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Bianchi Type-I Universe with wet dark fluid

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Abstract. The Bianchi Type-I Universe filled with dark energy from a wet dark fluid has been considered. A new equation of state for the dark energy component of the Universe has been used. It is modeled on the equation of state $p = \gamma(\rho - \rho_\star)$ which can describe a liquid, for example water. The exact solutions to the corresponding field equations are obtained in quadrature form. The solution for constant deceleration parameter have been studied in detail for both power-law and exponential forms. The cases $\gamma = 1$ and $\gamma = 0$ have also been analysed.

Keywords. Bianchi-type Universe; wet dark fluid; cosmological parameters.

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1. Introduction

The nature of the dark energy component of the Universe [1–3] remains one of the deepest mysteries of cosmology. There is certainly no lack of candidates: cosmological constant, quintessence [4], k-essence [5], phantom energy [6] etc. Modifications of the Friedmann equation such as Cardassian expansion [7] as well as what might be derived from brane cosmology [8] have also been used to explain the acceleration of the Universe.

In this work, we use wet dark fluid (WDF) as a model for dark energy. This model is in the spirit of the generalized Chaplygin gas (GCG) [9], where a physically motivated equation of state is offered with properties relevant for the dark energy problem. Here the motivation stems from an empirical equation of state proposed by Tait [10] and Hayword [11] to treat water and aqueous solution. The equation of state for WDF is very simple.

$$p_{\mathrm{WDF}} = \gamma(\rho_{\mathrm{WDF}} - \rho_\star) \tag{1.1}$$

and is motivated by the fact that it is a good approximation for many fluids, including water, in which the internal attraction of the molecules makes negative pressures possible. One of the virtues of this model is that the square of the sound speed, c_{s}^2, which depends on $\partial p/\partial \rho$, can be positive (as opposed to the case of phantom energy, say), even while giving rise to cosmic acceleration in the current epoch.

We treat eq. (1.1) as a phemenological equation [12]. Holman and Naidu [13] have shown that this model can be made consistent with the most recent SNIa data [14], the WMAP results [15] as well as constraints coming from measurements of the matter power spectrum [16]. The parameters γ and $\rho_\star$ are taken to be positive and we restrict ourselves to $0 \leq \gamma \leq 1$. Note that if c_{s} denotes the adiabatic sound speed in WDF, then $\gamma = c_{\mathrm{s}}^2$ (refer Babichev *et al* [17]).

To find the WDF energy density, we use the energy conservation equation

$$\dot{\rho}_{\mathrm{WDF}} + 3H(p_{\mathrm{WDF}} + \rho_{\mathrm{WDF}}) = 0. \tag{1.2}$$

From equation of state (1.1) and using $3H = \dot{V}/V$ in the above equation, we have

$$\rho_{\mathrm{WDF}} = \frac{\gamma}{1+\gamma}\rho_\star + \frac{C}{V^{(1+\gamma)}}, \tag{1.3}$$

where C is the constant of integration and V is the volume expansion.

WDF naturally includes two components: a piece that behaves as a cosmological constant as well as a standard fluid with an equation of state $p = \gamma\rho$. We can show that if we take $C > 0$, this fluid will not violate the strong energy condition $p + \rho \geq 0$:

$$\begin{aligned} p_{\mathrm{WDF}} + \rho_{\mathrm{WDF}} &= (1+\gamma)\rho_{\mathrm{WDF}} - \gamma\rho_\star \\ &= (1+\gamma)\frac{C}{V^{(1+\gamma)}} \geq 0. \end{aligned} \tag{1.4}$$

The wet dark fluid has been used as dark energy in the homogeneous, isotropic FRW case by Holman and Naidu [13], the early stage of expansion of the Universe exhibits substantially non-Friedmannian behaviour [18]. We, therefore, consider the simple case of a homogeneous but anisotropic Bianchi Type-I model with matter term with dark energy treated as dark fluid satisfying the equation of state (1.1). The solution has been obtained in the quadrature form. The models with constant deceleration parameter have been studied in detail.

