# Effects of molecular crowding on stretching of polymers in poor solvent

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(Received 24 June 2008; revised manuscript received 28 January 2009; published 11 May 2009)

We consider a linear polymer chain in a disordered environment modeled by percolation clusters on a square lattice. The disordered environment is meant to roughly represent molecular crowding as seen in cells. The model may be viewed as the simplest representation of biopolymers in a cell. We show the existence of intermediate states during stretching arising as a consequence of molecular crowding. In the constant distance ensemble the force-extension curves exhibit oscillations. We observe the emergence of two or more peaks in the probability distribution curves signaling the coexistence of different states and indicating that the transition is discontinuous unlike what is observed in the absence of molecular crowding.

DOI: 10.1103/PhysRevE.79.051801

PACS number(s): 82.35.Jk, 36.20.Ey, 64.90.+b

## I. INTRODUCTION

Every major change in cellular systems involves mechanical movement at a single molecule level. Recent advances in single molecule force spectroscopy (SMFS), e.g., optical tweezers, atomic force microscope, etc., have allowed cellular processes to be examined at the single molecule level [1-4]. Moreover, by applying a force of the order of pN on an isolated protein in vitro, the response of the force has been studied in order to understand the elastic, structural, and functional properties of proteins [2,5,6]. Modeling of proteins using simplified interactions amenable to statisticalmechanics analysis has been used extensively to theoretically understand the outcomes of these experiments [7–10]. Among the theoretical approaches lattice models despite their simplicity have proved to be quite predictive and have provided much important information about cellular processes [11–15].

New challenges have arisen in the area of protein folding when removed from an artificial, controlled environment in *vitro* and relocated to the cellular environment [16-19]. Cells have a very crowded environment because they are composed of many different kinds of biomolecules that may occupy a large fraction ( $\approx 40\%$ ) of the total volume (Fig. 1). This condition leads to a phenomenon called "volume exclusion" which is caused by the steric repulsion between different molecules [20,21]. It is now known that molecular crowding can influence the stability, dynamics, and function of proteins. Thus far, in theoretical modeling, the cellular environment has been considered as homogeneous with each monomer (amino acid) interacting with all its nearest neighbors [14,15] of which there is a fixed number for each given lattice. In vivo, the cellular environment and the interactions involved in the stability of proteins are no longer homogeneous. It is essential to somehow model this disordered media say via the introduction of randomness into the connectivity of the underlying lattice. Linear polymer chains trapped in a porous (random) media have been studied in detail because of the technological importance in filtration, gel permeation chromatography, etc. [22–24]. However, the response of a polymer to a force remains an elusive problem which has thus far not been studied in detail.

The aim of this paper is to study the effect of an applied force on a polymer in an artificially reproduced environment with molecular crowding similar to what is observed in a cell. In general the effect of the force on the reaction coordinate (end-to-end distance) [25] is mainly determined by the competition between a loss of configurational entropy and a gain in internal energy due to the stretching of the protein caused by the applied force. The confinement of the proteins to a restricted portion of a cell leads to a further loss in entropy because of the molecular crowding and this may affect the behavior of the proteins. Since SMFS experiments are performed on proteins with only a few monomers we expect that modeling the effect of a force on a *finite* chain in a disordered environment may provide a better understanding of the unfolding process in vivo. In disordered media information about the dependence of the reaction coordinate and the probability distribution of the reaction coordinate on the parameters of the system are difficult to obtain analytically and one therefore has to resort to numerical studies.

The freely jointed chain (FJC) and the wormlike chain (WLC) (developed for polymers) [9,10] are models which



FIG. 1. Schematic representation of a polymer chain in a disordered medium. Black and white circles represent available and unavailable sites, respectively. The imposed restriction of unavailability of certain sites gives rise to a "volume exclusion" of about 40% as seen in the cell. One end of the polymer chain is kept fixed while a force F is applied at the other end. Dashed line corresponds to nearest-neighbor attractive interaction among monomers.

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have been used extensively in order to understand the unfolding process and they correctly describe the force extension (F-x) curves in the intermediate and high force regime. However, these models ignore excluded volume effects in their description. Off lattice simulations do provide much important information about unfolding processes [11], but the exact density of states over the entire force regime is difficult to obtain. Recently, exact and complete information about the density of states was obtained using exact enumeration technique. Such studies can reproduce many qualitative features of the force-induced transitions as well as reveal many new insights into the mechanisms governing the cellular processes [12–14,26–28].

