Solar Phys (2007) 243: 163–169
DOI 10.1007/s11207-007-0464-7

An Evaluation of Possible Mechanisms for Anomalous Resistivity in the Solar Corona

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Received: 30 March 2007 / Accepted: 23 June 2007 / Published online: 8 August 2007

Abstract A wide variety of transient events in the solar corona seem to require explanations that invoke fast reconnection. Theoretical models explaining fast reconnection often rely on enhanced resistivity. We start with data derived from observed reconnection rates in solar flares and seek to reconcile them with the chaos-induced resistivity model of Numata and Yoshida (*Phys. Rev. Lett.* **88**, 045003, 2002) and with resistivity arising out of the kinetic Alfvén wave (KAW) instability. We find that the resistivities arising from either of these mechanisms, when localized over length scales of the order of an ion skin depth, are capable of explaining the observationally mandated Lundquist numbers.

1. Introduction

Transient events in the solar atmosphere can often be explained only with models of fast reconnection. Although the Petschek (1964) model of reconnection has long been invoked to account for fast reconnection, simulations as well as experiments (*e.g.*, Biskamp, 1986; Kulsrud, 2001; Ji *et al.*, 1998) seem to indicate that the reconnection geometry might be of the Sweet – Parker kind (Sweet, 1958; Parker, 1963). Anomalous resistivity has been often invoked to explain fast reconnection (*e.g.*, Biskamp and Welter, 1980; Yokoyama and Shibata, 1994). Nakariakov *et al.* (1999) have invoked an anomalous resistivity that is as much as 6 orders of magnitude larger than the classical value to explain the damping of coronal waves (see, however, Klimchuk, Tanner, and DeMoortel, 2004 for an alternative explanation). Based on observations of a variety of dynamic events, Dere (1996) concluded that the solar atmosphere is much more resistive than what can be accounted for by classical resistivity. Tsiklauri (2005) has invoked a novel mechanism to account for heating of coronal

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loops. However, it requires the loops to be composed of extremely small subthreads, with dimensions as small as a few proton Larmor radii. If the resistivity is larger than the classical value, it might alleviate this severe requirement on the thickness of the individual strands comprising coronal loops.

2. Candidate Mechanisms for Anomalous Resistivity

Several candidate mechanisms have been proposed for the microscopic origin of anomalous resistivity, including ion-acoustic turbulence (Bychenkov, Silin, and Uryupin, 1988; Uzdensky, 2003), the kinetic Alfvén wave (KAW) instability (Voitenko, 1995; Bellan, 1999, 2001), and chaotic particle motion near the null region (Numata and Yoshida, 2002, 2003). In this paper, we will examine in detail mechanism of Numata and Yoshida (2002, 2003), where the chaotic motion of particles in the relatively unmagnetized null region mimics collisions and can therefore be used as a basis for deriving an effective anomalous resistivity. The other mechanism we will consider is the current-driven KAW instability.

One of the important bases used for invoking current-driven instabilities such as the ion-acoustic instability or the KAW instability is the presence of magnetic-field-aligned currents. In turn, the presence of field-aligned currents is inferred from the fact that solar flares are usually produced from active regions with significant shear and that there is often a significant change in shear following the occurrence of a flare (*e.g.*, Sivaraman, Rausaria, and Aleem, 1992). The premise is that since the reconnecting fields are highly sheared, the classical reconnection geometry with antiparallel fields annihilating is no longer applicable, and there could well be significant magnetic field components along the reconnection-induced currents. However, such observational inferences arising from two-dimensional pictures of filaments should be treated with caution. First, as Venkatakrishnan (1993) demonstrates, redeployment of magnetic flux sources relative to the main sunspot(s) and/or emergence of new flux is a more satisfactory explanation for the observations of Sivaraman, Rausaria, and Aleem (1992). Furthermore, extensive vector magnetogram observations of flare-producing active regions have revealed that apparent photospheric magnetic shear is not really an essential condition for flare production (Wang, 1997). Their in-depth study shows that the photospheric magnetic shear does not change after several M-class flares, and it even *increases* after the occurrence of large X-class flares (in fact, it does so for all the X-class flares in their sample). These paradoxes can be understood only in the context of a three-dimensional reconnection process, of which photospheric shear provides only a partial, two-dimensional picture.