In this paper we study the Bianchi Type-I model with matter term with dark energy treated as a dark fluid satisfying the equation of state (1.1). The solution has been obtained in the quadrature form. The models with constant deceleration parameter have been studied in detail.

2. Basic equation

We take Bianchi Type-I metric in the form

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a_1^2\mathrm{d}x^2 - a_2^2\mathrm{d}y^2 - a_3^2\mathrm{d}z^2, \tag{2.1}$$

where the metric functions a_1, a_2, a_3 are functions of t only.

The Einstein field equations for the metric (2.1) are written in the form

$$\frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_2\dot{a}_3}{a_2a_3} = \kappa T_1^1, \tag{2.2}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_1\dot{a}_3}{a_1a_3} = \kappa T_2^2, \tag{2.3}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1\dot{a}_2}{a_1a_2} = \kappa T_3^3, \tag{2.4}$$

$$\frac{\dot{a}_1\dot{a}_2}{a_1a_2} + \frac{\dot{a}_2\dot{a}_3}{a_2a_3} + \frac{\dot{a}_3\dot{a}_1}{a_3a_1} = \kappa T_0^0. \tag{2.5}$$

Here κ is the gravitational constant and overhead dot denotes differentiation with respect to t.

The energy–momentum tensor of the source is given by

$$T_i^j = (\rho_{\mathrm{WDF}} + p_{\mathrm{WDF}})u_iu^j - p_{\mathrm{WDF}}\delta_i^j, \tag{2.6}$$

where u^i is the flow vector satisfying

$$g_{ij}u^iu^j = 1. \tag{2.7}$$

In a co-moving system of coordinates, from eq. (2.6) we find

$$T_0^0 = \rho_{\mathrm{WDF}}, \quad T_1^1 = T_2^2 = T_3^3 = -p_{\mathrm{WDF}}. \tag{2.8}$$

Now using eq. (2.8) in eqs (2.2)–(2.5) we obtain

$$\frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_2\dot{a}_3}{a_2a_3} = -\kappa p_{\mathrm{WDF}}, \tag{2.9}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_1\dot{a}_3}{a_1a_3} = -\kappa p_{\mathrm{WDF}}, \tag{2.10}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1\dot{a}_2}{a_1a_2} = -\kappa p_{\mathrm{WDF}}, \tag{2.11}$$

$$\frac{\dot{a}_1\dot{a}_2}{a_1a_2} + \frac{\dot{a}_2\dot{a}_3}{a_2a_3} + \frac{\dot{a}_3\dot{a}_1}{a_3a_1} = \kappa \rho_{\mathrm{WDF}}. \tag{2.12}$$

Subtracting eq. (2.10) from eq. (2.9), we get

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2}\right) + \left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2}\right)\left(\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3}\right) = 0. \tag{2.13}$$

Let V be a function of t defined by

$$V = a_1 a_2 a_3. \tag{2.14}$$

Then from eq. (2.13) we obtain

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2}\right) + \left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2}\right)\frac{\dot{V}}{V} = 0. \tag{2.15}$$

Integrating the above equation, we get

$$\frac{a_1}{a_2} = d_1 \exp\left(x_1 \int \frac{\mathrm{d}t}{V}\right), \quad d_1 = \text{ constant}, \quad x_1 = \text{constant}. \tag{2.16}$$

By subtracting eq. (2.11) from (2.9) and eq. (2.9) from (2.10), we obtain similarly

$$\frac{a_1}{a_3} = d_2 \exp\left(x_2 \int \frac{\mathrm{d}t}{V}\right), \tag{2.17}$$

$$\frac{a_2}{a_3} = d_3 \exp\left(x_3 \int \frac{\mathrm{d}t}{V}\right), \tag{2.18}$$

where d_2, d_3, x_2, x_3 are integration constants.

