Introduction and Literature Review

1.1 Introduction

The main objective of this thesis is to propose an analytical model for the prediction of electromagnetic field distribution and performance analysis of axial flux and radial flux SMPM machines. PM machines have been a topic of interest owing to their high efficiency, high power and torque density, superior torque to inertia ratio and fault tolerant features [El-Refaie, 2008], these machines are being used in a wide range of applications like EVs, ship propulsions, direct drive applications, and wind generators. The use of PMs in these machines induces the high magnetic fields in the air gap, without excitation currents, leading to high power density and high efficiency than DC motors, induction motors and synchronous reluctance motors. These motors potentially offer cost-effective energy-efficient alternatives to both domestic induction motors and automotive DC motors.

With the advent of rare-earth PMs in 1980's, the use of these machines in applications with demand of high efficiency, space constraint and less maintenance like HEVs, space craft applications and other safety critical applications are preferred over other electrical machines. The energy product of rare-earth NdFeB PMs is highest of all the other types of magnets and certain grades of NdFeB (N50) can be used upto 200°C. Compared to SmCo PMs, NdFeB offer compatible material properties and are essentially cheaper and mechanically stronger. Alnico PMs possess much lower energy product than the rare earth PMs though has highest working temperature of 550°C. The conventional ferrite magnets have very low energy product and are prone to demagnetization due to low thermal stability. Being cheapest of all, they have low thermal instability and less residual magnetism however the hard ferrite or ceramic magnets are an improvement with remanance of 0.2-0.4 Tesla and highest working temperature of 250°C. Table 1.1 gives a comparison of the properties of PMs. The higher values of both remanence and coercivity of NdFeB PMs gives the highest energy product that is practically 10 times higher than the Ceramic PMs.

Fig 1.1 shows the percentage of metric tonnes of rare-earth PMs produced globally with China yielding almost 88% of the total production. With the development of high performance NdFeB PM materials and grades, a trend towards the use of PM machines in large-scale industrial applications have started. The grade of a PM directly refers to the percentage of the Nd, Fe, B and maximum energy product, however physical properties of the material remains same. Table 1.2 depicts the available grades of NdFeB and SmCo rare-earth PMs with their corresponding coercive force, residual flux density and BH_{max} [Alliance LLC]. The higher the grade “the stronger” will be magnet. Fig 1.2 shows the use of PMs in various applications. The PMs have found their application mostly in hard disk drives and electric motors for industrial purposes. The automobile industry is another big sector employing PMs in large quantities. The material properties of NdFeB and SmCo make them very suitable to be used in electric motors, generators, clutch and brakes.

Due to the distinguished advantages of rare-earth magnets and decreasing cost of NdFeB PMs over years as seen from Fig. 1.3, the research interest and application of PM machines have found to be increasing. The thesis aims to develop an analytical method for the performance analysis of SMPM machines. Section 1.1 gives an introduction to PMs, their properties and applications. PM machines and their topologies have been discussed in Section 1.2. The necessity and importance of modeling of the PM machines and different methods of modeling of PM machines has been elaborated in Section 1.3. The detailed literature survey of the analytical modeling of magnetic field distribution and cogging torque of the PM machines has been reported in Section 1.4. Based on this literature, the motivation for this research and the objective of the thesis has been discussed in Section 1.5. The scope of the present work has been elaborated in Section 1.6. The Chapter is concluded in Section 1.7.

1.2 Permanent Magnet Machines

PM machines have received increasing research interest and are replacing induction machines in industrial and domestic applications due to their inherent high power density, high efficiency, low maintenance and easy manufacturing. The common applications are pumps, compressors, aircrafts, EVs, HEVs, etc. These machines have constant field excitation provided with PMs placed either on stator or rotor. They can produce higher output power for the same speed and active volume than that of the synchronous machine due to its lower reactance owing to the low permeability of the PM material compared with iron. PM motors offer better efficiency over induction motors due to the absence of windings in either stator or rotor. In addition to improved efficiency even at rated load, it provides high efficiency at light loads also. This is due to the reduced no-load current as the PMs provide the required flux at all loads [Melfi *et al.*, 2009]. An optimum level of air gap flux density around 0.8T can be attained with a relatively thin magnet (5-7 mm) mounted on a hollow, light weight rotor. In contrast, to attain this flux density level in a conventional synchronous machine requires more field ampere-turns owing to the inefficient rotor magnetic circuit.

Table 1.1 Properties of the Permanent Magnets

Parameters	**NdFeB**	**SmCo**	**Alnico**	**Ferrite**
Remanent Flux Density, B_r (T)	0.7–1.41	0.55–1.15	0.7–1.35	0.22–0.42
Coercivity, H_c (kA/m)	310–1500	360–820	44–151	151–254
Relative Permeability, μ_r	1.05	1.05	1.70-4.70	1-5
Operating Temperature, T (°C)	100–200	200–550	200–500	100–250
Conductivity, σ ($\Omega^{-1}\,m^{-1}$)	0.63×10^6	1.25×10^6	2×10^6	1×10^{-6}

Table 1.2 Rare Earth Material Grades

Material	**Grades**	**Residual Flux Density (Tesla)**	**Coercivity (kOe)**	**Maximum Energy Product (MGOe)**
NdFeB	N35	1.21	11.4	35
	N38	1.26	11.7	38
	N40	1.29	11.9	40
	N42	1.33	12.3	42
	N45	1.36	12.1	45
	N48	1.4	12.1	48
	N50	1.43	12.3	50
SmCo	18	0.85	8	18
	20	0.9	8	20
	24	0.1	8.5	24
	26	0.105	9.2	26
	28	1.1	9.5	28
	30	1.2	9.5	30
	32	1.2	11	32

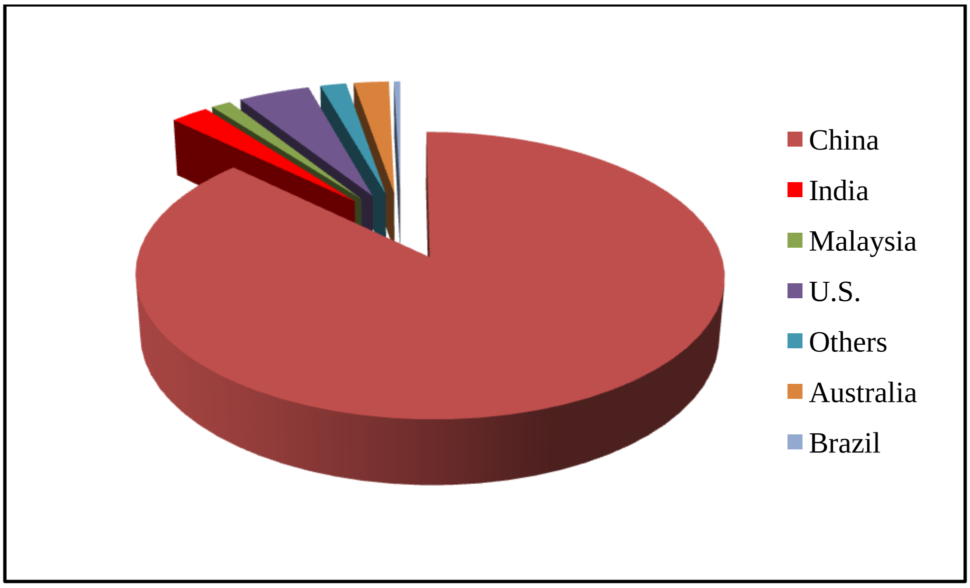


Fig. 1.1 Global Production of PMs [1]