The crucial difference between a scenario where the resistivity arising out of the current-driven KAW instability would be dominant and one where the chaos-induced resistivity would be so is that the former mechanism can proceed even when the reconnecting magnetic fields are not strictly antiparallel, and magnetic-field-aligned currents can therefore be present. As just discussed, the three-dimensional geometry of reconnecting fields is not immediately obvious from current observations. However, although the direct connection to observations of sheared filaments might be simplistic, it is possible that field-aligned currents will exist in reconnection regions. It therefore stands to reason that we should consider a general scenario where the reconnecting magnetic fields need not be exactly antiparallel, where the anomalous resistivity arises out of a current-induced instability. Of the two current-induced instabilities we have mentioned, Bellan (2001) has shown that the KAW instability has a lower threshold than the ion-acoustic one. In addition to the chaos-induced resistivity model of Numata and Yoshida (2002, 2003), we will therefore also examine the viability of the KAW instability-induced anomalous resistivity using the approach taken by Voitenko (1995).

3. Lundquist Number Comparison

The Lundquist number gives the ratio of the Lorentz ($J \times B$) force to the force due to resistive magnetic diffusion. We take this to be the figure of merit for evaluating the efficacy of the anomalous resistivity mechanisms we consider. We will derive Lundquist numbers for solar flare events reported in Isobe, Takasaki, and Shibata (2005) and Nagashima and Yokoyama (2006). We will compare these observationally mandated Lundquist numbers with those derived using the anomalous resistivity mechanisms of Numata and Yoshida (2002, 2003) and Voitenko (1995). The macroscopic Lundquist number is defined as

$$S = \frac{V_{\mathrm{A}} L}{D}, \tag{1}$$

where V_{A} is the Alfvén velocity, L is a suitable macroscopic scale length, and D is the magnetic diffusivity. In MKS units, the magnetic diffusivity is defined as

$$D = \frac{\eta}{\mu_0} \quad (\mathrm{m}^2\,\mathrm{s}^{-1}), \tag{2}$$

where η is the resistivity and μ_0 is the magnetic permeability of free space. Using Equation (2) in Equation (1) gives

$$S = \frac{V_{\mathrm{A}} L \mu_0}{\eta}. \tag{3}$$

For a given transient event in the solar atmosphere, the *observed* diffusivity is

$$D_{\mathrm{obs}} = \frac{L^2}{T} \quad (\mathrm{m}^2\,\mathrm{s}^{-1}), \tag{4}$$

where L is the observed length scale and T is the observed time scale. This gives the *required* Lundquist number as mandated by the observations,

$$S_{\mathrm{req}} = \frac{V_{\mathrm{A}} L}{D_{\mathrm{obs}}} = \frac{V_{\mathrm{A}} T}{L}. \tag{5}$$

3.1. Lundquist Numbers from the Numata – Yoshida Mechanism

We now turn our attention to the Lundquist number that can be realized by using the chaos-induced resistivity η_{eff} defined in Numata and Yoshida (2002). Using their anomalous resistivity prescription

$$\eta = \eta_{\mathrm{eff}} = \mu_0 \lambda_{\mathrm{i}}^2 \omega_{\mathrm{ci}}^2 \hat{\nu}_{\mathrm{eff}} \tag{6}$$

in Equation (3) we get

$$S = S_{Y1} = \frac{L \omega_{\mathrm{ci}}}{V_{\mathrm{A}} \hat{\nu}_{\mathrm{eff}}} = \frac{L}{\hat{\nu}_{\mathrm{eff}} \lambda_{\mathrm{i}}}, \tag{7}$$

where we have used the following expression for the ion skin depth λ_i:

$$\lambda_i = \frac{V_A}{\omega_{ci}} \quad (\mathrm{m}). \tag{8}$$

Using the expression $\omega_{ci} = eB/m_p$ for the ion cyclotron frequency, we can rewrite Equation (7) as

$$S_{Y1} = \frac{LeB}{m_p V_A \hat{\nu}_{eff}}. \tag{9}$$

The quantity $\hat{\nu}_{eff}$ is the effective collision frequency in units of the ion cyclotron frequency. Numata and Yoshida (2002) show that, in effect, $\hat{\nu}_{eff}$ is equal to the Alfvén Mach number M_A of the flow outside the reconnection region. This yields