In view of $V = a_1 a_2 a_3$ we find the following relation between the constants $d_1, d_2, d_3, x_1, x_2, x_3$:

$$d_2 = d_1 d_3, \quad x_2 = x_1 + x_3.$$

Finally from eqs (2.16)–(2.18), we write $a_1(t), a_2(t)$ and $a_3(t)$ in the explicit form

$$a_1(t) = D_1 V^{1/3} \exp\left(X_1 \int \frac{\mathrm{d}t}{V(t)}\right), \tag{2.19}$$

$$a_2(t) = D_2 V^{1/3} \exp\left(X_2 \int \frac{\mathrm{d}t}{V(t)}\right), \tag{2.20}$$

$$a_3(t) = D_3 V^{1/3} \exp\left(X_3 \int \frac{\mathrm{d}t}{V(t)}\right), \tag{2.21}$$

where D_i $(i = 1,2,3)$ and X_i $(i = 1,2,3)$ satisfy the relation $D_1 D_2 D_3 = 1$ and $X_1 + X_2 + X_3 = 0$.

Now, adding eqs (2.9)–(2.11) and three times eq. (2.12), we get

$$\left(\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3}\right) + 2\left(\frac{\dot{a}_1\dot{a}_2}{a_1 a_2} + \frac{\dot{a}_2\dot{a}_3}{a_2 a_3} + \frac{\dot{a}_3\dot{a}_1}{a_3 a_1}\right) = \frac{3\kappa}{2}(\rho_{\mathrm{WDF}} - p_{\mathrm{WDF}}). \tag{2.22}$$

From eq. (2.14) we have

$$\frac{\ddot{V}}{V} = \left(\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3}\right) + 2\left(\frac{\dot{a}_1\dot{a}_2}{a_1 a_2} + \frac{\dot{a}_2\dot{a}_3}{a_2 a_3} + \frac{\dot{a}_3\dot{a}_1}{a_3 a_1}\right). \tag{2.23}$$

From eqs (2.22) and (2.23) we obtain

$$\frac{\ddot{V}}{V} = \frac{3\kappa}{2}(\rho_{\mathrm{WDF}} - p_{\mathrm{WDF}}). \tag{2.24}$$

The conservational law for the energy–momentum tensor gives

$$\dot{\rho}_{\mathrm{WDF}} = -\frac{\dot{V}}{V}(\rho_{\mathrm{WDF}} + p_{\mathrm{WDF}}). \tag{2.25}$$

From eqs (2.24) and (2.25) we have

$$\dot{V} = \pm\sqrt{2(3\kappa\rho_{\mathrm{WDF}}V^2 + C_1)} \tag{2.26}$$

with C_1 being an integration constant.

Rewriting (2.25) in the form

$$\frac{\dot{\rho}}{\rho_{\mathrm{WDF}} + p_{\mathrm{WDF}}} = -\frac{\dot{V}}{V} \tag{2.27}$$

and taking into account that the pressure and the energy density obeying an equation of state of type $p_{\mathrm{WDF}} = f(\rho_{\mathrm{WDF}})$, we conclude that ρ_{WDF} and p_{WDF}, hence the right-hand side of eq. (2.24) is a function of V only.

$$\ddot{V} = \frac{3\kappa}{2}(\rho_{\mathrm{WDF}} - p_{\mathrm{WDF}})V \equiv F(V). \tag{2.28}$$

From the mechanical point of view, eq. (2.28) can be interpreted as equation of motion of a single particle with unit mass under the force $F(V)$. Then

$$\dot{V} = \sqrt{2[\varepsilon - U(V)]}. \tag{2.29}$$

Here ε can be viewed as energy and $U(V)$ as the potential of the force F. Comparing eqs (2.26) and (2.29) we find $\varepsilon = C_1$ and

$$U(V) = -3\kappa\rho_{\mathrm{WDF}}V^2. \tag{2.30}$$

Finally, we write the solution to eq. (2.26) in quadrature form

$$\int \frac{\mathrm{d}V}{\sqrt{2(C_1 + 3\kappa\rho_{\mathrm{WDF}}V^2)}} = t + t_0, \tag{2.31}$$

where the integration constant t_0 can be taken to be zero, since it only gives a shift in time.

