

## **LITERATURE REVIEW**

### **2.0 General**

A review of the application and limitations of basic physical laws employed in the literatures to study the flow behavior in a single cohesive crack is presented. Also a brief overview of fracture mechanics in concrete structures followed by the applicability of linear elastic fracture mechanics (LEFM) in concrete gravity dams has been carried out. LEFM approach has been investigated to estimate certain fracture parameters like residual crack mouth opening displacement (CMOD), fracture toughness, and crack length. An estimate of location of crack at upstream face of the dam is discussed. The effect of creep in fracture process zone (FPZ) on CMOD-rate is reviewed. The modeling of transient uplift pressure in cracks of the concrete gravity dam under dynamic loading conditions is explored in the literatures. The objectives of present research work are presented on the basis of this exhaustive literature review.

### **2.1 Hydraulics of Flow in Cracks**

Fractured porous media consist networks of interconnected fractures and pores. Fractures, generally considered as fast transport pathways (Zimmerman and Bodvarsson, 1996) are large ( $10^{-4}$  m to  $10^{-2}$  m) pores and crevices (Tsang and Tsang, 1987; Fischer and Fluhler, 1998). The study of transport processes in fractured porous media is important to understand the single and multi-phase flow of groundwater and to predict the fate of pollutants in aquifer contaminated by industrial, agricultural and radioactive wastes (Sung-Hoon et. al.2003). It also plays important roles in studying the

transient uplift pressure due to seismicity (Saouma et. al. 1991; Slowik and Saouma, 2000) or time varying reservoir level in concrete gravity dams, leakage and stability of dam foundations (Terzhagi,1936;Bazant, 1975), mine drainages, slope stability and hydraulic fracturing (Cristianovich and Zheltov, 1955; Cleary, 1980; Logan et. al. 2000; Jeffery et. al. 2001; Murdoch and Slack, 2002; Soliman et. al. 2004; Bahrami and Mortazavi, 2008; Bagherian et. al. 2010; Sarris and Papanastasiou, 2012 etc.).

## 2.1.1 Basic Equations of Flow in Single Crack

Flows within the fractures are governed by (1) mass conservation law and (2) momentum equation. While employing these basic equations to study flow in cracks, various assumptions are made and discussed separately to have a clear picture of the forms and applicability of these equations to crack flow problems.

### 2.1.1.1 Mass Conservation Law

This law requires that the rate of increase or decrease of fluid mass in a finite elemental control volume situated in the flow field be equal to the net rates of inflow or outflow. Based on the principle of conservation of mass, a 2-D continuity equation can be written according to continuity equation given by Illangasekare et. al. (1992).

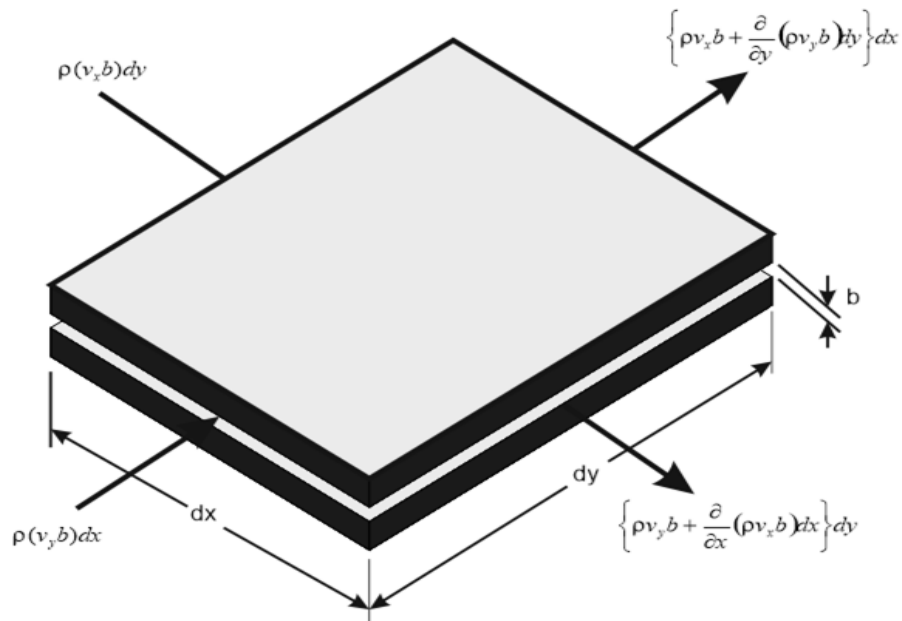
$$\frac{\partial(\rho b v_x)}{\partial x} + \frac{\partial(\rho b v_y)}{\partial y} = \frac{\partial(\rho b)}{\partial t} \quad (2.1)$$

Where,  $v_x$ ,  $v_y$  = velocity of fluid in  $x$  and  $y$  directions respectively, at a time  $t$ ;  $b$  = crack aperture;  $\rho$  = density of the fluid (Fig.2.1).

Equation (2.1) is derived without sink or source. The above equation can be modified by considering any one or combinations of following assumptions:

(i) flow is steady (ii) flow is incompressible (iii) fluid density changes with time but is independent of space and (iv) crack aperture is varying linearly with fluid pressure  $p$  as  $p = K_n b$ , where  $K_n$  is normal stiffness of crack wall.

1-D continuity equation is widely used in the literature (Slowik and Saouma, 2000; Sarris and Papanastasiou, 2012; Javanmardi et. al., 2005a, 2005b; Pekau and Zhu, 2008) with assumption that fluid leakage through walls of the cracks are negligible (i.e. walls of the crack are impermeable).



**Fig.2.1. Infinitesimal element used in the derivation of continuity equation (Illangasekare et. al. 1992).**

For any cross sectional area  $A$  normal to flow direction  $x$ , 1-D continuity equation can be written as

$$\frac{\partial(\rho Q)}{\partial x} = \frac{\partial(\rho A)}{\partial t} \quad (2.2)$$

Where,  $Q$  is the discharge through the crack.

Continuity equation for two phase flow in cracks are written and supplemented with additional conditions. The equation of continuity for a wetting fluid (e.g. water) and a non-wetting fluid (e.g. gas) can be derived on basis of Bear (1979), Bastian (1999), Helming (1997) and Reichenberger et. al. (2005) equations. These equations are completed with Brooks-Corey (Brooks and Corey, 1964) constitutive relations.

### 2.1.1.2 Momentum Equation and Darcy Law

The most general form of Navier-Stokes equations for fluid flow in a single fracture can be written as per the equation of White (1994).

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = \mathbf{F} - \frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{v} \quad (2.3)$$

Where,  $\mathbf{v} = (v_x, v_y, v_z)$  is the velocity vector;  $\mathbf{F}$  is the body force per unit mass;  $\nu$  is kinematic fluid viscosity;  $\nabla$  is gradient operator and  $p$  is the pressure. In most of the cases, the body force is gravity. Assuming that fluid density is uniform and replacing  $p$  by  $p + \rho g z$ , the steady state Navier-Stokes equation can be written as

$$\nu \nabla^2 \mathbf{v} - (\mathbf{v} \cdot \nabla) \mathbf{v} = \frac{1}{\rho} \nabla p \quad (2.4)$$

The Navier-Stokes equation (2.4) is non-linear due to the presence of advective acceleration term  $(\mathbf{v} \cdot \nabla) \mathbf{v}$  and so, it is very difficult to solve this equation. Consequently the Navier-Stokes equations are further simplified to Stokes or Reynolds equations.

The Stokes equations are derived from the Navier-Stokes equation after neglecting the advective acceleration terms. The reduced Reynolds number were derived, based on the basis of estimates of the order of magnitude of the various terms in the steady state Navier-Stokes equation and written as

the product of traditional Reynolds number and some geometric parameter. To ensure the negligible effect of advective acceleration term in Navier-Stokes equation, the reduced Reynolds number must be appreciably less than unity. Under this condition, the Navier-Stokes equation (2.4) reduces to Stokes equation

$$\rho \nu \nabla^2 \mathbf{v} = \nabla p \quad (2.5)$$

For few simulated fracture profiles, the Stokes equation (2.5) has been solved numerically (Brown et al. 1995; Mourzenko et. al. 1995). But it has limited application due to computational difficulty.

To overcome the limitations of Stokes equations Reynolds lubrication equations are derived under the assumption: (i) effect of geometric parameter used in reduced Reynolds number is negligible (ii) velocity profile is parabolic at every location in fracture plane and (iii) velocity satisfies the no-slip boundary condition at fracture walls. Under these assumptions, the 2-D Reynolds lubrication equation is written as

$$\frac{\partial}{\partial x} \left( b^3 \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left( b^3 \frac{\partial p}{\partial y} \right) = 0 \quad (2.6)$$

On the basis of some studies (e.g. Brown et al. 1995; Mourzenko et. al. 1995; Yeo et. al. 1998; Nicholl et. al. 1999), a consensus emerged that the Reynolds equation overestimates the flow rate as much as 100%. Later the Reynolds equation were improved after including fracture tortuosity in the definition of fracture geometry (Ge, 1997; Waite et. al. 1999). Yeo and Ge (2005) have discussed the issues about the range of applicability of Reynolds equation.