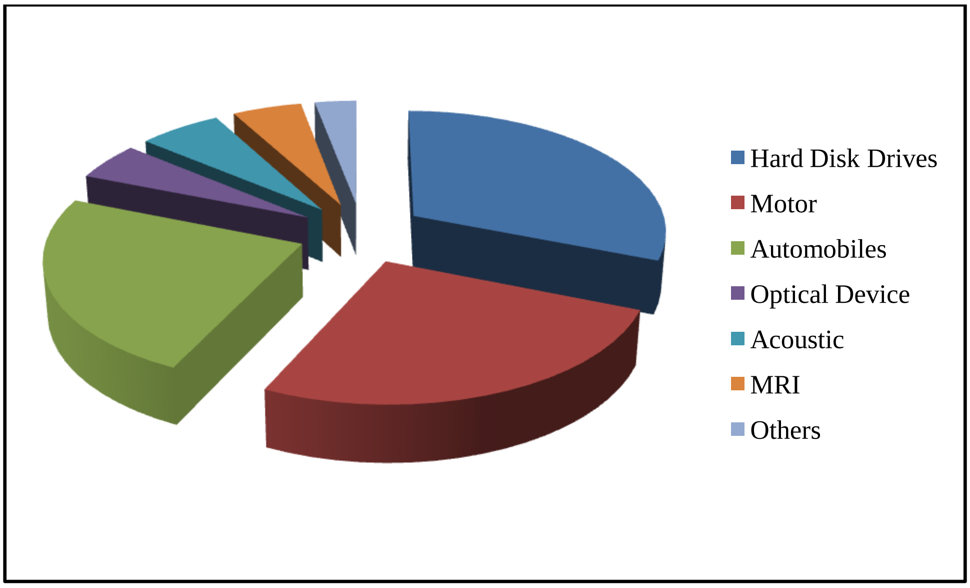


Fig. 1.2 Applications of PMs [2]

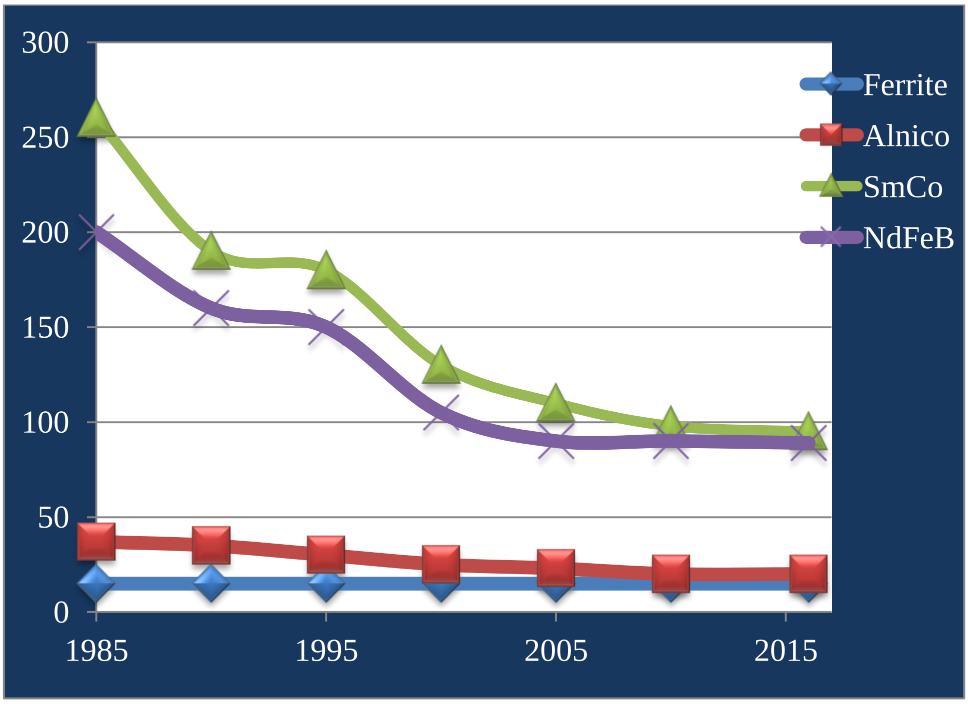


Fig 1.3 Cost of Permanent Magnets over years [3]

The absence of commutators, brushes, slip rings and field coils in the PM machines demands less or negligible maintenance. The simple rotor construction with a hollow or solid core has much lower inertia than that of the conventional machine offering high torque-to-inertia ratio. A wide speed range can be obtained using several rotor configurations of the PM machines. Introducing rotor saliency allows one to use flux weakening operation and thus speed can be varied above the base speed of machine. Different machine ratings can be achieved maintaining the same diameter and vary the machine active length to achieve as required for different applications. Because of the magnet permeability near to unity, the machine is less sensitive to air gap variations. Therefore, manufacturing tolerances could be relaxed to reduce manufacturing cost [Naidu *et al.*, 1995]. With modification in the rotor topology and geometry of the machine, it can be utilized for applications with different requirements. The classification of PM machines on the basis of construction, direction of flux and position of PMs has been shown in Fig. 1.4. In next section, merits and classification of PM machines on the basis of rotor topologies have been discussed.

1.2.1 Rotor Topologies of PM Machines

Depending on the position of PMs on the rotor, there are many possible rotor configurations of the PM machine. The common rotor topologies being SMPM, SIPM, IPM and LSPM have been elaborated here:

a) **Surface Mounted:** The SMPM machines with magnets pasted on the surface of the rotor have a small rotor diameter with low inertia. The magnetic structure of the rotor has circumferential symmetry as the permeability of the PM is almost equal to air permeability, thus the rotor has no saliency. The motor torque is produced due to the interaction of stator current with magnet flux. SMPM is the most frequently used configuration with ease of manufacturing and lower constructional cost as compared to other PM rotor topologies. With the introduction of rare-earth magnets having strong magnetic field, large demagnetizing resistance and operating temperature, the main drawback of SMPM machine i.e., the exposition of the permanent magnets to demagnetizing armature fields is no longer a problem. SMPM motor provides high torque to weight ratio with high peak torque capabilities. With low manufacturing cost, these machines find their application mostly in low power adjustable speed drives [Pellegrino *et al*., 2012].

b) **Surface Inset:** The PMs are located on the surface of the rotor with teeth present in between adjacent magnets creating saliency. This saliency allows an additional torque component .i.e., reluctance torque. Also because of the rotor saliency, the inductance of the stator has a different value as aligned with the rotor quadrature (q) axis, compared with that of the direct (d) axis. The SIPM structure provides a unique feature of flux regulation in the PM machine, thus providing additional advantages of high starting torque and wide power operation range.