$$S_{Y1} = \frac{LeB}{m_p V_A M_A}. \tag{10}$$

The expression for the Lundquist number S_{Y1} given by Equation (10) arises out of using the chaos-induced resistivity η_{eff} and assuming that the resistivity is operative over a *macroscopic* length scale L. However, Malyshkin, Linde, and Kulsrud (2005) and Malyshkin and Kulsrud (2006) suggest that it is not enough for the resistivity to be enhanced for the reconnection to occur at a fast rate; the resistivity also needs to be *localized* over small length scales. If the resistivity is spatially localized over a length scale l_η, the resulting Lundquist number is obtained by simply using $L = l_\eta$ in Equation (3). Numata and Yoshida's (2002) treatment suggests that the enhanced resistivity might be localized over length scales comparable to the ion skin depth λ_i. Using $L = l_\eta = \lambda_i$ in Equation (7) yields the following expression for the Lundquist number resulting from chaos-induced resistivity localized over an ion skin depth:

$$S = S_{Y2} = \frac{1}{\hat{\nu}_{eff}}. \tag{11}$$

As mentioned earlier, $\hat{\nu}_{eff}$ can be taken to equal to the Alfvén Mach number M_A (Numata and Yoshida, 2002), which yields

$$S = S_{Y2} = \frac{1}{\hat{\nu}_{eff}} = \frac{1}{M_A}. \tag{12}$$

In writing Equation (12) it may be noted that we have used the macroscopic Alfvén Mach number M_A, whereas Numata and Yoshida (2002) have referred to the microscopic Alfvén Mach number. However, Lin *et al.* (2007), have related a microscopic definition of the Lundquist number (the ratio of the resistive diffusion and Alfvén time scales, which is equal to the ratio of the width to thickness of the current sheet) to the macroscopic Alfvén Mach number. The use of the macroscopic Alfvén number is primarily because it is the only one that can be observationally estimated.

3.2. Lundquist Numbers from the KAW Instability Mechanism

We follow the approach of Voitenko (1995) in evaluating the Lundquist number S_{KAW} arising out of the KAW instability. For conditions applicable to the solar corona (in particular, we note that the Alfvén speed they use is similar to the values in Table 1), Voitenko (1995)

Table 1 Lundquist number ratios.

Number	L	T	B	V_A	V_{in}	M_A	$\frac{S_{req}}{S_{Y1}}$	$\frac{S_{req}}{S_{Y2}}$	$\frac{S_{req}}{S_{KAW}}$
1a	2.56×10^7	1.32×10^3	62×10^{-4}	4.25×10^6	4.8×10^3	1.1×10^{-3}	6.73×10^{-8}	0.24	0.72
1b	2.56×10^7	1.32×10^3	116×10^{-4}	8.0×10^6	4.8×10^3	6.0×10^{-4}	6.95×10^{-8}	0.25	0.71
1c	2.8×10^7	1.32×10^3	41×10^{-4}	2.8×10^6	1.3×10^5	4.7×10^{-2}	1.57×10^{-6}	6.204	0.66
2a	2.94×10^7	4.8×10^2	32×10^{-4}	2.2×10^6	1.5×10^4	7×10^{-3}	6.13×10^{-8}	0.25	0.23
2b	2.94×10^7	4.8×10^2	60×10^{-4}	4.2×10^6	1.5×10^4	3.7×10^{-3}	6.3×10^{-8}	0.25	0.22
2c	2.30×10^7	4.8×10^2	44×10^{-4}	2.1×10^6	3.2×10^4	1.5×10^{-2}	1.42×10^{-7}	0.66	0.42
3a	4.12×10^7	1.2×10^3	9.0×10^{-4}	6.2×10^5	8.6×10^3	1.4×10^{-2}	4.41×10^{-8}	0.25	0.40
3b	4.12×10^7	1.2×10^3	32×10^{-4}	2.3×10^6	8.6×10^3	3.9×10^{-3}	4.75×10^{-8}	0.26	0.39
3c	4.0×10^7	1.2×10^3	11×10^{-4}	9.4×10^5	6.7×10^4	7.1×10^{-2}	4.46×10^{-7}	2.0	0.33

Notes. The observational data for reconnection events are taken from Nagashima and Yokoyama (2006). All physical quantities are in MKS units. Column 1: For each event, we use observational data listed as method 1, method 2, and from Isobe, Takasaki, and Shibata (2005) in Nagashima and Yokoyama (2006). For instance, 1a refers to method 1, 1b refers to method 2 and 1c refers to data from Isobe, Takasaki, and Shibata (2005). Column 2: The observed length scale L of the reconnection event. Column 3: The observed time scale T of the event. Column 4: The inferred magnetic field B in the reconnection region. Column 5: The Alfvén speed V_A. Column 6: The observed inflow speed V_{in} in the reconnection region. Column 7: The Alfvén Mach number M_A $(= V_{in}/V_A)$. Column 8: Ratio of Lundquist numbers S_{req}/S_{Y1}, where S_{req} is given by Equation (5) and S_{Y1} by Equation (10). Column 9: Ratio of Lundquist numbers S_{req}/S_{Y2}, where S_{req} is given by Equation (5) and S_{Y2} by Equation (12). Column 10: Ratio of Lundquist numbers S_{req}/S_{KAW} given by Equation (15).