Most widely used conceptual model for fracture is two smooth parallel walls (Boussinesq, 1868) separated by a uniform aperture  $b$ . For one dimensional flow, the Navier-Stokes equation can be solved (Krantz et. al. 1979) exactly for this geometry because nonlinear term vanishes identically. Solution by

Tsang and Witherspoon (1981) yields a parabolic velocity profile with no-slip condition at walls. Total velocity flux, after integrating the velocity profile is (Zimmerman and Bodvarsson, 1996)

$$Q = -\frac{\rho w b^3}{12\nu} \frac{\partial p}{\partial x} \quad (2.7)$$

Where  $w$  is the depth of fracture. This result is known as local cubic law (LCL) because transmissivity ( $T_0 = \frac{w b^3}{12}$ ) is proportional to the cube of the aperture. Also, the equation (2.7) is known as Darcy law for fracture due to its similarity with Darcy equation used in porous media. This model, usually applied in 1-D flow in fractures having wide channel and smooth parallel walls, are found to be valid (Sausse and Genter, 2005). However many laboratory experiments (e.g. Raven and Gale, 1985; Pyrak-Nolte et. al. 1987; Durham and Bonner, 1994) and some field studies (e.g. Rasmuson and Neretnieks, 1986; Novakowski et. al. 1985, 1995; Raven et al. 1988) indicate that parallel plate concept for flow in fractures are not adequate.

Retaining the same assumptions, as was used for the derivation of LCL equation, Christian et. al. (2010) modified the LCL equation for penny shaped cracks after taking into account the displacement-length scaling laws for opening mode (Vermilye and Scholz, 1995; Olson, 2003; Schultz et. al. 2008) and showed that discharge varies as fifth power of crack aperture. Because of this high degree of non-linearity with crack aperture than LCL, they termed the derived equation as “quintic” law.

In addition to parallel plate model, approximate solutions are also obtained by perturbation methods for sinusoidal fracture walls (Basha and El-Asmar, 2003; Brush and Thompson, 2003; Sisavath et. al. 2003).

However, in real cracks, crack wall geometry is neither parallel nor smooth. Also crack walls may have leakages. Therefore to simulate the flow in real cracks, equation (2.7) has been widely discussed and modified in the

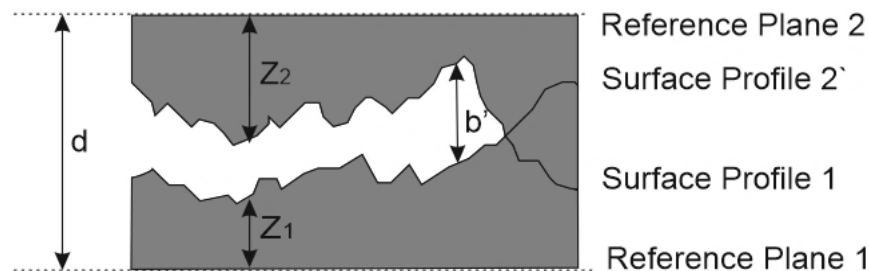
literatures (e.g. Zimmermann and Bodvarsson, 1996; Kim et. al. 2003) to address some of the following issues: (i) how the crack wall geometry is to be represented, (ii) how the crack aperture is to be measured, (iii) is the LCL or Darcy law valid for convergent or divergent crack profile, (iv) what is the range of validity of Darcy law, (v) how Darcy law is to be modified for rough crack walls and (vi) what is the modification in the Darcy law if no-slip boundary condition is violated etc.

### *Crack Wall Geometry*

Real fracture surfaces are very rough, unevenly distributed and are in contact with each other at some locations. Work of Thompson and Brown (1991) suggests that roughness significantly contribute to the hydro-mechanical behavior of fractures. The study of Zimmermann and Bodvarsson (1996), shows that the mean mechanical aperture of a fracture is greater than hydraulic aperture used in LCL equation. To measure the fracture aperture in  $x - y$  plane, two surface height functions,  $z_1(x, y)$  and  $z_2(x, y)$ , are used. These functions measure the distance of surface points from two parallel reference planes located above and below the crack walls in the matrix (Fig.2.2).

So aperture, measured perpendicular to the two reference plane, is defined as

$$b(x, y) = d - z_1(x, y) - z_2(x, y) \quad (2.8)$$



**Fig.2.2. Two rough fracture surface profile separated by distance  $b$  along with two reference plane separated by distance  $d$  (Zimmerman and Main, 2004).**

The information about surface height functions are not known hence aperture  $b$  in equation (2.8) is treated as random variable and fractures are characterized in terms of small number of statistical parameters.

Lognormal and Gamma probability distribution functions are commonly used to fit the aperture distributions (Tsang and Tsang, 1987; Renshaw, 1985; Zhan, 1998; Fetter, 1999). Since zero-aperture cannot be represented by Lognormal or Gamma distribution, these probability density functions (PDF) are not sufficient. Oron and Brian (1998) compared the numerical fracture sample to the measured histograms of Brown et al. (1986) and Hakami and Larsson (1996). They concluded that the skewed form of aperture distribution is the result of contact and deformation and it can be shown that the surface may quite accurately be described by "shifted" Gaussian model.

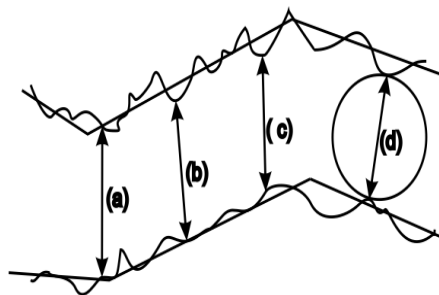
For stationary stochastic process (Box and Jenkins, 1976), two commonly used models of power spectra are exponential and Gaussian models. If  $z_0$  is a parameter which gives an indication of correlation length then, for exponential model the surface is effectively uncorrelated for  $z > 4 z_0$  whereas for Gaussian model the correlation is negligible for  $z > z_0$  Patir and Cheng (1978) solved the Reynolds equation (2.6) for Gaussian surface using finite difference technique.

A profile  $z(x)$  is said to be self-affine if  $z(\lambda x) = \lambda^{H_z} z(x)$ , where  $H_z$  a constant is called Hurst exponent (Zimmerman and Main, 2004). A self-affine profile has a power spectrum as  $P(f) = C f^{-\alpha}$ , Where  $\alpha = 2H_z + 1$  (Adler and Thovert, 1999);  $f$  is wave number and  $C$  is a constant. Self-affine power spectrum has been observed in various study of crack wall geometry (e.g. Brown and Scholz, 1985; Mandelbrot, 1982; Power and Tullis, 1991). Generally rough fracture surfaces possess self-affine fractal properties with Hurst exponent  $H_z = 0.8$  [Brown and Scholz, 1985; Poon et al. 1992; Schmittbuhl et. al. 1995). However in practice it is not possible to detect a dominant wavelength for self-affine curve (Oron and Brian, 1998).



### *Measurement of Crack Aperture*

In LCL equation (2.7), it is not clear how to measure aperture  $b$ . Equation (2.8) is used to measure  $b$  vertically. However, there may be the regions where both the surfaces are significantly inclined to global fracture plane. In these regions the vertical measurement may be erroneous. Ge (1997) measured the aperture normal to local orientation of centerline. Mourzenko et. al. (1995) drew a sphere around each point on centerline and increased the sphere until it touched both the walls. Oron and Brian (1998) addressed this issue through non-dimensional analysis of Navier-Stokes equation with Poiseuille flow as a leading order approximation for the solution of flow between parallel plates. On the basis of their analysis, they concluded that aperture for parallel oriented fracture should not be measured on point-by-point basis but rather as an average over certain characteristic length. Various definitions for fracture aperture have been shown in Fig.2.3 (The two rock surfaces are indicated by thick wavy lines. Aperture (a) is the distance between the two rock surfaces measured normal to the nominal macroscopic fracture plane, (b) is the distance between rock surfaces measured to the local normal fracture plane, (c) is the distance between two smoothed-out versions of the fracture surfaces, and (d) is the diameter of the largest sphere that can fit between the rock surfaces). They also showed that, for unmated (diagonal) walls, above method of aperture measurement hold true provided that mean half angle of the channel is limited to 0.5 radians.