c) **Interior PM:** These machines have PMs completely embedded in the rotor providing higher per unit inductance and thus more field weakening capability than SMPM and SIPM. The flux barriers are provided to increase the reluctance torque in the motor, thereby increasing the total torque produced. These motors are better option for high speed low torque applications while SMPM are very well suited for low speed high torque applications. The thickness of iron pole pieces can be optimized for proper flux weakening region. IPM motors with controlled flux density using suitable thickness of iron pole pieces and ribs can be used with flux-weakening operation for low torque wide speed range application [Vakil *et al*., 2010].

d) **Line start PM:** Line start PM motor can also be termed as self-starting PM motor. The inherent starting capability is due to the presence of either induction cage or conducting sleeve on the rotor. The motor starts as an induction motor and once synchronous speed is attained, runs as synchronous motor and induction cage or sleeve becomes invisible to the stator. The rotor configuration and PM volume in rotor mainly influences the performance of LSPM machine. These motors are well recognized in industries for their line start capability with high efficiency levels over conventional induction and PM motors [McElveen *et al.*, 2014].

1.2.2 Radial Flux SMPM Machines

Radial flux topology is the most common structure of electrical machines being used in industrial and domestic applications having simple construction and ease of manufacturing. RFPM machines are widely used in industrial applications such as pumps, fans, valve control, centrifuges, machine tools, robots and industrial equipment.

The current flows in axial direction while direction of flux is radial. The stator structure can be slot less or slotted and consists of a stack of laminated steel. Fig. 1.6 shows a conventional inner rotor RFSMPM machine. These machines can operate both as generator as well as motor. The windings facing the rotor poles are used for torque production in RFPMs. The portions of the windings on the outside surface of the stator on both sides are referred as end windings. This topology has long end windings when the aspect ratio D/L (diameter to axial length) is small. One important advantage of this topology is that the structure transfers the heat from stator frame very easily. Therefore, electrical loading of machine can be comparatively high. An important manufacturing shortcoming of RFPM machine is that PM must be carefully assembled on the surface of rotor so that it does not fly apart.

RFPM machines can be dual stator [Liu *et al.*, 2009], dual rotor [Hamadou *et al.*, 2009] or outer rotor [Umadevi *et al.*, 2011] apart from the conventional inner rotor outer stator motor. DSPM machine was introduced to perform the flux control however, the main role of these machines was to maintain the constant output voltage in the wind generation using multi-tasking in the event of change in speed or load, to increase the starting torque and to compensate energy of electric vehicles [Chai *et al.*, 2005]. The double stator structure is able to improve power density when it is used in wind power generation. However, double rotor or dual mechanical port (DMP) machine behaves as a two concentric motor with no electromagnetic coupling between the two rotors [Niu *et al.*, 2007]. The speed of two rotors is independent of each other. The DMP has two mechanical ports and combines the function of two electrical machines and a planetary gear [Zhang *et al.*, 2006].