From eqs (1.3) and (2.31) we obtain

$$\int \frac{\mathrm{d}V}{\sqrt{2\left\{\left(\frac{3\kappa\gamma}{1+\gamma}\rho_\star V^2 + 3\kappa C V^{(1-\gamma)}\right) + C_1\right\}}} = t + t_0. \tag{2.32}$$

3. Some particular cases

Case I. $\gamma = 1$ (Zeldovich fluid)

Equation (2.32) reduces to

$$\int \frac{\mathrm{d}V}{\sqrt{3\kappa\rho_\star V^2 + 6\kappa C + 2C_1}} = t \tag{3.1}$$

which gives

$$V = \sqrt{\frac{6\kappa C + 2C_1}{3\kappa\rho_\star}} \sinh(\sqrt{3\kappa\rho_\star} t). \tag{3.2}$$

From eqs (2.19)–(2.21) and (3.2), we get

$$a_1(t) = D_1 \left(\frac{6\kappa C + 2C_1}{3\kappa\rho_\star}\right)^{1/6} \sinh^{1/3}(\sqrt{3\kappa\rho_\star}\ t) \times \exp\left[-X_1 \frac{\cot^{-1}\left(\cosh\sqrt{3\kappa\rho_\star}\ t\right)}{\sqrt{6\kappa C + 2C_1}}\right], \tag{3.3}$$

$$a_2(t) = D_2 \left(\frac{6\kappa C + 2C_1}{3\kappa\rho_\star}\right)^{1/6} \sinh^{1/3}(\sqrt{3\kappa\rho_\star}\ t) \times \exp\left[-X_2 \frac{\cot^{-1}\left(\cosh\sqrt{3\kappa\rho_\star}\ t\right)}{\sqrt{6\kappa C + 2C_1}}\right], \tag{3.4}$$

$$a_3(t) = D_3 \left(\frac{6\kappa C + 2C_1}{3\kappa\rho_\star}\right)^{1/6} \sinh^{1/3}(\sqrt{3\kappa\rho_\star}\ t) \times \exp\left[-X_3 \frac{\cot^{-1}\left(\cosh\sqrt{3\kappa\rho_\star}\ t\right)}{\sqrt{6\kappa C + 2C_1}}\right], \tag{3.5}$$

where D_i $(i = 1, 2, 3)$ and X_i $(i = 1, 2, 3)$ satisfy the relation $D_1D_2D_3 = 1$ and $X_1 + X_2 + X_3 = 0$.

From eqs (1.3) and (3.2), we get

$$\rho_{\mathrm{WDF}} = \frac{\rho_\star}{2} + \frac{3C\kappa\rho_\star}{6\kappa C + 2C_1} \mathrm{cosech}^2(\sqrt{3\kappa\rho_\star}\ t) \tag{3.6}$$

and from eqs (1.1) and (3.6) we get

$$p_{\mathrm{WDF}} = -\frac{\rho_\star}{2} + \frac{3C\kappa\rho_\star}{6\kappa C + 2C_1} \mathrm{cosech}^2(\sqrt{3\kappa\rho_\star}\ t). \tag{3.7}$$

The physical quatities of observational interest in cosmology are the expansion scalar θ, the mean anisotropy parameter A, the shear scalar σ^2 and the deceleration parameter q. They are defined as

$$\theta = 3H, \tag{3.8}$$

$$A = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2, \tag{3.9}$$

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} H_i^2 - 3H^2\right) = \frac{3}{2}AH^2, \tag{3.10}$$

$$q = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{H}\right) - 1. \tag{3.11}$$