**Fig.2.3. Various definitions for fracture aperture  $b$  (Zimmerman and Main, 2004).**

Further in LCL equation, aperture is termed as hydraulic aperture and denoted here as  $b_H$  against the mechanical aperture  $b$  represented by equation (2.8). Elrod (1979) used Fourier transform to solve the Reynolds's equation for a fracture with aperture having sinusoidal ripples and showed for isotropic case

$$b_H^3 = \langle b \rangle^3 \left( 1 - \frac{3}{2} \frac{\sigma_b^2}{\langle b \rangle^2} + \dots \right) \quad (2.9)$$

Where  $\sigma_b$  is the standard deviation and  $\langle b \rangle$  is the mean value of mechanical aperture  $b$ .

Zimmerman et. al. (1991) showed that the above equation is valid up to second order for both sinusoidal and saw-tooth profiles. Renshaw (1995) derived an alternative expression which includes the second order terms of equation (2.9). Dagan (1993) has expressed the equation (2.9) in series form and showed that terms after fourth term vanish, indicating that geometric mean is a very good approximation for hydraulic aperture in lognormal case. The equation (2.9) is in good agreement with several numerical simulations and some laboratory data (Zimmerman and Main, 2004).

### ***Limitations and Modifications of LCL Equation***

The LCL equation was derived for laminar flow in smooth wall fracture. Various investigators (e.g. Hasegawa and Izuchi, 1983; Oron and Brian, 1998; Skjetne et. al. 1999; Zimmerman and Yeo, 2000) using different approaches, concluded that LCL is valid for Reynolds numbers less than about 10. At higher Reynolds number, the relationship between pressure gradient and flux is nonlinear. This nonlinearity is not necessarily due to turbulence, which occurs only at much higher Reynolds numbers. For Reynolds number between 10 to 100, the observed nonlinearity is merely due to the effects of curvature of streamlines (Phillips, 1991) and occurs in the laminar flow

regime. Non-Darcian flow is more likely in single fracture (Sharp and Maini, 1972; Quien et. al. 2005, 2007, 2010; Wen et. al. 2006) due to high flow velocity and hence the high Reynolds number.

When crack walls contact each other at discrete points then fluid will flow through a few preferred paths (channels) which are tortuous and having variable aperture along their length and may or may not intersect each other. Under these conditions LCL equation does not provide the correct result (Gangi, 1978; Brown and Scholz, 1985; Brown, 1987, 1995) at low velocity due to formation of tortuous path at contact points (Iwai, 1976; Brown, 1987; Sisavath et.al. 2003) and aperture regions of recirculation due to streamline separation in flow field (Brush and Thompson, 2003; Bout et. al. 2006). If an equivalent hydraulic aperture found for the open regions of the fracture, the effect of asperity regions can be modeled by assuming that the fracture consists of regions of aperture  $b = b_0$ . Many researchers have found the equivalent hydraulic apertures to be generally valid for LCL equation (Liu, 2005 ; Zimmermann et. at. 1992).

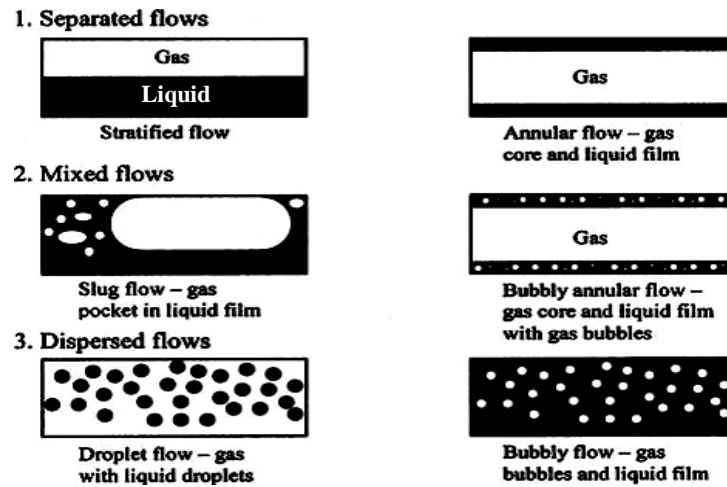
When walls of fractures contain cracks and fissures then no-slip boundary condition is violated and LCL equation does not provide the correct result for volumetric flow. Berman (1953) investigated the effects of wall porosity for the first time. Berkowitz (1989) studied the permeable walls incorporating Brinkman slip boundary conditions in 1-D modeling of fractures having permeable walls and concluded that no-slip underestimates the volumetric flow by as much as 19%. Similar deviations were also noticed by Crandall et. al. (2010). Flow experiments have shown that in such situations slip boundary condition should be applied (Beavers et.al. 1970). Therefore the LCL equation is modified after incorporating slip boundary condition. Study of Tilton and Cortelezzi (2006) have shown that wall permeability can significantly destabilize flows in channels with permeable walls compared to those with impermeable walls. Chang et. al. (2006) and Liu et. al. (2006) have

studied the instability of the flow and used a two-layer approach by coupling governing equations in the separate Newtonian fluid and Darcy/Brinkman porous region by appropriate interfacial boundary conditions. Beavers and Joseph (1967) hypothesized that the tangential component of free fluid velocity at the boundary of the permeable wall is considerably higher than mean seepage velocity within the permeable body and depends on the structure of the permeable material within the boundary region, flow direction at the interface and Reynolds number (Sahraoui and Kaviany, 1992).

This Beaver-Joseph slip flow hypothesis is valid for common viscous fluid at low Reynolds number (Neale and Nader 1974). Mohais et. al. (2011) presented an analytical solution incorporating Beaver-Joseph slip boundary conditions, using perturbation expansion to determine flow. Further Mohais et. al. (2012) extended this study and concluded that the hydraulic conductivities are modified by a numerical factor greater than one.

In two-phase flow, the determination of volumetric flow rate is a challenging problem. The applicability of cubic law for flow rate calculations were investigated by some researchers (e.g. Iwai, 1976; Tsang and Witherspoon, 1981, 1983; Brown, 1987). Fourar et. al. (1993) observed different flow pattern in their parallel plate model experiments. The two-phase air- water flow undergoes different modes of flow under the influences of pressures and different flow conditions. The classification of possible flow pattern by Ishii (1975) and Fourar and Bories (1995) is given in Fig.2.4.

In two-phase-stratified flow, the air-water interface depends on aperture and pressure variations and therefore there is a need of identifying this interface (Nichole and Glass, 2001). Rasmussen (1991) applied the capillary pressure criteria for determining the air-water interface to study the fracture flow under the conditions of partial fluid saturation. Pruess and Tsang (1990)



**Fig.2.4.** Different modes of two-phase flow (Ishii, 1975; Fourar and Bories, 1995).

conducted a numerical analysis based on relative permeability of two-phase flow subject to pressure difference between air and water. The interrelation of multiphase flows, relative permeability, and pressure variation through rock fractures has not been properly identified (Indraratna et. al. 2003). Indraratna et. al. (2003) after following the idea of Keller et. al. (2000), argued the existence of two-phase stratified flow in cracks having greater than 50 micron size and thus neglected the effect of capillary pressure. To compute the flow rate, LCL equation was applied separately for each zone of fracture aperture occupied by air and water. To determine these zones, Indraratna et. al. (2003 ) have derived the expression for locating the air-water interface after taking into account the effects of solubility of air in water, compressibility and density of air and water and fracture deformation in the two-phase momentum conservation equation by Wallis (1969).

To describe flow regime in rough wall fracture it is important to estimate the boundary layer thickness near the fracture wall (Quin et. al. 2011). When the thickness of the boundary layer is greater than the roughness of the fracture, then surface is assumed to be hydraulically smooth. On the other hand, if the thickness of the boundary layer is smaller than the roughness of the

fractures, then surface is treated as hydraulically rough. Under hydraulically rough condition recirculation zones inside the roughness cavities rather than the boundary layer would be the controlling parameter for flow behavior.

The earliest comprehensive work on parallel plate flow of water was conducted by Lomize (1951). He studied the effect of aperture, surface roughness, and aperture variability. Similar experimental studies were also performed by Louis (1969) with nearly identical results. After that flow regime chart which identified five distinct flow regions was formulated from these results (Fig.2.5).