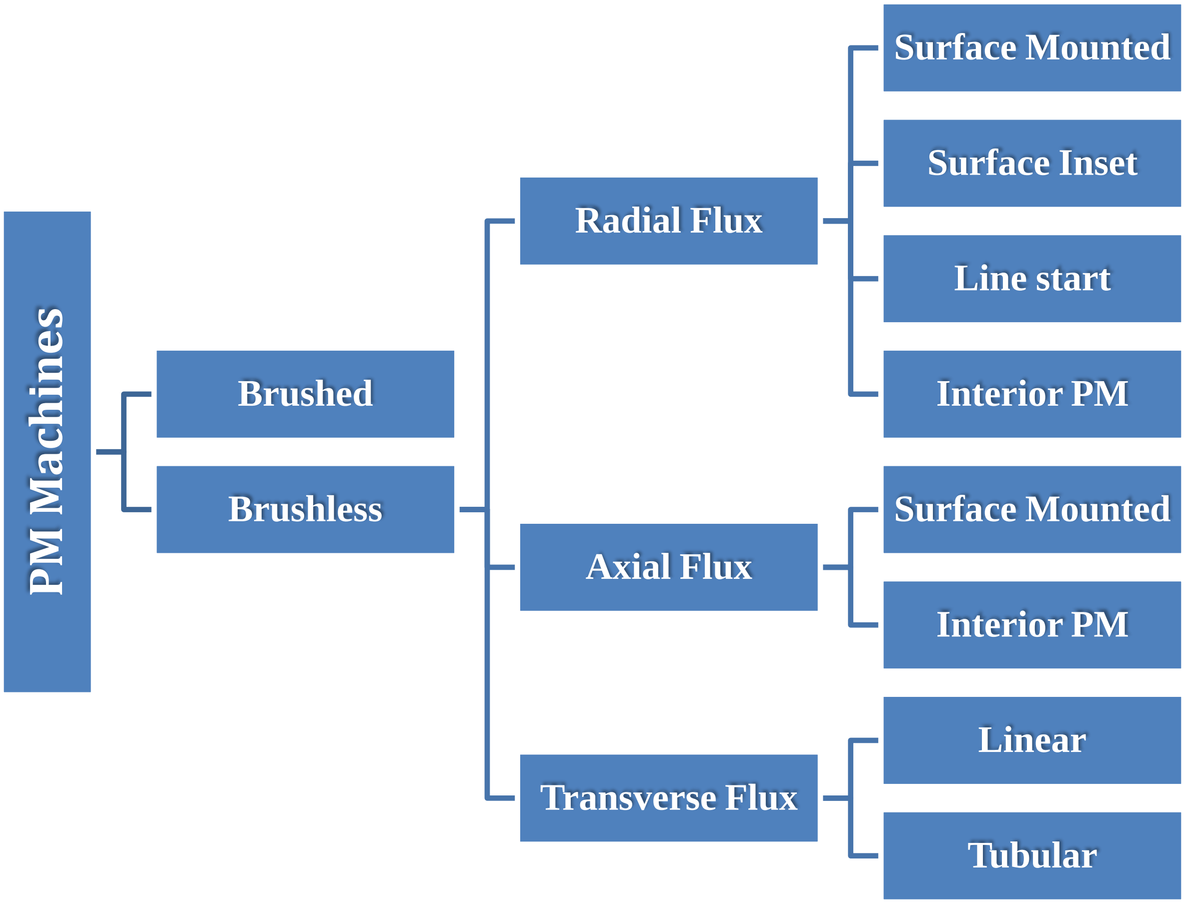


Fig. 1.4 Classification of PM Machines

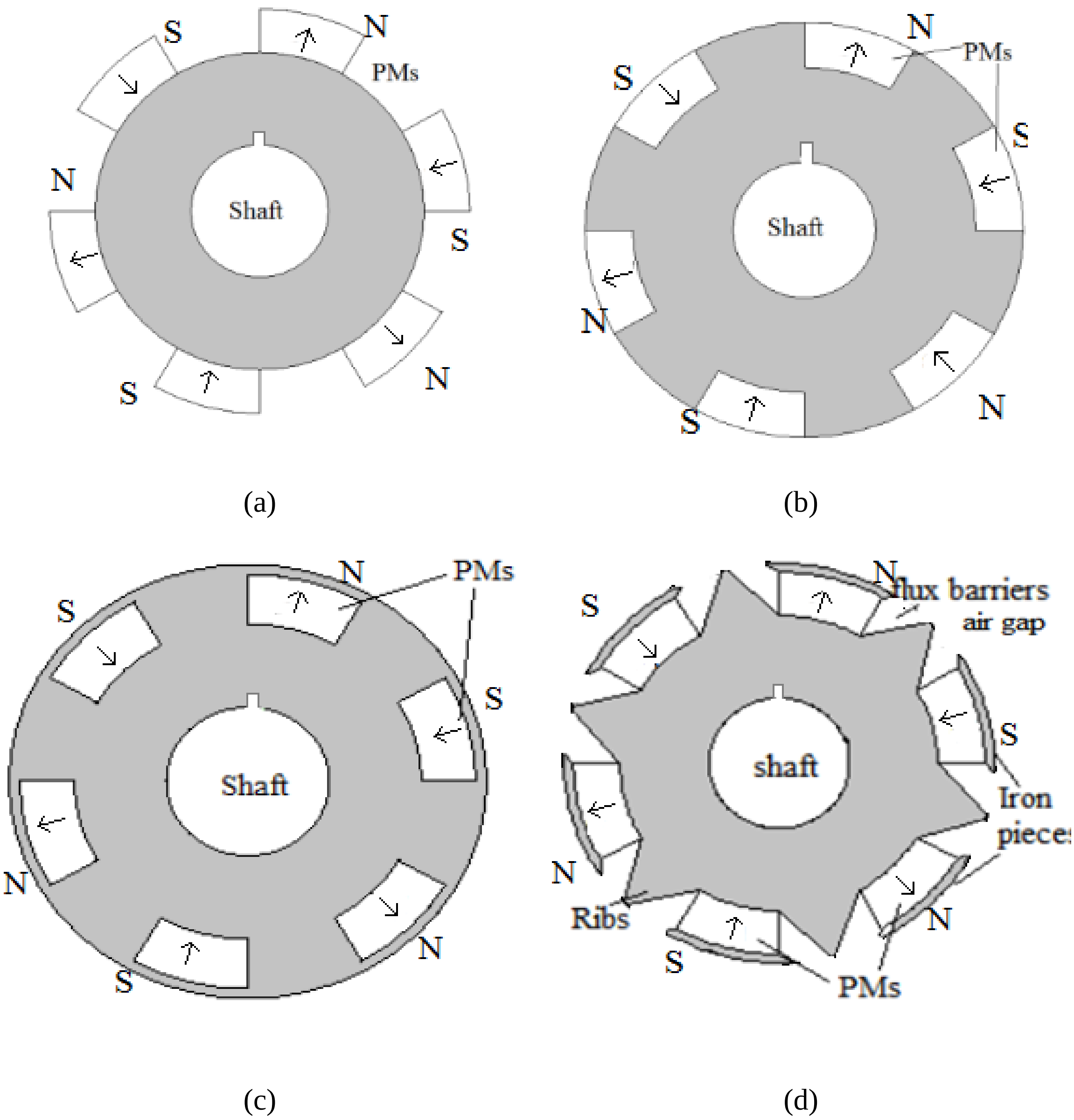
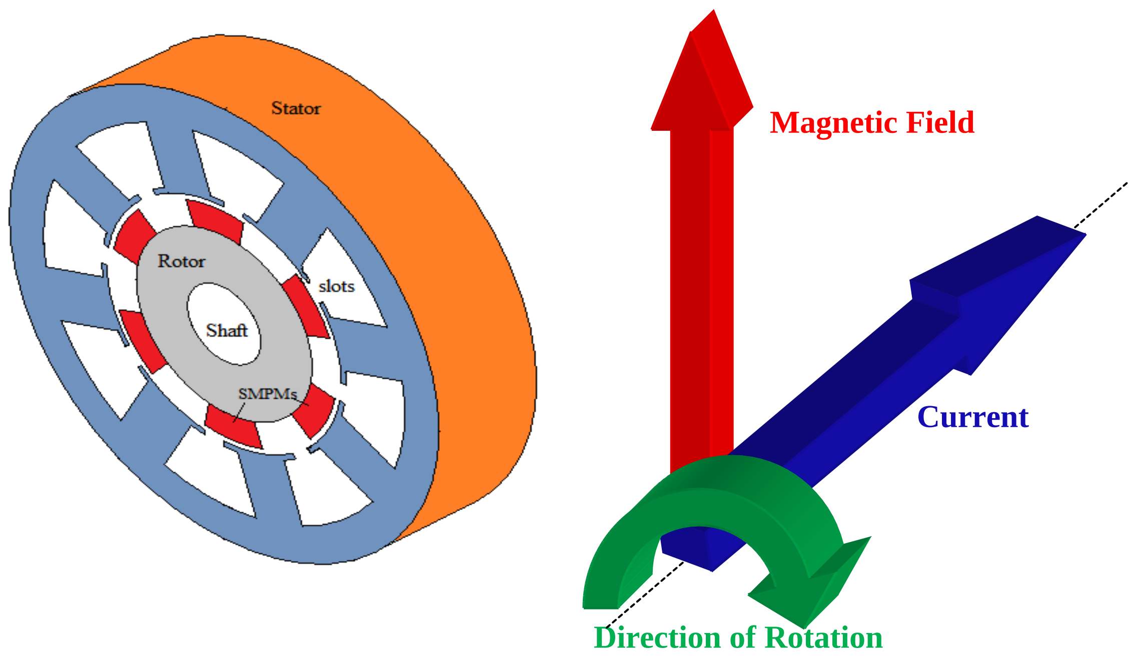


Fig 1.5 Rotor Topologies (a) SMPM (b) SIPM (c) IPM without flux barriers (d) IPM with flux barriers

