Chapter 5

# Intuitionistic fuzzy rough set model based on $(\alpha, \beta)$ -indiscernibility and its application to feature selection

## 5.1 Introduction

Feature selection or attribute reduction technique addresses the dimensionality reduction problem by governing a subset of original features to construct a good model for classification or prediction task. The classical rough set model, introduced by Pawlak [67, 68, 69], has been effectively applied as a feature selection or attributes reduction and rule learning tool. In the rough set model, a crisp equivalence relation as well as crisp equivalence classes is applied to define the dependency function between decision and conditional attributes available in the information system. The dependency function is effectively used to establish the relevance between the decision and conditional features and to assess the classification potential of the features [77]. However, the classical rough set model could not apply directly on the real-valued datasets due to its limited requirement of nominal data. Therefore, many generalizations of rough set model have been presented to avoid the information loss. Fuzzy rough set is one of the most efficient extensions of rough set, which can be directly applied to the real-valued datasets without any modification in the information system. Predominately, fuzzy rough set can efficiently tackle both fuzziness and vagueness available in the datasets with continuous features. By combining rough and fuzzy sets [67, 100] as presented by Dubois and Prade [20, 21] allows the notion of fuzzy rough sets, which gives a powerful means of dealing with the problem of discretization and can be effectively implemented to the reduction of continuous attributes. Fuzzy rough sets are mostly implemented to direct the inconsistency between conditional attributes and decision attributes [42], i.e. a few samples have similar or having the same conditional attribute values but distinct labels. With lower approximations in fuzzy rough sets, each sample can be assigned to a membership in the form of a decision class to evaluate this inconsistency, and feature selection techniques based on fuzzy rough sets focus to obtain a reduct to retain the membership of every sample. In recent years, IF set theory has been effectively applied in the field of pattern recognition, decision analysis, medical image processing [11, 83, 101, 104], etc. In spite of the fact that rough sets and IF sets both capture specific aspects of the same idea-imprecision, the combination of IF set theory and rough set theory are rarely discussed by the researchers [26, 54, 55, 60]. In the current paper, first, we propose the concept of $(\alpha, \beta)$-indiscernibility of two objects in an IF information system. Second, we define a novel IF tolerance relation using this $(\alpha, \beta)$-indiscernibility concept. Third, we establish a $(\alpha, \beta)$-indiscernibility based IF rough set model, which is grounded on the substitution of the indiscernibility relation in traditional rough set theory with our proposed IF tolerance relation. Fourth, a positive region based feature selection technique is developed by using our proposed model. Moreover, an algorithm based on our proposed approach is given to calculate the reduct of an IF decision system. Finally, our approach is applied on example dataset and the reduct is calculated.

## 5.2 Intuitionistic Fuzzy Rough set based approach for feature selection

In the literature [36, 37, 39, 40, 41, 42, 43, 44], different feature selection approaches have been developed by many researchers but they only considered fuzzy tolerance relation, characterized by its membership function. However, hesitancy is available in almost every information system, so it is required to consider non-membership grades of objects. All the above proposed techniques do not consider non-membership grades, which is a limitation of these approaches.

In this paper, we define a new kind of lower and upper approximations by considering both membership and non-membership grades as follows:

**Definition 5.2.1** *For each pair of objects $x, y \in U$, we can define an $IF$ relation $R$ in $U \times U$ by*

$$\begin{aligned} \mu_R(x,y) &= \frac{1}{n}\sum_{i=1}^{n} \mu_{\hat{R}}(x_i,y_i) \\ \nu_R(y,x) &= \frac{1}{n}\sum_{i=1}^{n} \nu_{\hat{R}}(x_i,y_i) \end{aligned} \tag{5.1}$$

*where,* $\mu_{\hat{R}}(x_i,y_i) = 1 - |\mu(x_i) - \mu(y_i)|$ *and* $\nu_{\hat{R}}(x_i,y_i) = |\nu(x_i) - \nu(y_i)|$