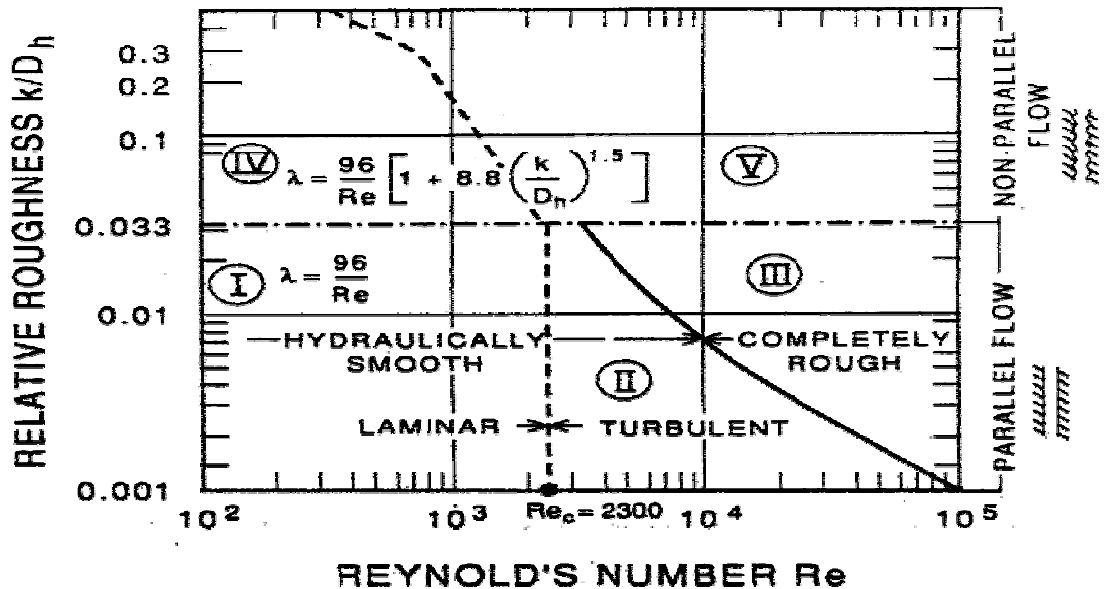


Fig.2.5. Flow regime chart showing flow in cracks with varying degree of roughness (Louis, 1969).

For this chart, Reynolds number and relative roughness is defined respectively as

$$Re = \frac{D_h v}{\nu} \quad (2.10)$$

and

$$\text{Relative Roughness} = \frac{\varepsilon}{D_h} \quad (2.11)$$

Where  $\varepsilon$  is average asperity height or absolute roughness and  $D_h$  is hydraulic radius of the crack which is equal to  $2b$ . For hydraulically smooth condition, the relative roughness is less than 0.033 and transition from laminar to turbulent flow occurs when Reynolds number is approximately equal to 2300. Louis (1969) also developed relationships between velocity  $v$  and hydraulic gradient  $J$  for these regimes in the form

$$v = -KJ^n \quad (2.12)$$

The values of hydraulic conductivity  $K$  and exponent  $n$  are given in the table 2.1 for different flow conditions.

**Table 2.1 Values of Hydraulic Conductivity  $K$  and exponent  $n$  (Louis, 1969).**

Hydraulic Zone	Hydraulic Conductivity, $K$	Exponent, $n$	Flow Condition
I	$\frac{gb^2}{12v}$	1.0	Laminar
II	$\frac{1}{b} \left[ \frac{g}{0.079} \left( \frac{2}{v} \right)^{0.25} b^3 \right]^{4/7}$	$\frac{4}{7}$	Turbulent
III	$4\sqrt{g} \log \left[ \frac{3.7}{\varepsilon/D_h} \right] \sqrt{b}$	0.5	Turbulent
IV	$\frac{gb^2}{12v[1 + 8.8(\varepsilon/D_h)^{1.5}]}$	1.0	Laminar
V	$4\sqrt{g} \log \left[ \frac{1.9}{\varepsilon/D_h} \right] \sqrt{b}$	0.5	Turbulent

For zone IV, nearly same equation was obtained by Witherspoon et. al. (1980). These equations have also been discussed by Wittke (1990). Illangaskareet.al. (1992) developed a finite element computer programme, called CRFLOOD, to compute uplift pressure distribution along the cracks of the concrete gravity dams for all types of flow.

## 2.2 Fracture Mechanics in Concrete Structures

There are three approaches i.e. linear elastic fracture mechanics (LEFM), non-linear elastic fracture mechanics (NLFM) and energy criterion generally applied to study the fracture problems in concrete structures. LEFM and energy approaches are precludes for understanding the fracture phenomenon. NLFM approach is important due to presence of fracture process zone (FPZ) near the crack tip.

In LEFM, the equations of elasticity and Airy stress functions are used to describe the stress and displacement field near the crack tip. For two-dimensional (2-D) problems in polar coordinates, the bi-harmonic Airy stress equation and related stress fields are written in terms of Airy functions (Shi, 2009). These set of equations are supplemented by constitutive relations for plane stress and plane strain conditions and solved for stress and displacement field under given boundary conditions.

Williams (1952, 1957) solutions of elastic stress fields to a traction free edge-cracked problems revealed crack tip stress singularity and played an important role in the early development of LEFM. After that, complex stress function approaches (Westergaad, 1939; Muskhelishvili, 1953) were used to solve Airy equation to find the stress and displacement fields near the crack tip for all three independent modes of deformation at crack tip (Fig.2.6).

Stress field solution near the crack tip for each mode of deformation can be expressed as (Shi, 2009).

$$\sigma_{ij}^{m_0} = \frac{1}{\sqrt{2\pi r}} K_{m_0} f_{ij}^{m_0}(\theta) \quad (2.13)$$



Where  $m_0 = I, II, III$  are the modes of deformations,  $i, j = x, y$  and  $f_{ij}^{m_0}(\theta)$  is the trigonometric function. Fig.2.7 shows the rectangular and polar components stress field near the crack tip.

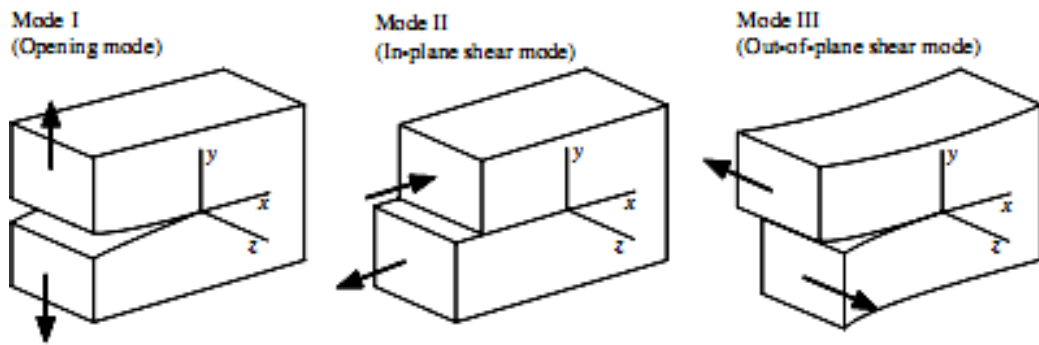


Fig.2.6. Three independent modes of deformation at crack tip (Shi, 2009).

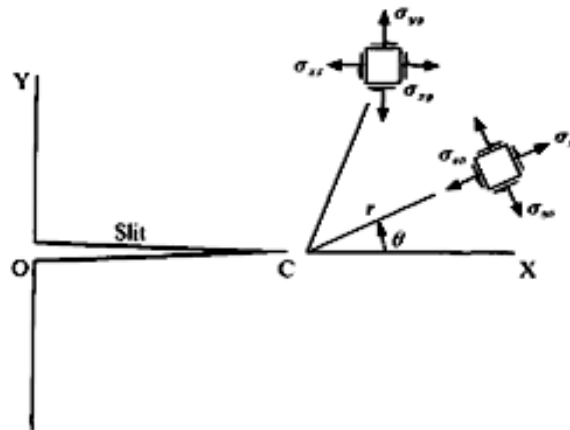


Fig.2.7. Stress field at crack tip C, showing rectangular and polar-coordinate components.

Above equation clearly shows the existence of infinite stresses at crack tip, known as inverse-square-root singularity, which does not reflect the physical reality. However, Irwin (1957) envisioned the physical significance of the constants contained in these solutions and coined the term stress intensity factor (SIF) to characterize the singularity at the crack tip, which turned out to be one of the most important concepts in fracture mechanics. These stress intensity factors for all the three modes of crack deformation is defined in Shi

(2009). Analytical and numerical methods for calculating SIF have been extensively discussed in many text books of fracture mechanics (e.g. Broek, 1984; Anderson, 2005) and hand books (e.g. Tada et. al. 2000) for different crack geometries and loading conditions. SIF is a measure of all stresses and strains. Crack extension will occur when the stresses and strains at the crack tip reach a critical value. This means that fracture must be expected to occur when SIF reaches a critical value called critical stress intensity factor or fracture toughness.

With the crack-tip stress and displacement field now being completely defined in terms of SIF, an energy description of the fracture process needs to be described using Griffith's (1921, 1925) fracture theory. The Griffith energy criterion for fracture states that crack growth can occur if the energy required to form an additional crack can just be provided by the system.

Griffith energy balance for an incremental increase in the crack area  $dA$  (Anderson, 2005), under equilibrium conditions, can be expressed as

$$\frac{dE_T}{dA} = \frac{d\Pi}{dA} + \frac{dW_s}{dA} = 0 \quad (2.14)$$

Where  $E_T$  is total energy;  $\Pi$  is potential energy, and  $W_s$  is the work required to create new surfaces.