1.2.3 Axial Flux SMPM Machines

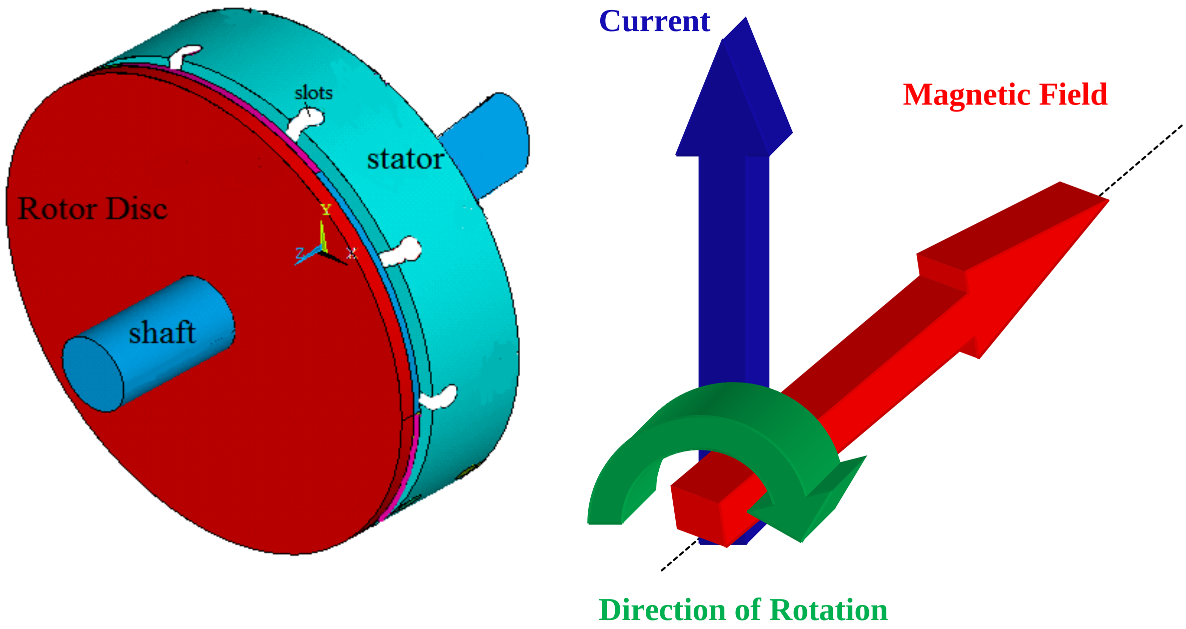
AF machines have disc shaped construction and planar air gaps. Flux crosses the air gap in the axial direction and passes through the cores in the circumferential direction. AFPM machines are prominent solutions for applications with high torque to inertia ratio, high efficiency, compactness, less maintenance like electric traction, aircrafts, electric vehicles, wind generators and in small- to medium-scale power generators [Gieras, 2004]. These machines have better utilization of active material, thus offering higher power density as compared to their radial flux counterpart. The compact design of the AF machine allows the operation with small current density and therefore, the power losses are less as compared to conventional induction motors. This increases the torque and power density of the AFPM machine for low speed high torque applications [Aydin, 2007]. They are particularly appropriate for application in electrical drives devoted to ship propulsion, since they eliminate the requirement of the large-power gearbox used in conventional systems. Conventional machines, either AC excited or brushed DC, are not well suited for application as direct-drive wheel motors due to their poor torque density and low overload capability as compared to PMBL machines. Therefore, AFPM machines designed for low-speed high-torque operation are one of the best options to serve the requirements posed by wheel direct drive.

AFPMs have some distinct advantages over RFPMs. First, they can be designed to have a higher power-to-weight ratio resulting in less core material and higher efficiency. Second, they are smaller in size than their radial flux counterparts and have disc shaped rotor and stator structures. This is an important feature of AF machines because suitable shape and size match the space limitation needed for space constraint applications. Third, they have planar and adjustable air gap [Mignot *et al.*, 2012].



(a) (b)

Fig. 1.6. Radial Flux PM Machine (a) Construction (b) Direction of flux



(a) (b)

Fig. 1.7 Axial Flux SMPM Machine (a) Construction (b) Direction of flux

AF machines have disc type construction producing higher torque-per-unit volume and also torque-per-unit weight as that of RFPM machines. The possibility to obtain a large axial length makes these machines very attractive for elevators in which the axial length of the machine is a limiting design factor. AF machines have usually been used in integrated high-torque applications. A single stator single rotor AFPM machine has been shown in Fig. 1.7.

With the increasing applications of RFPM and AFPM machine, the accurate prediction of electromagnetic performance, such as back electromotive force (EMF) waveform and torque capability, as well as for the parasitic effects in the PM machines including cogging torque, stator and rotor iron losses, magnet eddy current loss, acoustic noise, and vibration, etc., a precise modeling technique for these machines is required. The next section gives an overview of various modeling techniques present for the analysis of PM machines. The merits and demerits of these methods have been discussed.

1.3 Modeling of PM Machines

Mathematical modeling of electromagnetic devices is essential for computation of electromagnetic behavior and performance parameters of the machine. The magnetic field distribution in PMBL motors originates from two sources: the PMs and the armature winding current. The prediction of magnetic field distribution from these two sources within PM machine is a prerequisite for the computation of performance characteristics like force, torque-speed characteristics, back-EMF calculation and cogging torque. The methods reported for the analysis of PMBL motors can be mainly classified as Analytical methods, Numerical methods and Lumped parameter methods involving generalized machine theory as shown in Fig. 1.8.