## II. MODEL

We model the polymer chain as self-attracting selfavoiding walks (SASAWs) whose one end is kept fixed and a force is applied at the other end. The confinement imposed by molecular crowding has been modeled by generating site percolation clusters [23,24,29] on the underlying square lattice of M sites with a fixed density p of available sites. In order to model the "volume exclusion," which is about 40% of the total volume [20,21], we choose a density p of available sites just above the percolation threshold  $p_c$ (=0.5928) [24]. It means that a SASAW cannot access about 40% of the volume as happens in the cell. The percolation process gives rise to a distribution of clusters, which will vary in size from isolated disorder sites up to clusters with an extent spanning the entire system. Due to the randomness of the clusters the ground state of the system will not be fixed (as is the case on a regular lattice), but may vary with each realization. Because of the steric repulsion between the molecule of interest and the surrounding biomolecules [20,21], in the present model we do not consider interactions between the polymer and the disorder sites. The model can readily be extended to three dimensions (3D) [14]. The qualitative nature of the F-xcurve should remain the same. But in the case of 3D lattices the density of states can be calculated exactly for walks of length up to only 20 steps and one therefore has to use other methods, e.g., Monte Carlo simulation [14], molecular dynamics [11] etc., to study such systems.

Since molecular crowding is a dynamical phenomenon involving the appearance, disappearance, and movement of voids, the internal structure of the cell changes continuously. To model this we perform an averaging over many realizations while keeping  $p_c$  constant. This ensures that the concentration of the crowding agent does not change, but the internal structure may change as happens in the cell. Since we are near to percolation threshold, every disorder realization constitutes the partition function which is nonzero. The partition function of the *i*th realization of the disorder configuration may be written as

$$Z_{i} = \sum_{(N_{p}, |x|)} C_{N}^{(i)}(N_{p}, |x|) u^{N_{p}} \omega^{|x|}.$$
 (1)

Here  $C_N^{(i)}(N_p, |x|)$  is the number of distinct conformations of walks of length N in the *i*th realization of the disordered with  $N_p$  nonbonded nearest-neighbor pairs and whose end points



FIG. 2. (Color online) (a) and (b) show the variation of  $\ln Z$  for two different sets of realizations (5000 each) at T=0.3 and F=0.3. In (c) we plot a histogram of  $\ln Z$ . Note that all the three histograms overlap within statistical error bar. (d)–(f) are for T=1.0 and F=1.0.

are a distance x apart.  $\omega$  is the Boltzmann weight for the force defined as  $\exp[\beta(\mathbf{F}\cdot\hat{x})]$ , where  $\hat{x}$  is the unit vector along the x axis.  $\beta$  is defined as  $\frac{1}{kT}$ , where k is the Boltzmann constant and T is the temperature.  $u = \exp(-\beta\epsilon)$  is the Boltzmann weight of nearest-neighbor interactions with energy  $\epsilon$ . In the following, we set  $\epsilon/k=1$  and focus our discussion on the force-induced globule-coil transition on the percolative lattice. Throughout this paper  $[\cdots]$  denotes an average over various realizations and  $\langle \cdots \rangle$  denotes thermal averaging. In the present study we enumerate all possible walks of length N=45 steps on given percolation clusters using 15 000 realizations  $(N_{tot})$  of the percolation process. For the sake of comparison we also enumerate walks of the same length on the square lattice with no disorder. It has been shown in previous studies that the chain length considered here is sufficient to predict the correct qualitative behavior and while increasing the chain length yields better estimates of say the phase boundary; the qualitative features of the phasediagram remain the same [27,28].

The limit  $T \rightarrow \infty$  corresponds to pure self-avoiding walks (SAWs), i.e., polymers in a good solvent [8]. We reproduced the scaling proposed by Blavatska and Janke [24] for SAWs on a percolation cluster as well as for the square lattice. In Figs. 2(a) and 2(b) we plot  $\ln Z_i$  with *i* for two different sets (5000 each) of realizations. In Fig. 2(c), we have plotted the histogram of  $\ln Z$ . The resulting distributions overlap (within statistical error bar) with each other having a common peak.

### **III. RESULTS**

In studying properties of disordered systems one encounters two types of averages in the literature, namely, the an-



FIG. 3. (Color online) Figures show the variation of extension with force. In (a) we show some of the representative *F*-*x* curves for different disorder realizations at T=0.3. (b) and (c) show the approximate annealed average and sample-averaged quenched disorder at low T (0.3) and high T (1.0). In these plots, we also show the pure case for comparison. The individual realization and approximate annealed average exhibit multistep plateaus which are absent in the sample-averaged quenched and pure cases.