Irwin (1956) defined an energy release rate (also called crack driving force)  $G$  as  $G = -\frac{d\Pi}{dA}$ . Similarly the term  $\frac{dW_s}{dA}$  is called crack resistance force (Broek, 1984) and denoted by symbol  $R$ . The crack extension occurs when  $G$  reaches a critical value called fracture toughness (Anderson, 2005).

But crack growth may be stable or unstable, depending on how  $G$  and  $R$  vary with crack size. The condition for stable and unstable crack growth are

$$\frac{dG}{dA} \leq \frac{dR}{dA} \text{ and } \frac{dG}{dA} > \frac{dR}{dA} \text{ respectively.}$$

In LEFM,  $G$  is calculated from elastic energy and is called elastic energy release rate. The relation between  $G$  and  $K$  has been derived in the literature [e.g. Anderson, 2005]. As an example for mode-I this relationship can be written as  $G = \frac{K_I^2}{E'}$ ; where  $E' = E$  (Young modulus) for plane stress and  $E' = E/(1 - \nu_0^2)$  for plane strain and  $\nu_0$  is the Poisson ratio. Compliance techniques are used in LEFM to compute the fracture energy  $G$  and SIF.

To remove the crack tip singularity in the above equations, it is necessary to take into account inelastic material behavior, such as plasticity in metals and fracture process zone (FPZ) in concrete by introducing inelastic-zone concept to the crack tip. Concrete is a heterogeneous material that consists of aggregates and cement pastes bonded together at the interface, and the material is inherently weak in tension due to the limited bonding strength and various preexisting micro cracks and flaws formed during hardening of the matrix. Under external loading, an inelastic zone known as fracture process zone (Fig.2.8) at the crack tip is extensively developed due to micro-failure mechanisms.

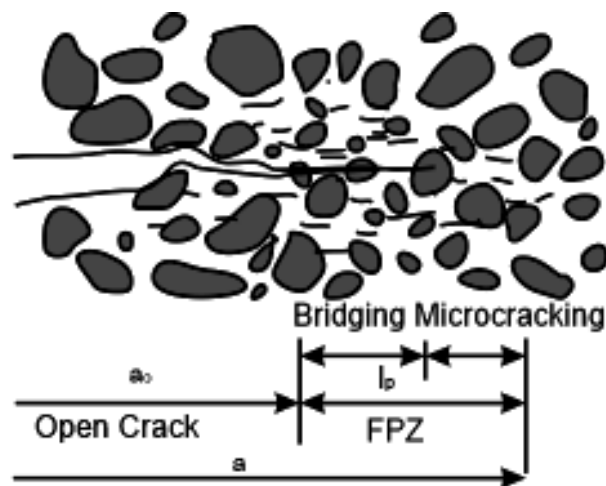
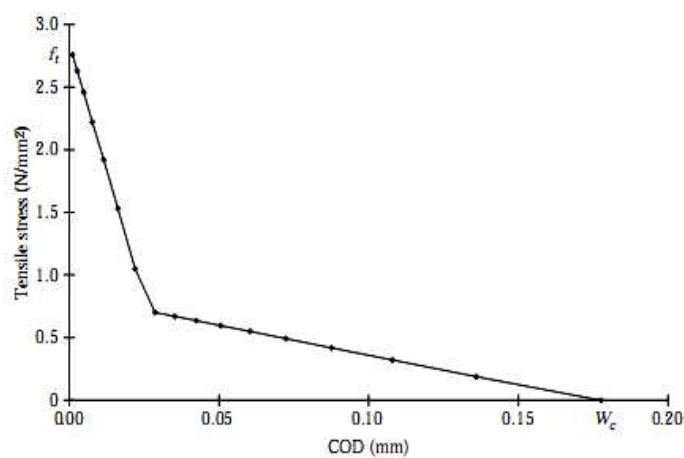


Fig.2.8. Fracture Process Zone (FPZ) in concrete (Shi, 2009).

Hillerborg et. al. (1976) envisioned a fictitious crack in place of physical FPZ and subjected it to closure traction. The closure stress is maximum (tensile strength) at the tip of the FPZ and decreases to zero at a point along the crack

where the crack opening displacement (COD) reaches its critical value beyond which an open crack forms. Known as tension-softening phenomenon, the relation between the closure stress and the COD with the fracture energy of concrete completely defines the local material behavior inside the FPZ when fracture takes place in concrete. Just like the constitutive relationship of a continuous material, the tension softening law is the constitutive relationship for material in the FPZ that describes the transitional material behavior from continuous state to the discontinuous state (Fig.2.9). The area enclosed by this tension softening curve with horizontal axis is the fracture energy  $G$ . As shown in the Fig.2.9, the tension-softening relation of concrete possesses two distinctive features: (i) the descending slope caused by the rapid loss of tensile strength in the initial stage of softening and (ii) a long tail with the increasing COD.



**Fig.2.9. The relation between the tensile stress and the COD along the FPZ (Shi, 2009).**

The choice of tension softening curve influences the prediction of structural response significantly, and local fracture behavior. Many different shapes of this curve, including linear (Hillerborg et. al. 1976), bilinear (Roelfstra and Wittman, 1986; Figueiras and Owen, 1984; CEB-FIP Model Code, 1990), trilinear (Liaw et. al. 1990), exponential (Footer et. al. 1986; Reinhardt, 1985;

Gopalaratnam and Shah, 1985; Cedolin et. al. 1987), and power (Du et. al. 1990) functions have been reported in the literatures.

### **2.3 Estimation of Crack Location and Residual Crack Mouth Opening Displacement**

Historical (Hansen and Roehm, 1979; ICOLD, 2001), experimental (NRC, 1990; Donolon and Hall, 1991; Lin et. al. 1993) and numerical (Leger and Leclerc, 1996) evidences have shown that gravity dams subjected to strong earthquake ground motions are likely to develop cracks at slope changing points of upstream and downstream face and also near the foundation of concrete gravity dams. Kanenawa et. al (2004) analyzed the FEM model of a 100 m concrete gravity dam under deadweight, static hydrostatic pressure and different seismic loading for location of crack. They also studied the effect of seismic acceleration, tensile strength and fracture energy on crack length. They concluded that the cracks tend to occur in the bottom of the dam and near the slope changing point of upstream face of the concrete gravity dam. For given fracture energy and earthquake acceleration, they found that the relationship between crack length and tensile strength is almost linear.

The crack wall motion occurs during earthquake or reservoir level variations. A complete hydraulic closure of crack walls during its motion under the influence of varying reservoir level is almost impossible due to crack wall asperities and existence of free sedimentary material (local crushing of concrete material from crack wall). Existence of residual crack mouth opening displacement (CMOD) was reported by EPRI (1995) during their wedge- splitting (WS) experiment to evaluate the various fracture parameters. Javanmardi et. al. (2005a) showed that this residual crack opening may be taken as 0.5 mm for structural concrete. However, the residual crack opening in mass concrete used in concrete gravity dam is not

found in the literatures. Overturning moment under the influence of dead storage reservoir level always act on concrete gravity dams which increases the residual crack opening by more than 0.5 mm. This increased residual crack opening can be estimated by following formula assuming linear elastic behavior of the mass concrete:  $p = K_n b'_{mr}$ ; where,  $p$  is the uniform pressure over the crack which may be taken as equal to reservoir pressure;  $K_n$  is the normal stiffness and  $b'_{mr}$  is the increased residual crack opening. The final residual crack opening  $b_{mr}$  is then calculated after adding 0.5mm to increased residual crack opening.

## **2.4 Applicability of LEFM and Estimate of Fracture Toughness in Cracked Concrete Gravity Dams**

Concrete dams are often referenced as a major application of fracture mechanics, however little work has been performed to evaluate the fracture properties of dam concrete. Most of the research works carried out, on the fracture of concrete, has concentrated on laboratory testing of small specimens of structural concrete. Based on these test results, a basic understanding of the fracture processes occurring in concrete has been developed. The most reliable technique for fracture toughness (denoted by  $K_{IC}$ ) determination would be based on field testing. However, appropriate experimental techniques have not yet been developed. The second recommended approach is based on recovered dam-concrete cores, using a technique described in Bruhwiler (1991) and evaluating the fracture toughness using the compliance method. Under third approach, the fracture toughness could be extrapolated from the size effect law, preferably on the basis of geometrically identical specimens, which may be difficult for specimens recovered from structures.

The size effect law (SEL) was first proposed by Bazant (1984, 1993) and Carpinteri (1984) for fractured concrete structures. Bazant (1993) developed a

graph between nominal strength  $\sigma_N$  and reference structural size  $D$  based on SEL and dimensional analysis for cracked as well as continuum structures.