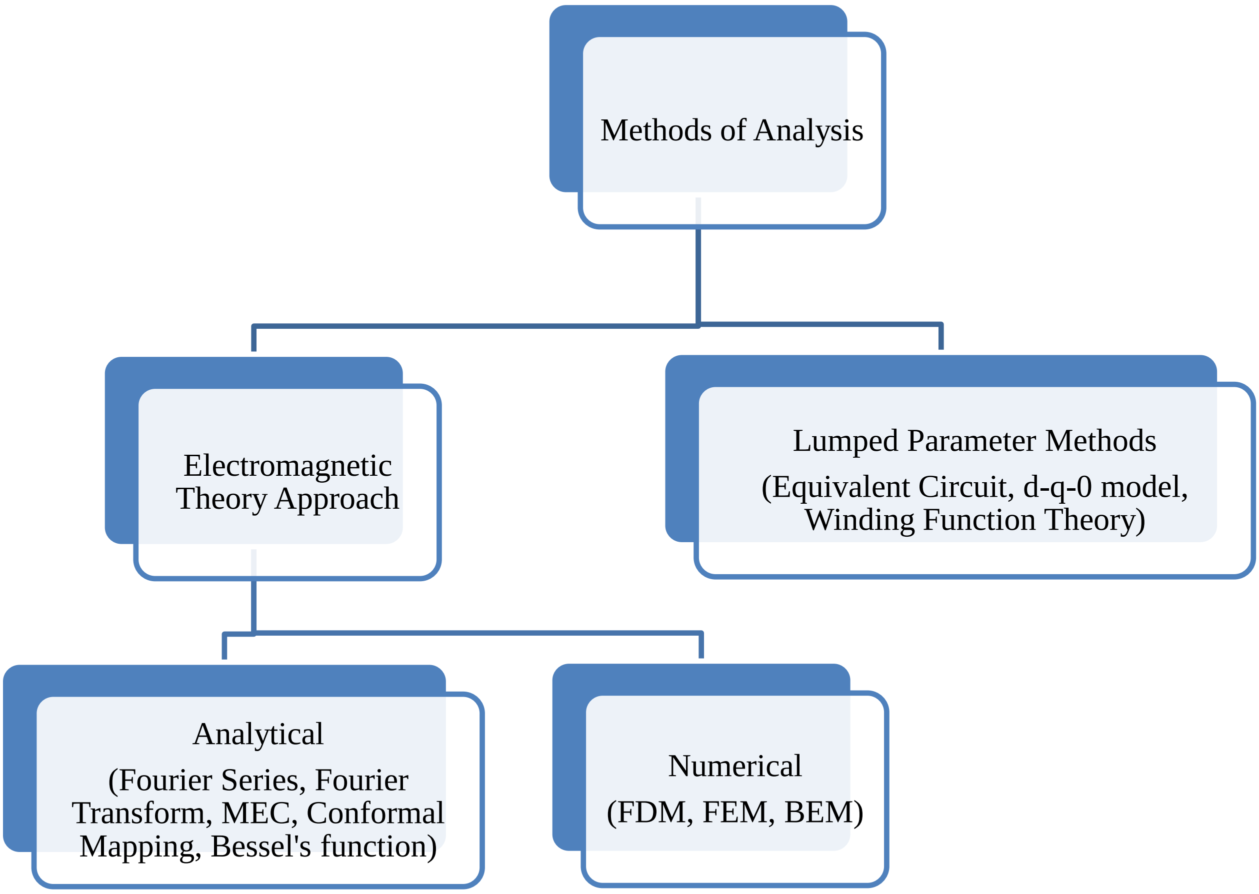


Fig. 1.8 Methods of Analysis for PM Machines

Using numerical FEM models and computing facilities, the motor design may be optimized by a process of repeated simulations, thus avoiding expensive experiments until a final prototype design is ready. The simulation process is inconsistent yielding more accurate results which require long computing times, large memory space and costly software making it difficult for the designer to sustain the creative process in his mind. Most of the reported methods like Magnetic Equivalent Circuit (MEC), Finite Element Method (FEM) and Boundary Element Method (BEM) require discretization of geometry .i.e., meshing before calculation of the electromagnetic field distribution and thus, the solutions only at the particular points can be obtained [Rasmussen *et al.*, 1997]. An increased mesh density though improves the accuracy however increases the computational time. In ironless structures, without concentrated magnetic fields, or machines with a small air gap and a large outer size, these methods become even more challenging due to the necessity of a high mesh density for accurate computation of magnetic fields and forces in the machine.

Analytical methods are important tool for design analysis and optimization of electrical machines since they do not require field solutions at a predetermined point. These models are useful for initial design and optimization of the machines while the numerical methods are good for the validation and adjustment in the design. Along with the computation of electromagnetic fields, the parameters like back-EMF, inductance, cogging torque, etc. needs to be calculated for PM machine. Cogging torque is an important phenomenon which occurs in permanent-magnet motors. It is caused by the affinity of the PMs in the rotor to line up with the stator in a particular direction, where the reluctance is lowest. Combined with the ripple torque caused by the harmonics in the current waveforms, it produces torque pulsations which are highly undesirable in applications like servo drives or electric steering. The next section summarizes the work reported in the area of analytical modeling of the PM motors for computation of magnetic field and performance parameters.

1.4 Literature Review

N. Boules introduced an analytical model for the analysis of a cylindrical SMPM machine for predicting the magnetic field distribution at no-load. The armature slotting and saturation effects are included by modifying the air-gap length using Carter's coefficient and the saturation factor respectively. The effect of air gap length, pole numbers and magnetization of PMs on air gap flux density on the flux density and instantaneous torque has been computed. The effect of curvature of PMs on the magnetic field distribution is also included [Boules, 1984, 1985]. A series of paper has been published on the complete analytical determination of magnetic field including open circuit analysis, effect of armature reaction, effect of stator slotting, on-load magnetic field distribution and the effect of magnetization pattern on the air gap magnetic field of radial flux PMDC machines. The technique proposed uses superposition of two fields produced by PM and stator currents [Zhu *et al.*, 1993a, 1993b, 1993c, 1993d] for calculation of total magnetic field. The stator slotting effect is included using relative permeance model and the modifying effective air gap using Carter's coefficient. However, the performance characteristics like torque-speed and current–speed has not been reported in this approach.

Two-dimensional (2-D) approach is an easy way to compute the parameters of the PM machine. A number of 2-D models have been presented for calculating armature reaction field and stator inductances [Atallah *et al*., 1998], radial and tangential components of the magnetic field and optimization of different design parameters [Wang *et al.,* 2003]. Kim and Lieu introduced an analytical technique for determination of the instantaneous magnetic field distribution in the air gap region of PM motors as a function of rotor eccentricity with and without slotting effect [Ungtae *et al.* 1998]. This work has been extended further by Proca in 2003 for the calculation of cogging torque, back-EMF and electromagnetic torque are performed [Proca *et al.*, 2003].

Numerical methods have always been in vogue for the electromagnetic field analysis of PM machines. FEM is a powerful numerical tool for this purpose because of their high accuracy and capability of non-linear computation. However, they are relatively slow and time-consuming and the results obtained from numerical methods are sensitive to the FE mesh density. Both two-dimensional and three dimensional FEM are in trend for the analysis of PM machines. The two dimensional analysis are mostly reported for the analysis of RFPM machines [Ionel *et al.*, 2011; Zaki *et al.*, 2005], AFPM machines [Novinschi *et al.*, 2004; Gair *et al.*, 1996; Caricchi *et al.*, 1998], for rotor losses estimation [Nerg *et al.*, 2002; Xuan *et al.*, 2010; Klötzl *et al.*, 2010] and in combination with the MEC for the analysis of flux focusing PM machines [Nedjar, 2012]. The 3-D FEM is used these days for the magnetic field computation of PM machines [Demerdash *et al.*, 1996], generally for AFPM machines [Aydin *et al.*, 2003; Parviainen, *et al.*, 2004] since they have inherent 3-D structure. 3-D FEM analysis of AFPM machines is highly time-consuming.

The MEC method is a lumped parameter analytical method similar to FEM with two differences. The number of elements deployed for the MEC method is much less than FEM which reduces its accuracy of the method. The direction of flux in each element of MEC model must be decided before the method is applied, in contrast to FEM where it is a result. MEC is a lumped parameter electrical network where resistive components represent reluctances whose values depend on the geometry, and for ferromagnetic materials, the flux density in the region. The modeling of SMPM through

MEC has been introduced for the calculation of magnetic field distribution, losses, back-EMF and cogging torque [Rasmussen *et al.,* 2003]. Later the method has also been used for the magnetic field density and losses in linear PMSM motor [Ghalavand *et al.,* 2010], AFPM motor [Abbaszadeh *et al.*, 2013; Kano *et al.*, 2007], IPM [Tariq *et al.*, 2010] and flux focusing PM motor [Nedjar, 2012].