nealed average and the quenched average [22,30]. In the case of annealed averaging, conformational changes in the disorder have time scales which are very fast compared to the motion of the polymers. Therefore, in the annealed case, averaging has to be done over all possible conformations of the disorder implying that  $N_{tot} \rightarrow \infty$ . In the case of quenched disorder the biopolymers unfold very fast compared to the conformational changes (if any) of the disorder sites. In fact neither type of averaging is entirely appropriate as far as the unfolding of a biopolymer in the presence of continuously moving crowding agents is concerned. It may be noted that unfolding takes place on time scales on the order of microseconds to a few seconds. In such a relatively short time span the internal structure of a cell can change substantially but not enough to access all possible conformations and hence increasing the number of realization toward the limit of infinity is not an experimental prerequisite. In view of the time scale involved in unfolding, we restrict ourselves to averaging over a finite but large set of realizations which may roughly mimic a real system in vivo. The number of realizations considered here is almost 20 times more than previous studies [23]. The approximate annealed average of the reaction coordinate in this case may be defined as

$$\langle [x] \rangle_D = \frac{\frac{1}{N_{tot}} \sum_i \sum_{(N_p, |x|)} x^{(i)} C_N^{(i)} (N_p, |x|) u^{N_p} \omega^{|x|}}{Z}, \quad (2)$$

where  $Z = \sum_i Z_i / N_{tot}$  and the summation is over  $N_{tot}$  realizations of the disorder. It may be noted that in the limit  $N_{tot} \rightarrow \infty$  the annealed average and the pure system will give the same results and the two summations of Eq. (2) can be interchanged. In this case there are roughly  $2^M$  possible conformations and hence summation over all of them is computationally impossible. The suffix "D" in Eq. (2) corresponds to an approximate annealed averaging that depends on the disorder realization. The quenched average of the reaction coordinate for the *i*th realization is given by

$$\langle x^{(i)} \rangle_Q = \frac{\sum_{(N_p, |x|)} x^{(i)} C_N^{(i)} (N_p, |x|) u^{N_p} \omega^{|x|}}{Z_i}.$$
 (3)

The sample average over Eq. (3) can be written as

$$[\langle x \rangle_Q] = \sum_i \langle x^{(i)} \rangle_Q / N_{tot}, \tag{4}$$

which we call an average over quenched disorder.

## A. Analysis in constant force ensemble

In Fig. 3(a) we show some of the representative plots of quenched averaged reaction coordinate obtained from Eq. (3)for different realizations. In Fig. 3(b) we plot the extension versus the force for the approximate annealed disorder average  $(\langle [x] \rangle_D)$  and the sample averaged quenched disorder  $([\langle x \rangle_{\Omega}])$  at low temperature (T=0.3). For the sake of completeness we also plot  $\langle x \rangle$  versus F for the pure case at the same temperature. In the approximate annealed case we find multistep plateaus which are absent in the pure case. Such plateaus have been observed in the pure case at much lower T during force induced unfolding [14,28]. It appears that disorder reduces the entropy of the system (making the solvent poorer) which causes the emergence of such plateaus. We expect qualitatively similar behavior for other sets of realizations with a finite  $N_{tot}$  and these plateaus will vanish and approach the pure case in the limit  $N_{tot} \rightarrow \infty$ . Notably every specific realization shows such plateaus, which are induced by the disorder. However sample averaging [Eq. (4)] over quenched disorder [Eq. (3)] smoothen the plateaus. At high temperature the entropy of the system is high enough that any effects of disorder vanish. As a consequence of this the force-extension curve overlaps with the pure case as can be seen from Fig. 3(c).

#### B. Analysis in constant distance ensemble

It is known that in protein folding, the protein acquires a unique (native) conformation among a large number of con-



FIG. 4. (Color online) Same as Fig. 3, but at constant distance ensemble. The strong oscillations can be seen here for individual disorder realizations and in the approximate annealed case, but absent in the pure and sample-averaged quenched cases.

formations and many biological functions depend on this. However, in a "disordered protein" [31,32] there is no unique native conformation and diseases such as Alzheimer are the result of that. In fact the ground-state entropy of such molecules is comparatively larger than the native conformations (unique in the case of a protein) but much less than the entropy associated with the globule of the same length [31]. It is therefore important to have a reference calculation which predicts the shape of these curves in quasiequilibrium in order to precisely estimate kinetic effects in the experiments or in molecular or Langevin dynamics simulations [13]. In the case of polymers we find that the residual entropy of the globule on the percolation cluster is much less than the entropy of the globule (pure case) on the square lattice. Moreover, on the percolation cluster, the number of interactions per monomer is not the same as on a pure square lattice and this induces a heterogeneity in interactions along the chain even for a homopolymer. The number of nearest neighbors in this case varies between 0 and 2 in comparison to the pure lattice where this number is always 2. Because of this the present model may give some intrinsic features close to such molecules because of the induced heterogeneity in the interaction. Since atomic force microscopy (AFM) works in the constant distance ensemble (CDE) [26,28], we also calculate  $\langle F \rangle$  in the CDE and plotted it against x (Fig. 4). It is interesting to note that the model presented here shows oscillations in the *F*-*x* curve for the approximate annealed case. However, for the sample-averaged quenched disorder and pure cases, such oscillations are absent.