In the plot of  $\log \sigma_N$  versus  $\log D$ , the linear-elastic fracture mechanics failures are represented by a straight line of slope  $-1/2$ , while all stress- or strain-based failure criteria correspond to a horizontal line. Some studies (Bazant, 1983, 1984, 1987; Bazant and Pfeiffer, 1987; Bazant and Kazemi, 1990) have shown that the scaling law represents a gradual transition from the strength theory to LEFM. The transition curve was experimentally obtained by Walsh (1979) for notched three-point-bend (TPB) specimen. This curve approaches asymptotically the horizontal line for the strength theory when the size is becoming very small and the inclined straight line for LEFM when the size is becoming very large. A general exact expression for this curve cannot be obtained, however, under certain simplifying assumptions the Bazant (1983, 1984) derived an equation for it.

Based on laboratory Wedge-split (WS) test data and size- effect laws (SEL), Bruhwiler and Souma (1991) concluded that the SEL were valid with respect to specimen sizes and it can be used to extrapolate a fracture toughness (critical stress intensity factor, CSIF) value for infinitely large specimens with characteristic structural dimension taken as thickness of a dam through which a potential crack would propagate. However, dependency on aggregate sizes of concrete was found to be invalid. Using Bazant (1984) brittleness, Carpinteri (1982) brittleness and Hillorborg (1978) characteristic length models, Bruhwiler and Souma (1991) showed that the minimum structural dimension for LEFM validity is 25 m and models accounting for the fracture process zone (FPZ) should be used in the fracture analysis of arch and buttress dams as well as in the upper part of gravity dams. However, LEFM is applicable primarily for the bottom part of large gravity

dams. They also reported that compliance method based fracture toughness value (Souma, 1991) is more appropriate for engineering uses.

An expression for scale factor associated with SIF (Broz et.al. 1991) can be derived using LEFM based SIF,  $K_I$  (Broek, 1986).

$$K_I = \omega_0 \sigma_u \sqrt{\pi L} \quad (2.15)$$

Where  $\omega_0$  is some numerical correction factor that takes into account loading conditions, boundary conditions, and specimen geometry;  $\sigma_u$  is uniform boundary stress acting on the boundary and  $L$  is the corresponding crack length. Designating  $P$  for prototype and  $M$  for model, scaling law can be written as

$$\frac{(K_I)_P}{(K_I)_M} = \left( \frac{\omega_{0P}}{\omega_{0M}} \right) \left( \frac{\sigma_{uP}}{\sigma_{uM}} \right) \left( \sqrt{\frac{L_P}{L_M}} \right) \quad (2.16)$$

Assuming  $\frac{\omega_{0P}}{\omega_{0M}} = \frac{\sigma_{uP}}{\sigma_{uM}} = 1$  and denoting  $\frac{L_P}{L_M} = \lambda_0$  (linear dimension) above equation can be written as

$$\frac{(K_I)_P}{(K_I)_M} = \sqrt{\lambda_0} \quad (2.17)$$

Same scale factor for SIF can be derived using Buckingham  $\pi$ -theorem to perform a dimensional analysis.

Plizzari and Souma (1995) performed centrifuge modeling to simulate the gravitational effects on crack growth in concrete gravity dams. The numerical values of equivalent fracture toughness with uplift pressure of 0.53Mpa were obtained from finite element analysis for maximum water level for four specimens under constant scale factor  $\lambda_0 = 87$ . In their experiment,  $(K_I)_M$  range reported is 0.74-0.88Mpa.m<sup>1/2</sup>. So using above scaling law range of  $(K_I)_P$  is calculated as 6.90-8.20Mpa.m<sup>1/2</sup>. But they have not reported the effect of concrete tensile strength on fracture toughness.



Plizzari (1997) performed a LEFM based parametric study of concrete gravity dam with crack on dam-foundation interface. He expressed the SIF for opening mode in non-dimensional form. As an example parametric study under full uplift pressure conditions in the crack located at dam-foundation interface of a triangularly shaped dam were performed using MERLIN (Reich et. al. 1994) software. For chosen values of non-dimensional parameters, Plizzari (1997) reported the range of SIF as  $1.25\text{-}8.2\text{Mpa}\cdot\text{m}^{1/2}$  for dam base width in the range of 20-100 m. But this model cannot be applied for general dam profile and crack in the body of the dam.

Souma et.al (1991) and EPRI (1995) conducted WS test on mass concrete and used the data to determine the SIF using finite element (FE) modeling of compliance method used in LEFM and suggested extrapolating the result by using SEL. Souma et.al (1991) proposed a method to calculate the fracture toughness using WS test data in FE modeling of compliance method used in LEFM, assuming complete crack closure during loading-unloading cycle of test specimens which was at variance with reported experimental results. For mass concrete with 76 mm maximum aggregate size specimens, they obtained an average fracture toughness value of  $1.04\text{Mpa}\cdot\text{m}^{1/2}$  without uplift pressure. EPRI (1995) conducted the WS test on mass concrete with 76mm maximum aggregate size specimens and calculated the fracture toughness values using same method as used by Souma et. al. (1991) under both with and without full uplift conditions and reported an average of maximum value of fracture toughness as  $1.11\text{Mpa}\cdot\text{m}^{1/2}$  under full uplift pressure of 0.1Mpa and concrete strength of 1.91Mpa. These test results for mass concrete with or without maximum aggregate size and uplift pressure is summarized in the table 2.2.

**Table 2.2. Values of fracture toughness for mass concrete as reported in the literature**

Authors	Test	Max.Max. aggr.size (mm)	$K_{IC}$ (Without uplift) Mpa.m <sup>1/2</sup>	$K_{IC}$ (With uplift)Mpa.m <sup>1/2</sup>	
				Uplift Pressure of 0.1 Mpa	Uplift Pressure of 0.53 Mpa
(1)	(2)	(3)	(4)	(5)	(6)
Souma et. al. (1995)	WS	76	1.04	-	-
Plizzari and Souma (1995)	Centrifuge	-	0.63	-	0.83
EPRI (1995)	WS	76	1.19	1.11	

Table 2.2, shows that the fracture toughness values without uplift pressure is quite low in centrifuge modeling of Plizzari and Souma (1995) compared to WS tested corresponding values of Souma et. al. (1991) and EPRI (1995). Also, as reported by EPRI (1995), the increased uplift pressure reduces the fracture toughness (Table 2.2).

Under cyclic loading (fatigue), a cyclic plastic zone forms at the crack tip, and the growing crack leaves behind a plastic wake. If the plastic zone is sufficiently smaller than elastic singularity zone, then in the presence of constant amplitude cyclic stress intensity, the crack growth rate (Anderson, 2005) is dictated by

$$\frac{dL}{dN} = f_1(\Delta K_d, R_0) \quad (2.18)$$

Where,  $\Delta K_d = (K_{max} - K_{min})$ ,  $R_0 = K_{min}/K_{max}$ ,  $K_{max}$  and  $K_{min}$  are maximum and minimum fracture toughness values in each cycle, and  $N$  is the number of cycles.

Under variable amplitude loading, particularly when there are occasional overloads and under loads during the loading history, the fatigue crack growth analyses become considerably more complicated. Consequently, equations of crack growth rate under constant amplitude are applied whenever possible. It must be recognized, however, that such analyses are potentially subjected to error in the case of variable amplitude loading.

The sigmoidal curve, obtained from schematic log-log plot of  $dL/dN$  versus  $\Delta K_d$ , contains three distinct regions. At intermediate  $\Delta K_d$  values, the curve is linear, but the crack growth rate deviates from the linear trend at high and low  $\Delta K_d$  levels. At the low end,  $dL/dN$  approaches zero at a threshold  $\Delta K_d$ , below which the crack will not grow. The linear region of the log-log plot can be described by Paris Law (Anderson, 2005). In concrete structures the amplitude of cyclic loading rarely remains constant during the service period. Concrete like quasi-brittle materials when subjected to repeated loads with different amplitude, exhibits complicated behavior due to the presence of large size process zone ahead of the crack tip. Softening curves were obtained experimentally by Slowik et. al. (1996), based on a cyclic load, with pre-defined spike amplitude and frequency.

Very few attempts have been made to predict the crack growth analytically in plane concrete when subjected to variable amplitude fatigue loading. Slowik et. al. (1996) have proposed an empirical law to study the crack propagation, based on linear elastic fracture mechanics (LEFM) concept. For concrete, the knowledge of fatigue fracture is limited. This is due to the fact that fracture behavior in concrete is more complicated because of its heterogeneous nature and presence of large size fracture process zone (FPZ) at the crack tip. The crack propagation study under fatigue loading began with the well-known Paris law which is found to exhibit a weak form of scaling. The earlier assumption about the Paris law coefficients as material constants has been proved to be invalid by various workers (e.g. Spagnoli,

2004; Carpinteri and Paggi, 2007; Ciavarella et. al. 2008; Ray and Chandra Kishen, 2011). One of the earliest attempts, made by Bazant and Xu (1991), who introduced the size effect into the Paris law using the size dependent fracture toughness. Recently, various authors (e.g. Spagnoli, 2004; Carpinteri et. al. 2009; Carpinteri and Spagnoli 2004; Ciavarella et. al. 2008; Carpinteri and Paggi, 2007, 2009; Paggi, 2009 and Ray and Chandra, 2010, 2011) have investigated the fatigue behavior of concrete by adopting dimensional analysis and self-similarity approach.