By using eqs (3.8)–(3.11) we can express the physical quantities as

$$\theta = \sqrt{3\kappa\rho_\star}\coth(\sqrt{3\kappa\rho_\star}\ t), \tag{3.12}$$

$$A = \frac{3X^2}{(6\kappa C + 2C_1)}\frac{\sinh^4(\sqrt{3\kappa\rho_\star}\ t)}{\cosh^2(\sqrt{3\kappa\rho_\star}\ t)[1+\cosh^2(\sqrt{3\kappa\rho_\star}\ t)]}, \tag{3.13}$$

$$\sigma^2 = \frac{3\kappa\rho_\star X^2}{2(6\kappa C + 2C_1)}\frac{\sinh^2(\sqrt{3\kappa\rho_\star}\ t)}{[1+\cosh^2(\sqrt{3\kappa\rho_\star}\ t)]}, \tag{3.14}$$

$$q = 3\ \mathrm{sech}^2(\sqrt{3\kappa\rho_\star}\ t) - 1, \tag{3.15}$$

where $X^2 \equiv X_1^2 + X_2^2 + X_3^2$ is a constant.

Case II. $\gamma = 0$ (Dust)

Equation (2.32) reduces to

$$\int \frac{\mathrm{d}V}{\sqrt{6\kappa CV + 2C_1}} = t \tag{3.16}$$

which gives

$$V = \frac{3\kappa C}{2}t^2 - \frac{C_1}{3\kappa C}. \tag{3.17}$$

Case IIa. When $t > \frac{\sqrt{2C_1}}{3\kappa C}$

From eqs (2.19)–(2.21) and (3.17), we get

$$a_1(t) = D_1\left(\frac{3\kappa C}{2}t^2 - \frac{C_1}{3\kappa C}\right)^{1/3} \times \exp\left[-X_1\sqrt{\frac{2}{C_1}}\coth^{-1}\left(\frac{3\kappa C}{\sqrt{2C_1}}t\right)\right], \tag{3.18}$$

$$a_2(t) = D_2\left(\frac{3\kappa C}{2}t^2 - \frac{C_1}{3\kappa C}\right)^{1/3} \times \exp\left[-X_2\sqrt{\frac{2}{C_1}}\coth^{-1}\left(\frac{3\kappa C}{\sqrt{2C_1}}t\right)\right], \tag{3.19}$$

$$a_3(t) = D_3\left(\frac{3\kappa C}{2}t^2 - \frac{C_1}{3\kappa C}\right)^{1/3} \times \exp\left[-X_3\sqrt{\frac{2}{C_1}}\coth^{-1}\left(\frac{3\kappa C}{\sqrt{2C_1}}t\right)\right], \tag{3.20}$$

where D_i $(i = 1, 2, 3)$ and X_i $(i = 1, 2, 3)$ satisfy the relation $D_1 D_2 D_3 = 1$ and $X_1 + X_2 + X_3 = 0$.

From eqs (1.3) and (3.17), we get

$$\rho_{\text{WDF}} = C\left(\frac{3\kappa C}{2}t^2 - \frac{C_1}{3\kappa C}\right)^{-1} \tag{3.21}$$

and from eqs (1.1) and (3.21), we get

$$p_{\text{WDF}} = 0. \tag{3.22}$$

By using eqs (3.8)–(3.11) we can express the physical quantities as

$$\theta = \frac{3\kappa C t}{\frac{3\kappa C}{2}t^2 - \frac{C_1}{3\kappa C}}, \tag{3.23}$$

$$A = 27\frac{X^2\kappa^2 C^2}{t^2}, \tag{3.24}$$

$$\sigma^2 = \frac{81}{2}\frac{X^2\kappa^4 C^4}{t^2(\frac{3\kappa C}{2}t^2 - \frac{C_1}{3\kappa C})}, \tag{3.25}$$

$$q = \frac{1}{2} + \frac{C_1}{3\kappa^2 C^2}\frac{1}{t^2}, \tag{3.26}$$

where $X^2 \equiv X_1^2 + X_2^2 + X_3^2$ is a constant.