Conformal mapping is an efficient method for geometries with elongated regions like a series of slots in the air gap of motor or for the slot geometry which is different from the slots having parallel slot openings. Using this technique, the effect of curvature of the rotor can be considered by transforming the cylindrical rotor into a square region so that the effective air gap length determined by flux path in PM can be calculated. This method solves the magnetic field in RFPM motor with slotted stator and takes care of the effect of curvature. Wang derived a relative air gap permenace function for including the effect of stator slotting using Schwarz-Christoffel (SC) Transformation for calculation of no-load field, armature reaction and back-EMF [Wang *et al.*, 2003]. SC transformation has been in use for the analysis of SMPM motors [Boughrara *et al.*, 2010; Zarko *et al.*, 2009], SIPM motors [Jian *et al.*, 2009], IPM motors [Fang *et al.*, 2006], AFPM motors [Alipour *et al.*, 2013]. SC Transformation has been used along with the quasi 3-D modeling of the AFPM motor to reduce the time consumed in the analysis of AFPM machine.

Though different types of methods for air gap field distribution of PM machines are present in the literature, still the research is more oriented towards the solution of analytical methods using Maxwell's equations. A complete review on the analytical methods for the slotless PM machines is presented in [Pfister *et al.*, 2011]. These methods are mostly used for the field analysis of SMPM, SIPM, AFPM motors and for the determination of cogging torque. The literature survey on the analytical methods for

PM motors has been done and some of the important contributions have been summarized here.

An analytical sub-domain model has been presented to calculate the magnetic field in SMPM machine having semi-closed slots and radially magnetized PMs. The proposed model has equally been useful for PM machines with different pole and slot number combinations including fractional-slot machines having distributed or concentrated windings. The governing field equations are solved separately for each sub-domain .i.e., air gap, magnets, slot-openings and slots [Lubin *et al.*, 2011]. Rahideh wrote a series of two papers concerning analytical magnetic field distribution in slotless PM AC or DC machines with any number of phases. The first part explicate the calculation of armature reaction field and losses in the PM motor while the second part extends the same analytical model for the computation of open circuit field and torque calculations. The calculations are accomplished for six different magnetization patterns and for three different current waveforms. Back-EMF calculated from the proposed method for all the magnetization patterns has also been reported [Rahideh *et al.*, 2012a, 2012b].

A sub-domain model for the calculation of no-load magnetic field has been discussed by Zhu for fractional-slot machines with the effect of stator slotting in RFPM machines. The solution is derived using governing field equations in the regular sub-domain model and using boundary conditions. The relationship between this exact subdomain model and the conventional subdomain model, which is based on a simplified one slot/pole machine model has also been discussed. The back-EMF, electromagnetic torque, cogging torque and unbalanced magnetic force has also been obtained based on the field model. The study shows that both the exact and conventional sub-domain model has similar accuracy for machine having small slot opening width. While the exact subdomain model shows comparable accuracy for back-EMF and electromagnetic torque, but higher accuracy for cogging torque with machines having relatively large slot opening width. [Zhu *et al.,* 2010; Wu *et al.,* 2011].

An analytical model for air gap field distribution of PM machines with Halbach magnetized PMs in polar co-ordinates has been proposed by Xia. The developed model is appropriate for external and internal rotor PM machines having either iron or air core. The effect of number of poles on flux density distribution with optimum combination of poles and magnet thickness for maximum air gap flux density is investigated. The developed model takes significantly less computing time than FEM [Xia *et al.,* 2004]. Many analytical methods for SMPM motors taking into account different parameters like magnetization, inner and outer rotor topology, effect of slotting, PM width, etc. has been reported [Bianchi *et al.,* 2005; Boroujeni *et al.,* 2015; Tutelea *et al.,* 2012; Deng *et al.,* 1987; Tessarolo *et al.,* 2013].

A number of analytical methods has been proposed for the magnetic field contributions of SIPM and AFPM machines. Some of the important contributions are highlighted. A sub-domain model for the air gap field distribution of surface-inset motors with semi-closed slots has been suggested. Two dimensional Laplace's and Poisson's equations are used to solve the magnetic field distribution of the machine and computation of back-EMF and electromagnetic torque at no-load. The method is reported to have less computation time than FEM [Lubin *et al.,* 2012]. An analytical magnetic field calculation for slotless brushless PM machines equipped with surface inset magnets is proposed to calculate both open-circuit and armature reaction field distributions. The open-circuit magnetic field is calculated for three different magnetization patterns: Radial, Parallel and Halbach. For magnetic field distribution during load conditions overlapping winding, non-overlapping with all teeth wound and non-overlapping winding with alternate teeth wound are considered [Rahideh *et al.,* 2012c, 2013]. The method is further extended and reported for the slotted PM machine [Rahideh *et al.,* 2012d]. A series-slot analytical model is proposed to describe the magnetic field behavior by a set of partial differential equations in terms of scalar magnetic potentials. The approach included the determination of cogging torque using Maxwell stress tensor [Jian *et al.,* 2009]. The analytical models has equally been useful for calculating the eddy current losses in PMs, rotor iron, retaining sleeve for a slotless PMSM with surface inset PMs on rotor. A 2-D subdomain method for the solution of Maxwell's equation has been used for the calculation of air gap magnetic field in SIPM motors. The eddy currents derived from the air gap field is then used to calculate the eddy current reaction field. The current distribution in the stator is modeled using equivalent current sheet at slot openings. The results obtained are validated using the FEA [Dubas *et al.,* 2014].

Kumar and Bauer introduced an improved analytical model for the calculation of instantaneous air gap field distribution which can be further used to determine the cogging torque, back EMF and losses in the motor. The proposed model is valid for radial, parallel, sinusoidal magnetization. The model takes into account that the stator yoke has finite permeability and finite thickness. The major advantage of the analytical method is that it is fast as compared to FEM and can be used for initial multi-objective optimization of the BLDC motor [Kumar *et al.,* 2008]. A 3-D analytical model for the analysis of an air-cored axial flux machine using magnetic scalar potential and modified Bessel's function is proposed in cylindrical co-ordinate system. The cylindrical system allows the decomposition of residual magnetization of PMs in radial, circumferential and axial directions [Jin *et al.,* 2014].

The analytical magnetic field calculation for slotted brushless permanent-magnet (PM) machines equipped with surface inset magnets is proposed to calculate both open-circuit and armature reaction field distributions. The open-circuit magnetic field is calculated for three different magnetization patterns: Radial, Parallel and Halbach. For magnetic field distribution during load conditions overlapping winding, non-overlapping with all teeth wound and non-overlapping winding with alternate teeth wound are considered. The method can be used for the surface inset magnet but cannot be for surface mounted magnets. Slotting and slot-opening effects are also included using the subdomain technique [Rahideh *et al.,* 2012a]. The other paper discussed the analytical armature reaction magnetic field calculation of brushless machines equipped with surface-inset magnets is proposed using the subdomain technique. The rotor/stator back-irons are assumed to have infinite permeability and therefore the analytical solutions are obtained for the winding, air-gap, and magnets regions [Rahideh *et al.,* 2012b].