## **2.5 Crack Mouth Opening Displacement (CMOD) Rate**

Fracture properties are affected by loading rate. The loading rate can be categorized as high and low. Studies of effect on fracture properties have been conducted by Mindess and Shah (1986) under high loading rate, in which the maximum load is reached in one second. Under this high loading rate, the creep effect of the material in fracture process zone (Shah and Chandra, 1970; Wittmann and Zaitsev 1971; Liu et. al. 1989) cannot be accounted. Bazant and Gettu (1992) conducted three-point bending experiments to obtain the load versus CMOD and load versus load-line displacement curves under high and slow loading rates. They observed that peak load of faster curve is more than that of lower rate. Post-peak curve in faster rate is steeper than slower rate in load deflection curve. They concluded that difference in peak values in load-CMOD curve is the creep phenomenon occurring in the FPZ, while same phenomenon in the bulk of specimen is responsible for difference in the steepness of the post-peak curves under load-deflection plot. The detrimental effect of moisture is more significant at slower rates (Harsh et. al. 1990; Rossi and Boulay, 1990). The wedge splitting tests on dam concrete by Bruhwiler and Wittmann (1990) were carried out to study the influence of CMOD-rate on the specific fracture

energy and the strain softening diagram under fast monotonic loading rate. They proposed the following CMOD-rate relation.

$$\dot{b}_m = c(\dot{b}_m)^{n_0} \quad (2.19)$$

Where,  $\dot{b}_m$  is CMOD rate,  $b_m$  is CMOD and  $c$  and  $n_0$  are constants. The main drawback of the experiment is that specimen is provided with a longitudinal reinforcement in order to prevent shear failure of the cantilevers. Bazant and Gettu (1990) have derived the material properties using size-effect law. The same has not been reported by Bruhwiler and Wittmann (1990). Therefore, applicability of above equation to dams needs further investigation. Studies of fracture properties of dam concrete (He et. al. 1992) show that fracture behavior of dam concrete agrees well with size effect law and as the loading rate decreases, the fracture behavior becomes closer to LEFM. It is interesting to note that for long time loading, the concretes are more brittle than for short time loading. Because the stress relaxation in notched concrete specimens are very significant due to creep in fracture process zone (FPZ) as compared to creep in the bulk of concrete specimens (Bazant and Gettu, 1992). However, the effects of loading rate on fracture properties of concrete have been studied by various workers (e.g. Liu et. al. 1989; Mindess, 1985; Reinhardt, 1985, 1986; Wittmann, 1985; Ross and Kuennen, 1989) and it is observed that the creep is non-linear within the fracture process zone. Bazant (1994) derived the CMOD rate using activation theory for creep in FPZ. At constant temperature, his equation can be written as

$$\dot{b}_m = C_0 \sinh \left\{ k_0 \left[ \sigma_{br} - f_0(b_m) \right] \right\} \quad (2.20)$$

Where  $C_0$  and  $k_0$  are constants,  $\sigma_{br}$  is crack bridging stress,  $f_0(b_m)$  is a function of  $b_m$ . If  $\sigma_{br} < f_0(b_m)$  then loading and reloading (fatigue) is not considered. De Borst et. al. (1993) proposed a simple and alternative

expression, in which the cracking process is considered to be viscous and activation controlled process is described with a viscosity term which acts only in FPZ. However Van Zil et. al. (2001) showed that this alternative expression is unable to reflect the fatigue effect over the entire range of loading rates. Wu and Bazant (1993), Chaimoon et.al. (2008) and Giovanni (2009) applied the rate dependent fracture model with creep model for bulk of material in the study of crack propagation in quasi-brittle materials. They concluded that inclusion of both creep in the bulk of specimen and rate dependence of crack opening is important and if only one of these phenomenon is modeled, good agreement with experimental data cannot be obtained. Carpinteri et.al. (1997) conducted tensile and flexural creep rupture tests of concrete specimens to study the creep CMOD versus time behavior. They concluded that both tensile and flexural creep CMOD behavior are almost same at given sustained load level and are similar to creep behaviors in metals.

## **2.6 Transient Uplift Pressure Modeling in Crack**

The studies of concrete gravity dams in past decades were mainly focused to address the fracture failure of the materials and its influence on dynamic response of the dam without much concern for safety considerations (Bhattacharjee and Leger, 1993; Ayari and Souma, 1990; El-Aidi and Hall, 1989). Further, study of the concrete gravity dams with penetrated cracks attracted much attention to some of the researchers (Peakau and Cui, 2004; Peakau and Zhu, 2006).

A static triangular uplift pressure at the base of the dam with no-drainage condition is assumed in safety guidelines (ICOLD, 1986; USBR, 1987; USACE, 1995). However, there are no unified considerations for uplift pressure developed during dynamic pressure variations in the reservoir when cracks develop either in the body or at the dam-foundation interface of the concrete

gravity dams. Chopra and Zhang (1991) and Chavez and Fenves (1995) have studied the safety of concrete gravity dam under seismic condition after assuming a constant triangular uplift pressure at the base of the dam with cracks.

To consider the transient nature of the uplift pressure, Hall (1998) assumed that pressure at the mouth of the crack varies in accordance with hydrodynamic pressure due to dam reservoir interaction and uplift pressure remains constant with triangular variation. Zhang and Ohmachi (1998) used this model for seismic crack analysis without crack wall motion to validate experimental results of Ohmachi et. al. (1998). Yi et. al. (1997) developed a one dimensional fluid pressure relation for the growing crack in brittle materials under hydrodynamic pressure conditions and combined their results with fracture mechanics model to compute crack tip stress intensity. However, this model was developed for small crack length which is not applicable for the case of cracks in concrete gravity dams where crack lengths are much longer.

There are only few experimental and theoretical studies for uplift pressure distribution in cracks during hydrodynamic loading. Slowik and Souma (1994) conducted dynamic wedge- splitting (WS) test to measure the water pressure during propagating cracks for slow and rapid opening velocities of crack walls. They observed that the water front and crack front velocities during slow opening were almost equal while the same are quite different during rapid wall movement. They also reported the results for sudden crack closing and cyclic opening and closing of existing cracks. Further their result indicates that in sudden crack closure case, the water is trapped in the crack resulting in a temporary over pressurization. This over pressurization induces additional stresses causing failure of the specimen. During cyclic opening and closing mode, pressure variations along the crack were different with maximum pressure (which was more than initial hydrostatic pressure)

reached during closing of crack walls. Slowik and Souma (2000) proposed a theoretical model for opening mode considering only laminar flow in the cracks. Laminar flow based models is used extensively by researchers for hydromechanical coupling analysis of rock masses (e.g. Jing et. al. 2001). Tinawi and Guizani (1994) developed a theoretical model for hydrodynamic model for preexisting cracks in concrete gravity dams using laminar flow in cracks. These methods are not applicable for uplift pressure computations in cracks of concrete dams because turbulent and unsteady flows occur in cracks during hydrodynamic condition. Independent experimental works by Louis (1968) and Lomiz (1951) provided almost identical results for relationships between velocity and hydraulic gradient in cracks after taking into account the different flow conditions ( i. e. turbulent/laminar) and crack wall roughness. Illangaskare et. al. (1992) developed a finite element computer programme, called CRFLOOD, to compute uplift pressure distribution along the cracks of the concrete gravity dams for all types of flow characterized by Louis (1968).

## **2.7 Dam-crown Deflection**

In the design of dam it is generally assumed that there is no tensile stress across the any section of the dam. It means there is no possibility of occurrence of cracks in the dam body. By assuming so, it is believed that no-tension plastic design of the dam is on safer side. However, this has been never proved mathematically (Bazant, 1994). Bazant (1994) analyzed rectangular dam geometry and showed that there exists a critical size of the dam above which fracture mechanics gives a smaller failure load than classical no-tension plastic analysis. Gioia et. al. (1992) analyzed the problem in greater details using finite element formulations of complex dam geometries. They showed that differences between plasticity and fracture mechanics are more pronounced for the case when water penetrates into the crack and applies pressure on the concrete. Some results are taken from Gioia



et. al. (1992). Fig.2.10 shows that fracture mechanics yields a lower resistance, for the same horizontal displacement at the top of the dam, than plasticity solution with negligible tensile strength and water pressure on a pre-existing crack. Fig.2.11 shows a comparison of plasticity solution when realistic values of tensile strength and critical stress intensity factors are compared.