For large t, the model tends to be isotropic.

Case IIb. When $t < \frac{\sqrt{2C_1}}{3\kappa C}$

From eqs (2.19)–(2.21) and (3.17), we get

$$a_1(t) = D_1\left(\frac{3\kappa C}{2}t^2 - \frac{C_1}{3\kappa C}\right)^{1/3} \times \exp\left[-X_1\sqrt{\frac{2}{C_1}}\tanh^{-1}\left(\frac{3\kappa C}{\sqrt{2C_1}}t\right)\right], \tag{3.27}$$

$$a_2(t) = D_2 \left(\frac{3\kappa C}{2} t^2 - \frac{C_1}{3\kappa C} \right)^{1/3} \times \exp\left[-X_2 \sqrt{\frac{2}{C_1}} \tanh^{-1}\left(\frac{3\kappa C}{\sqrt{2C_1}} t \right) \right], \tag{3.28}$$

$$a_3(t) = D_3 \left(\frac{3\kappa C}{2} t^2 - \frac{C_1}{3\kappa C} \right)^{1/3} \times \exp\left[-X_3 \sqrt{\frac{2}{C_1}} \tanh^{-1}\left(\frac{3\kappa C}{\sqrt{2C_1}} t \right) \right], \tag{3.29}$$

where D_i $(i = 1, 2, 3)$ and X_i $(i = 1, 2, 3)$ satisfy the relation $D_1 D_2 D_3 = 1$ and $X_1 + X_2 + X_3 = 0$.

From eqs (1.3) and (3.17), we get

$$\rho_{\mathrm{WDF}} = C \left(\frac{3\kappa C}{2} t^2 - \frac{C_1}{3\kappa C} \right)^{-1} \tag{3.30}$$

and from eqs (1.1) and (3.21), we get

$$p_{\mathrm{WDF}} = 0. \tag{3.31}$$

By using eqs (3.8)–(3.11) we can express the physical quantities as

$$\theta = \frac{3\kappa C t}{\frac{3\kappa C}{2} t^2 - \frac{C_1}{3\kappa C}}, \tag{3.32}$$

$$A = 27 \frac{X^2 \kappa^2 C^2}{t^2}, \tag{3.33}$$

$$\sigma^2 = \frac{81}{2} \frac{X^2 \kappa^4 C^4}{t^2 (\frac{3\kappa C}{2} t^2 - \frac{C_1}{3\kappa C})}, \tag{3.34}$$

$$q = \frac{1}{2} + \frac{C_1}{3\kappa^2 C^2} \frac{1}{t^2}, \tag{3.35}$$

where $X^2 \equiv X_1^2 + X_2^2 + X_3^2$ is a constant.

For large t, the model tends to be isotropic.

4. Models with constant deceleration parameter

Case I. Power-law

Here we take

$$V = at^b, \tag{4.1}$$

where a and b are constants.

From eqs (2.19)–(2.21) and (4.1), we get

$$a_1(t) = D_1 a^{1/3} t^{b/3} \ \exp\left(\frac{X_1}{a(1-b)} t^{1-b}\right), \tag{4.2}$$

$$a_2(t) = D_2 a^{1/3} t^{b/3} \ \exp\left(\frac{X_2}{a(1-b)} t^{1-b}\right), \tag{4.3}$$

$$a_3(t) = D_3 a^{1/3} t^{b/3} \ \exp\left(\frac{X_3}{a(1-b)} t^{1-b}\right), \tag{4.4}$$

where D_i ($i = 1, 2, 3$) and X_i ($i = 1, 2, 3$) satisfy the relation $D_1 D_2 D_3 = 1$ and $X_1 + X_2 + X_3 = 0$.