quotes the following approximate value for the magnetic diffusion coefficient D_{KAW} arising from this mechanism:

$$D_{\mathrm{KAW}} \simeq 10^5 \mathrm{~m}^2 \mathrm{~s}^{-1}. \tag{13}$$

Following Equation (1), and since the KAW anomalous resistivity is naturally localized over a thickness of the order of an ion skin depth (Voitenko, 1995), we write the Lundquist number S_{KAW} as

$$S_{\mathrm{KAW}} = \frac{V_{\mathrm{A}} \lambda_{\mathrm{i}}}{D_{\mathrm{KAW}}}, \tag{14}$$

where we have used $L = \lambda_{\mathrm{i}}$. Equations (13), (14), (8), and (5) and $\omega_{\mathrm{ci}} = eB/m_p$ yield

$$\frac{S_{\mathrm{req}}}{S_{\mathrm{KAW}}} \simeq 10^5 \frac{T}{L} \frac{1}{V_{\mathrm{A}}} \frac{eB}{m_p}. \tag{15}$$

4. Results

We use the formalism developed in the previous section to derive the ratios S_{req}/S_{Y1}, S_{req}/S_{Y2}, and $S_{\mathrm{req}}/S_{\mathrm{KAW}}$ for several reconnection events, using observational data given in Nagashima and Yokoyama (2006). They have compiled a statistical study of flares observed with the soft X-ray telescope aboard the *Yohkoh* spacecraft. We have listed the observed length scale L and time scale T, inferred magnetic field B, and reconnection inflow speed V_{in} for each of these events in Table 1. The inferred ambient density for each of the events is $n = 10^{15} \mathrm{~m}^{-3}$. Using these quantities, we have derived the Alfvén speed V_{A} and the Alfvén Mach number $M_{\mathrm{A}} \equiv V_{\mathrm{in}}/V_{\mathrm{A}}$. We have compared S_{req}/S_{Y1}, S_{req}/S_{Y2}, and $S_{\mathrm{req}}/S_{\mathrm{KAW}}$ for each of these events using Equations (5), (10), (12), and (15). S_{req} is the Lundquist number mandated by the observations, whereas S_{Y1} is the Lundquist number obtained by assuming that the resistivity is due to the Numata – Yoshida mechanism (Numata and Yoshida, 2002, 2003). S_{Y2} is the Lundquist number obtained by assuming that the resistivity is due to the Numata – Yoshida mechanism and is localized over a length scale equal to the ion skin depth λ_{i}. S_{KAW} is the Lundquist number arising from the KAW instability and is naturally localized over an ion skin depth. For all the events, it is evident from Table 1 that S_{Y2} is much closer to S_{req} than S_{Y1} is. Unlike Nagashima and Yokoyama (2006), Dere (1996) does not explicitly list an inflow velocity V_{in} for the events he has considered. By using $V_{\mathrm{in}} = L/T$, the Alfvén Mach number is $M_{\mathrm{A}} = V_{\mathrm{in}}/V_{\mathrm{A}} = L/V_{\mathrm{A}}T$. For each of the events listed in Dere (1996), this yields values of S_{req}/S_{Y1} that are similar to those for the events listed in Table 1. However, when this definition of M_{A} is used in the definition of S_{Y2} [Equation (12)], it works out to be exactly the same as S_{req} [Equation (5)].

5. Summary

The numbers in Table 1 show that the Lundquist number arising from the Numata – Yoshida resistivity localized over an ion skin depth (S_{Y2}) as well as that from the KAW instability (S_{KAW}) are fairly close to the Lundquist number S_{req} mandated by observations. It is also evident that to explain the observations the resistivity must not only be enhanced but also well localized.

Acknowledgements K.A.P.S. acknowledges the support from the University Grants Commission, New Delhi, for the award of the senior research fellowship. We acknowledge constructive criticism from an anonymous referee that has helped us significantly improve the paper.

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