**Lemma 5.2.1** *For $x, y \in U$, $\mu_R(x,y) = 1$ and $\nu_R(x,y) = 0$ if and only if $x = y$*

**Proof:** If $x = y$, then $\mu_R(x,y) = \mu_R(x,x) = \frac{1}{n}\sum_{i=1}^{n} \mu_{\hat{R}}(x_i,x_i)$
$= \frac{1}{n}\sum_{i=1}^{n}(1 - |\mu(x_i) - \mu(x_i)|) = 1$ (Since, $\sum_{i=1}^{n} 1 = n$)
and $\nu_R(x,y) = \nu_R(x,x) = \frac{1}{n}\sum_{i=1}^{n} \nu_{\hat{R}}(x_i,x_i)$
$= \frac{1}{n}\sum_{i=1}^{n}(|\nu(x_i) - \nu(x_i)|) = 0$
Conversely, if $\mu_R(x,y) = 1$, then $\frac{1}{n}\sum_{i=1}^{n} \mu_{\hat{R}}(x_i,y_i) = 1$,
which implies that

$$\sum_{i=1}^{n}(|\mu(x_i) - \mu(y_i)|) = n \tag{5.2}$$

Since $\sum_{i=1}^{n} 1 = n$, hence Eq.(5.2) reduces to $n - \sum_{i=1}^{n}|\mu(x_i) - \mu(y_i)| = n$
which gives $\sum_{i=1}^{n}|\mu(x_i) - \mu(y_i)| = 0$.
After expansion, each term in left hand side is a non-negative quantity and hence $\mu(x_i) = \mu(y_i)$ for all $i$. If $\nu_R(x,y) = 0$, then $\frac{1}{n}\sum_{i=1}^{n}(|\nu(x_i) - \nu(y_i)|) = 0$
which implies that $\sum_{i=1}^{n}|\nu(x_i) - \nu(y_i)| = 0$.
Again each term in left hand side is a non-negative quantity and hence
$\nu(x_i) = \nu(y_i)$ for all $i$.
From above, we get $x = y$. Therefore, $IF$ relation as defined in Eq.(5.1) is an $IF$ tolerance relation.

Let R be an $IF$ relation in $U$ characterized by $\mu_R : U \times U \to [0,1]$ and $\nu_R : U \times U \to [0,1]$, then for given $\alpha, \beta \in (0,1]$, objects x and y in U will be $(\alpha,\beta)$-indiscernible if $\mu_R(x,y) \geq \alpha$ and $\nu_R(x,y) \leq \beta$.

Now, lower and upper approximations can be defined as follows:

$$\underline{R}_{\alpha,\beta}(X) = \{x \in U | \mu_R(x,y) < \alpha \text{ and } \nu_R(x,y) > \beta, \forall y \in X^C\}$$
$$\overline{R}_{\alpha,\beta}(X) = \{x \in U | \mu_R(x,y) \geq \alpha \text{ and } \nu_R(x,y) \leq \beta, \text{ for some } y \in X\}$$

where, $\alpha, \beta \in (0,1]$ refers the level up to which the compatibility among the objects is to be considered so that lower approximation includes only those objects which are not related up to a degree $\alpha$ and related up to a degree $\beta$ to any object outside $X$.

Let $U$ be an universe of discourse, $R$ be an $IF$ relation in $U$ and $X \subseteq U$ then following holds:

**Theorem 5.2.2** *Let $\alpha_1, \alpha_2, \beta_1, \beta_2 \in (0,1]$ be such that $\alpha_1 \leq \alpha_2$ and $\beta_1 \geq \beta_2$, then $\underline{R}_{\alpha_1,\beta_1}(X) \subseteq \underline{R}_{\alpha_2,\beta_2}(X)$ and $\overline{R}_{\alpha_2,\beta_2}(X) \subseteq \overline{R}_{\alpha_1,\beta_1}(X)$*

**Proof:** For any $x \in \underline{R}_{\alpha_1,\beta_1}(X)$, $\mu_R(x,y) < \alpha_1$ and $\nu_R(x,y) > \beta_1$ for all $y \in X^C$. Since $\alpha_1 \leq \alpha_2$ and $\beta_1 \geq \beta_2$, we have $\mu_R(x,y) < \alpha_2$ and $\nu_R(x,y) > \beta_2$ for all $y \in X^C$ and hence $x \in \underline{R}_{\alpha_2,\beta_2}(X) \Rightarrow \underline{R}_{\alpha_1,\beta_1}(X) \subseteq \underline{R}_{\alpha_2,\beta_2}(X)$.

Similarly, if $x \in \overline{R}_{\alpha_2,\beta_2}(X)$, then $\mu_R(x,y) \geq \alpha_2$ and $\nu_R(x,y) \leq \beta_2$, for some $y \in X$. Since $\alpha_1 \leq \alpha_2$ and $\beta_1 \geq \beta_2$, we have $\mu_R(x,y) \geq \alpha_1$ and $\nu_R(x,y) \leq \beta_1$ for some $y \in X$ and hence $x \in \overline{R}_{\alpha_1,\beta_1}(X) \Rightarrow \overline{R}_{\alpha_2,\beta_2}(X) \subseteq \overline{R}_{\alpha_1,\beta_1}(X)$.