From eqs (1.3) and (4.1), we have

$$\rho_{\mathrm{WDF}} = \frac{\gamma}{1+\gamma}\rho_\star + \frac{C}{a^{(1+\gamma)}} t^{-(1+\gamma)b} \tag{4.5}$$

and from eqs (1.1) and (4.5), we get

$$p_{\mathrm{WDF}} = \gamma\left(\frac{C}{a^{(1+\gamma)}} t^{-(1+\gamma)b} - \frac{1}{1+\gamma}\rho_\star\right). \tag{4.6}$$

Using eqs (3.8)–(3.11) we can express the physical quantities as

$$\theta = \frac{b}{t}, \tag{4.7}$$

$$A = \frac{6X^2}{a^2 b^2} \frac{1}{t^{2(b-1)}}, \tag{4.8}$$

$$\sigma^2 = \frac{X^2}{a^2} \frac{1}{t^{2b}}, \tag{4.9}$$

$$q = \frac{3}{b} - 1, \tag{4.10}$$

where $X^2 \equiv X_1^2 + X_2^2 + X_3^2$ is a constant.

For large t, the model tends to be isotropic when $b > 1$.

Case II. Exponential type

Here we take

$$V = \alpha \mathrm{e}^{\beta t}, \tag{4.11}$$

where α and β are constants.

From eqs (2.19)–(2.21) and (4.11), we get

$$a_1(t) = D_1\alpha^{1/3}\ \exp\left(\frac{\beta t}{3} + \frac{X_1}{\alpha\beta}\mathrm{e}^{-\beta t}\right), \tag{4.12}$$

$$a_2(t) = D_2\alpha^{1/3}\ \exp\left(\frac{\beta t}{3} + \frac{X_2}{\alpha\beta}\mathrm{e}^{-\beta t}\right), \tag{4.13}$$

$$a_3(t) = D_3\alpha^{1/3}\ \exp\left(\frac{\beta t}{3} + \frac{X_3}{\alpha\beta}\mathrm{e}^{-\beta t}\right), \tag{4.14}$$

where D_i $(i = 1, 2, 3)$ and X_i $(i = 1, 2, 3)$ satisfy the relation $D_1D_2D_3 = 1$ and $X_1 + X_2 + X_3 = 0$.

From eqs (1.3) and (4.11), we have

$$\rho_{\mathrm{WDF}} = \frac{\gamma}{1+\gamma}\rho_\star + \frac{C}{\alpha^{(1+\gamma)}}\mathrm{e}^{-(1+\gamma)\beta t} \tag{4.15}$$

and from eqs (1.1) and (4.15), we get

$$p_{\mathrm{WDF}} = \gamma\left(\frac{C}{\alpha^{(1+\gamma)}}\mathrm{e}^{-(1+\gamma)\beta t} - \frac{1}{1+\gamma}\rho_\star\right). \tag{4.16}$$

By using eqs (3.8)–(3.11) we can express the physical quantities as

$$\theta = \beta, \tag{4.17}$$

$$A = \frac{6X^2}{\alpha^2\beta^2}\mathrm{e}^{-2\beta t}, \tag{4.18}$$

$$\sigma^2 = \frac{X^2}{\alpha^2}\mathrm{e}^{-2\beta t}, \tag{4.19}$$

$$q = -1, \tag{4.20}$$

where $X^2 \equiv X_1^2 + X_2^2 + X_3^2$ is a constant.

For large t, the model tends to be isotropic.

5. Conclusion

The Bianchi Type-I Universe has been considered for a new equation of state for the dark energy component of the Universe (known as dark wet fluid). The solution has been obtained in quadrature form. The models with constant deceleration parameter have been discussed in detail. The behaviour of the models for large time have been analysed.

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