic equations, through which the desired solution of nonlinear system of fractional differential equations. Further fractional systems of different nature are considered to show the effectiveness of the presented method.