### C. Probability distribution

It has been shown that the probability distribution curves P(x) provide important information about cellular processes [26,27]. P(x) can be calculated from the following expressions for the approximate annealed and averaged quenched cases:

$$P_{A}(x) = \frac{\sum_{i} \sum_{N_{p}} C_{N}^{(i)}(N_{p}, |x|) u^{N_{p}} \omega^{|x|}}{\sum_{i} Z_{i}}$$
(5)

 $P_{Q}(x) = \frac{1}{N_{tot}} \sum_{i} \left[ \frac{\sum_{N_{P}} C_{N}^{(i)}(N_{P}, |x|) u^{N_{P}} \omega^{|x|}}{Z_{i}} \right].$ (6)

For the pure, approximate-annealed, and sample-averaged quenched disorder cases, the probability distribution curves shown in Figs. 5 and 6 display qualitatively similar behavior at low and high forces with T=0.3. For F=0 we find a peak position corresponding to the collapsed state. At high F it peaks around the stretched state. The collapse transition is of second order in two dimensions (2D) [33] which can be seen from the probability distribution curve shown in Fig. 5(b) for the pure case. Near the transition point the peak broadens, which indicates that the transition is continuous [Fig. 5(b)]. However, in the approximate annealed case [Figs. 6(a)-6(1)], one can see the emergence and disappearance of peaks for different F at T=0.3. With increasing F we see that the height of one peak increases while others decrease. We find at many different forces that the height of two peaks (at different positions) becomes equal indicating the coexistence of two states. These features are observed for each and every realization of disorder. This gives the signature that the transition is no longer continuous.



FIG. 5. (a)–(c) show the probability distribution curves for the pure and sample-averaged quenched cases at T=0.3 at different forces. (a) For F=0 (pure case), the peak position corresponds to the folded state, while at F=1.1 the peak position corresponds to the stretched state. (b) Near the transition temperature the peak broadens indicating the transition is continuous. (c) is for the sample averaged quenched disorder. One can see the effect of disorder induced entropy on the sample-averaged quenched disorder which shows the broadening of the peaks at F=0 and F=1.1.

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FIG. 6. (a)–(l) show the variation in the probability distribution curves for the approximate annealed case for different forces at fixed temperature T=0.3. The appearance of peaks at different F shows the existence of intermediate states induced by the force. The coexistence of peaks at different F gives a clear signature that the transition is no longer continuous.

#### **IV. CONCLUSIONS**

Recently, Yuan *et al.* [34] studied the effect of concentration of dextran (disorder sites) which is quite below the  $p_c$  on the mechanical stability of protein molecules. They found that average force increases with concentration in presence of crowding agents (dextran). For the low concentration, force increases linearly, but above than 30% concentration it is no more linear. Our early calculation for small realization does show such behavior and we find that there is a crossover when concentration approaches to  $p_c$  [35].

In this work we have reproduced most of the studied behavior of a polymer chain at force F=0, e.g., scaling of the probability distribution curve (SAWs) for pure and disordered lattices [24]. Our results are also consistent with earlier studies, i.e., at F=0, x is less than the pure case for approximate annealed averaging while greater for sample averaged quenched case [Fig. 3(b)] [22,23]. The approximate annealed average in fact represents the typical effect of the quenched disorder case. However, entropy induced by sample averaging over quenched realization smoothens the reaction coordinate and we therefore observe a monotonic increasing effect in the force-extension curve. Similar features have also been seen for individual disorder at high temperature where entropy smoothens such plateaus.

We have modeled the unfolding process in a cell where the polymer is surrounded by noninteracting biomolecules. Our results based on exact enumeration technique clearly show that disorder induces intermediate states because of the heterogeneity in the interaction. The occurrence of peaks of equal height in the probability distribution shows the coexistence of two states at different forces suggesting that the transition is discontinuous in the case of finite chains. This may be because the underlying structure (percolation clusters) is no longer a regular lattice but a fractal [24]. At this stage long chain simulation is required to verify this. It is important to recall that experimentally observed coil-globule transition is also first order. In constant distance ensemble system probes local ground state and average of force is calculated from them. Therefore, heterogeneity in nonbonded nearest-neighbor interaction induced by disorder shows such strong oscillation in the F-x curve. At this stage additional work is needed to understand the effect of a force in the cellular environment.

### ACKNOWLEDGMENTS

We thank D. Dhar, S. M. Bhattacharjee, and K. P. N. Murthy for many helpful discussions related to disorder averaging. We also thank I. Jensen for a critical reading of this paper and his comments. Financial support from DST New Delhi and UGC, New Delhi are gratefully acknowledged. We also acknowledge the generous computer support from MPIPKS, Dresden, Germany.

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