A combination of FEA and analytical method using Fourier series has been presented for the modeling of AFPM motor. The main feature of the proposed method is the saving of computation time in comparison with the 3-D FEM method. The computational time of this method is also lesser than the analytical methods present in literature [Zhilichev *et al.,* 1998]. MEC method has been reported for the design of AFPM motor with single rotor double stator configurations. The method is used to study electromagnetic behavior as well as thermal design of the AFPM machines [Parviainen *et al.,* 2004]. AFPM machines possess an intrinsic 3-D electromagnetic nature. With this in mind, a quasi-3-D analytical model which consumes very less time is developed. The model gives sufficient accuracy as compared to the three-dimensional FEM [Tiegna *et al.,* 2014]. The AFPM machine divided into several annular slices (submachines) so that in each slice magnet-width to pole-pitch ratio and slot-width to pole-pitch ratio can be considered constant. For each of the slices a linear machine and 2-D model can be used. The performance of AFPM machine can be obtained after analyzing individual 2-D models. Different approaches have been developed for analysis of 2-D models in quasi-3-D method. After conversion to 2-D models, various analytical methods can be used for performance calculation. The important analytical methods reported for the analysis of AFPM machines [Alipour *et al.*, 2013; Choi *et al.*, 2011; Jin *et al.*, 2014; Mendrela *et al.*, 2004; Egea *et al.*, 2010; Mardaneh *et al.*, 2006; Zhilichev *et al.*, 1998; Tiegna *et al.*, 2014; Huang 2012; Virtic 2009; Kurronen 2007; Virtic 2008; Tiegna 2012; Sung 2012; Egea 2012; Furlani 1994; Hemeida *et al.*, 2014]. Apart from the magnetic field distribution, the analytical methods are also used for cogging torque computation.

For computation of cogging torque, mainly three methods are reported; First is to calculate the torque as a derivative of co-energy inside the air gap. The second method includes the integration of the lateral forces along the slot sides and the third method is to integrate the tangential component of Maxwell stress tensor along a circular contour inside the air gap. Ree and Boules in 1989 introduced an analytical method for the calculation of cogging torque in PM machines. The cogging torque was calculated as rate of change of total energy in the air gap w.r.t. the rotor position [Ree *et al.*, 1989]. Later Gieras also proposed a method based on the rate of change of co-energy with the classical equations to calculate cogging torque [Gieras *et al.*, 2004]. A method to calculate the cogging torque using conformal mapping by calculating both the radial and tangential components of the flux density based on the integral of Maxwell Stress Tensor has been presented [Zarko *et al.*, 2008]. Zhu and Howe proposed an analytical method in cylindrical co-ordinate system for the analysis of cogging torque based on the calculation of the airgap field distribution and the resulting lateral forces which act on the stator teeth. The method can be used for both internal and external rotor PM machines and offers high accuracy. The effect of design parameters on cogging torque can also be observed [Zhu *et al.*, 1992]. An effective way of predicting cogging torque has been described in [Liu *et al.*, 2005]. A pole-slot transition model is presented and the method is based on the analysis of the torque due to transition of magnet of alternate polarity over the slot openings. This method can be applied for cogging torque analysis for a PM motor with a pole and slot number combination that is relatively difficult to be analyzed using FEM. The approach is easy to implement and can achieve good accuracy with much reduced computation time budget and uses superposition/synthesis principle to calculate cogging torque. The basic principle is that the resultant cogging torque can be obtained through superposition or synthesis of the cogging torque generated by a single slot or by a single pole transition model. These methods can significantly reduce the calculating work.

A state of art of numerical and analytical methods is presented for the analysis of cogging torque of PM machines. Cogging torque calculated from the flux density expression including the effect of stator slotting by permeance function. The virtual work method and Maxwell stress Tensor method is elaborated and their advantages and drawbacks has also been summarized [Guo *et al.*, 2009].

1.5 Motivation for Research and Problem Statement

Though most of the analytical and numerical methods proved effective for computing electromagnetic field distribution, however the performance calculation of the PM motors with less computation time is still a challenging task. Further, ease of computation and computational time is the main factor affecting prediction of electromagnetic fields and forces. This requires an analytical method for determination of the magnetic field as well as performance characteristics simultaneously in early design stages.

The commonly used FEA provides accurate electromagnetic field distribution considering geometrical details and nonlinear effects of magnetic material. However, computation time required for geometrical modeling and computation of electromagnetic fields and performance parameter is extremely large. Since AFPM machines possess inherent three-dimensional electromagnetic structure, the 3-D FEM can give accurate results however, are very time consuming. This time required has to be remarkably improved. Also numerical approaches like FEM and BEM do not allow one to perform numerous parametric studies quickly. For this reason, the analytical method needs to be introduced in the early design stages for design, optimization, electromagnetic field analysis and performance calculation. Both the methods provide precise results, but the analytical method using a magnetic vector potential is much faster than the FEM analysis. Although analytical models are fast, and provide a good physical insight for designers, the research on the development of analytical models for analysis of PM machines over FEM has increased over years.

The analytical methods based on the solution of Maxwell's equations uses Fourier series and Fourier Transform since the electric machines generally have a periodic characteristic on geometry. The MMF in winding, air-gap permeance and slot effect can be easily represented by the Fourier series that can be expressed as a periodic function. However, the results obtained from the Fourier series are dependent on speed and the computation time increases at high speeds. The application of Fourier Transform for the determining magnetic fields in PM machines has not been reported in the literature.

To overcome the difficulties of these methods, a new analytical method is developed and proposed for the design analysis of RFPM and AFPM SMPM motors using Fourier Transform as analytical tool for the solution of Maxwell's equations. The basic idea of the proposed method is to simplify the analysis of the performance characteristics of the PMBL motor and in the meantime obtaining the field distribution of the machine with a minimum computational time and better accuracy. The Fourier transform applicable to LIM and introduced by S. Yamamura [Yamamura, 1972] is modified and adapted for analyzing PMBL motor.

1.6 Scope of Present Work

The salient contributions of the thesis have been summarized as:

i. Analysis of Surface Mounted Permanent Magnet motors using a two-dimensional analytical and space Fourier Transform.

ii. Representation of PMs as a continuous current sheet producing same MMF as that of the PMs in real space.

iii. Performance Analysis of the SMPM motor with almost negligible computation time.

iv. The proposed method is able to compute the electromagnetic field distribution as well as performance characteristics simultaneously.

v. The two-dimensional proposed method for SMPM motors is applicable to both RFPM and AFPM motors.

vi. The cogging torque can also be determined easily with almost negligible computational time.

1.7 Conclusions

The chapter has discussed the construction of RFPM and AFPM machines. A detailed literature survey on the analytical methods has been presented and studied for the magnetic field computation and performance calculation of the PM machines. A review on the prediction of cogging torque using analytical methods has also been summarized. Based on the literature review on the present research topic, the motivations and objectives of this thesis have been decided and reported. The objectives are realized and presented systematically in steps of Chapters and discussed briefly in the outline of the thesis.

The different methods of analysis of SMPM machines are discussed in detail in the next chapter and based on the drawbacks and outcome of other methods, the basis of the proposed analytical method has been pointed out and elaborated for the analysis of slotless and slotted stator in the SMPM motor.