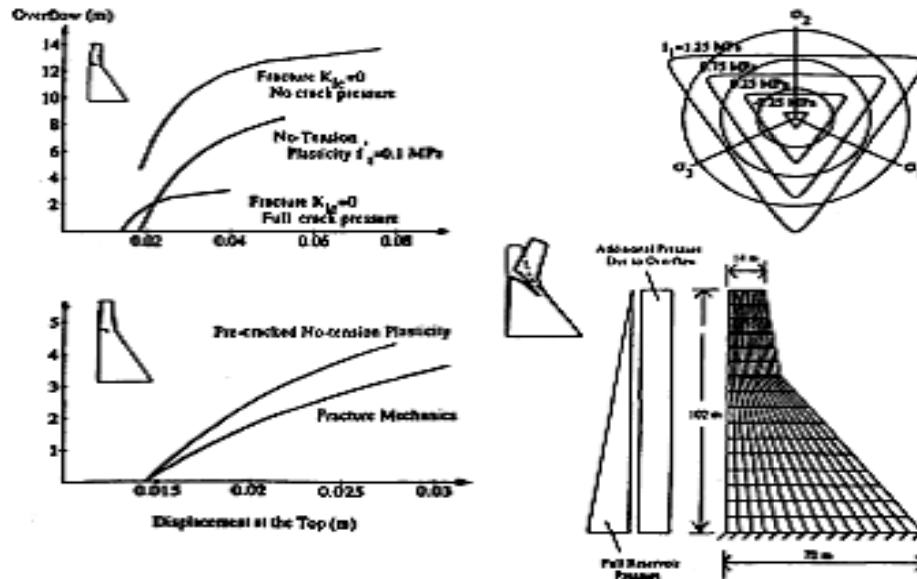


Fig.2.10. Left: Comparison of plots of overflow height of water versus horizontal displacement at the top of dam for no-tension plastic analysis of a pre-cracked dam and for fracture analysis (full hydrostatic pressure is considered on pre-cracked portion of the dam cross section). Top right: Ottosen yield surface used. Lower right: Dam analyzed and meshes (Gioia et. al. 1992).

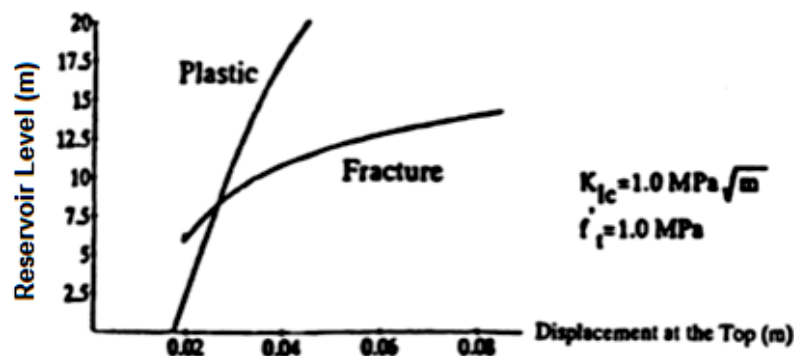


Fig.2.11. Comparison of overflow-displacement curves for plastic and fracture analysis using realistic values of tensile strength and fracture toughness (Gioia et. al. 1992).

However, the effect of transient pressure variation along the crack on dam-crown deflection has not been reported in the above studies.

## 2.8 Dam Stability against Sliding

A static triangular uplift pressure at the base of the dam with no-drainage condition is assumed in safety guidelines (ICOLD, 1986; USBR, 1987; USACE, 1995). Review of dam safety guidelines for assumptions of uplift pressure in cracks during seismic condition are summarized below.

- USBR (1987) and CDSA (1997) recommend a zero uplift pressure in cracks of the concrete gravity dam. Its recommendation is based on the study of rapid opening and closing of the cracks which do not allow water to penetrate into the crack and hence pressure does not develop there.
- ICOLD (1986) assume that uplift pressure in the crack is equal to reservoir head.
- USACE (1995) and FERC (2002) recommend that uplift pressure in crack should be assumed to be unaffected by the earthquake.
- IS 6512 (2000) is silent about the uplift forces in cracks and its distribution during earthquake conditions.

Further, Chopra and Zhang (1991) and Chavez and Fenves (1995) studied the safety of concrete gravity dam under seismic condition after assuming a constant triangular uplift pressure at the base of the dam with crack. However, there are no unified considerations for uplift pressure developed during dynamic pressure variations in the reservoir when cracks develop either in the body or at the dam-foundation interface of the concrete gravity dams.

## 2.9 Objectives of Present Research Work

The objectives of the present research work are as follows.

- To develop a model for transient uplift pressure in cracks of constant length over given opening-closing cycle of the crack mouth under constant crack mouth pressure.
- Calculation of transient uplift pressure, under constant crack mouth pressure, using MATLAB software and its validation from the available data in literature.
- To propose crack-mouth-opening-displacement (CMOD) rate generating functions, caused due to creep effect in fracture process zone (FPZ) of the crack based on the results available in the literatures.
- To develop the model for calculating the transient uplift pressure in cracks of concrete gravity dam under varying reservoir levels.
- Calculation of dam-crown deflection under varying reservoir level using plane strain finite element programme in ANSYS and its validation from field data.
- Calculation of factor of safety in sliding using result of uplift pressure in cracks for present study and its comparison to USACE recommendations.

## 2.10 Scope of Present Study

The research work is focused on the development of a suitable uplift pressure, and dam-crown deflection modeling and applying it to predict the safety of concrete gravity dam. The research is limited to two-dimensional (2-D) analysis of concrete gravity dams. The following areas are not covered in this research.

- Dynamic cracking.
- The coupling between different crack modes, and different cracks.
- Behavior of creep and shrinkage in the body of the dam.

## 2.11 Summary

Studies of fractures are important to understand and predict a variety of hydraulic and mechanical phenomenon. Flow field in the single crack is obtained using combined solution of mass and momentum equations. One dimensional (1-D) mass conservation equation without sink or source is used to derive the discharge in the cracks of concrete gravity dams under certain simplifying assumptions. However for two phase flow, the continuity equation is supplemented with additional equations describing the constitutive relation of effective saturation of wetting phase. Because of presence of advective acceleration terms, momentum equations are difficult to solve. Therefore, Stokes equation were derived after neglecting the advective terms in momentum equations. To overcome the computational difficulty of Stokes equation, Reynolds equation, based on the lubrication theory, were used but found limited applications because, the solution of these equations overestimates the flow rate by as much as hundred percent. Most widely used conceptual model for fracture, known as local cubic law (LCL), is that of two smooth parallel walls separated by a uniform aperture. To simulate the flow in real cracks, the LCL equation has been modified to take into account the crack geometry, crack wall roughness and leakages. The LCL equation is found to be valid for laminar flow when Reynolds number is less than ten. Lomize (1951) and Louis (1969) derived the momentum equations for both type (laminar and turbulent) of flow regimes and smooth/rough walls of the cracks.

Linear, nonlinear and energy based fracture mechanics are applied to study the crack propagation in concrete structures. In linear elastic fracture

mechanics (LEFM), the solution of equations of elasticity and Airy stress functions for all modes of deformations at crack tip, shows the existence of infinite stress at crack tip. Irwin (1957) coined the term stress intensity factor (SIF) to characterize this singularity at the crack tip. The fracture is expected to occur when SIF reaches a critical value called critical stress intensity factor or fracture toughness. Griffith's (1921, 1925) energy criterion for fracture led Irwin (1956) to define an energy release rate. The crack extension occurs when energy release rate reaches a critical value called fracture toughness. To remove the singularity at the crack tip, Hillerborg et. al. (1976) envisioned a fictitious crack in place of physical fracture process zone (FPZ). Known as tension-softening phenomenon, the relation between the closure stress and the crack opening displacement (COD) completely defines the local material behavior inside the FPZ when fracture takes place in concrete.

Historical, experimental and numerical evidences have shown that gravity dams subjected to strong earthquake ground motions are likely to develop cracks at slope changing points of upstream and downstream face and also near the foundation of concrete gravity dams.

The crack wall motion occurs during earthquake or reservoir level variations. A complete hydraulic closure of crack walls during its motion under the influence of varying reservoir level is almost impossible. Literature study reveals that: (i) the size-effect law (SEL) is valid with respect to specimen sizes and it can be used to extrapolate a fracture toughness and (ii) the minimum structural dimension for LEFM validity is 25 m and models accounting for the fracture process zone (FPZ) should be used in the fracture analysis of arch and buttress dams as well as in the upper part of gravity dams. However, the determination of fracture toughness under fatigue and uplift condition is difficult and unpredictable.

Various authors have found that creep phenomenon, active in the FPZ, is responsible for the crack mouth opening displacement (CMOD) rate.

The study for uplift pressure in cracks, under moving crack wall motion, has been reported by very less researchers. Dam-crown deflection and stability analysis, under transient uplift pressure in the cracks of concrete gravity dams, are not found in the literatures. On the basis of literature review, a set of inter-related research objectives has been framed to simulate the field data.