**Theorem 5.2.3** $\underline{R}_{\alpha,\beta}(X) \subseteq X \subseteq \overline{R}_{\alpha,\beta}(X)$

**Proof:** For any $x \in \underline{R}_{\alpha,\beta}(X)$, we have $\mu_R(x,y) < \alpha$ and $\nu_R(x,y) > \beta, \forall y \in X^C$.

Since $\mu_R(x,x) = 1 \geq \alpha$ and $\nu_R(x,x) = 0 \leq \beta$, hence $x \notin X^C$. This gives $x \in X$ and hence $\underline{R}_{\alpha,\beta}(X) \subseteq X$

Now, if $x \in X$, then $\mu_R(x,x) = 1 \geq \alpha$ and $\nu_R(x,x) = 0 \leq \beta$ for $x \in X$, which implies that $x \in \overline{R}_{\alpha,\beta}(X)$ and hence $X \subseteq \overline{R}_{\alpha,\beta}(X)$.

**Theorem 5.2.4** $\underline{\phi}_{\alpha,\beta}(X) = \phi = \overline{R}_{\alpha,\beta}(\phi)$.

**Proof:** $\underline{R_{\alpha,\beta}}(\phi) = \{x \in U | \mu_R(x,y) < \alpha \text{ and } \nu_R(x,y) > \beta, \forall y \in U\} = \phi$

$\overline{R}_{\alpha,\beta}(\phi) = \{x \in U | \mu_R(x,y) \geq \alpha \text{ and } \nu_R(x,y) \leq \beta, \text{ for some } y \in \phi\} = \phi$

**Theorem 5.2.5** $\underline{R}_{\alpha,\beta}(U) = U = \overline{R}_{\alpha,\beta}(U)$.

**Proof:** From theorem 5.2.3, we have $\underline{R}_{\alpha,\beta}(U) \subseteq U$ and $U \subseteq \overline{R}_{\alpha,\beta}(U)$.

Remain to show that $U \subseteq \underline{R}_{\alpha,\beta}(U)$ and $\overline{R}_{\alpha,\beta}(U) \subseteq U$

Let $x \in U$, then $\mu_R(x,y) < \alpha$ and $\nu_R(x,y) > \beta, \forall y \in U^C (= \phi)$

This implies that $x \in \underline{R}_{\alpha,\beta}(U)$, hence $U \subseteq \underline{R}_{\alpha,\beta}(U)$

Now, $\overline{R}_{\alpha,\beta}(U) \subseteq U$ is obvious, hence $\overline{R}_{\alpha,\beta}(U) \subseteq U$.

**Theorem 5.2.6** $\underline{R}_{\alpha,\beta}(X^C) = (\overline{R}_{\alpha,\beta}(X))^C$, *where* $X^C$ *is complement of set* $X$.

**Proof:** Let $x \in \underline{R}_{\alpha,\beta}(X^C) \iff \mu_R(x,y) < \alpha$ and $\nu_R(x,y) > \beta, \forall y \in (X^C)^C (= X) \iff \mu_R(x,y) < \alpha$ and $\nu_R(x,y) > \beta, \forall y \in X \iff x \notin \overline{R}_{\alpha,\beta}(X) \iff x \in (\overline{R}_{\alpha,\beta}(X))^C \Rightarrow \underline{R}_{\alpha,\beta}(X^C) = (\overline{R}_{\alpha,\beta}(X))^C$.

**Theorem 5.2.7** $\overline{R}_{\alpha,\beta}(X^C) = (\underline{R}_{\alpha,\beta}(X))^C$

**Proof:** Replace $X$ by $X^C$ in theorem 5.2.6 and take complement on both sides.

**Theorem 5.2.8** *For* $Y \subseteq U$, $X \subseteq Y \Rightarrow \underline{R}_{\alpha,\beta}(X) \subseteq \underline{R}_{\alpha,\beta}(Y)$ *and* $\overline{R}_{\alpha,\beta}(X) \subseteq \overline{R}_{\alpha,\beta}(Y)$

**Proof:** For any $x \in \underline{R}_{\alpha,\beta}(X), \mu_R(x,y) < \alpha$ and $\nu_R(x,y) > \beta, \forall y \in X^C$

Since $Y^C \subseteq X^C$, hence $\mu_R(x,y) < \alpha$ and $\nu_R(x,y) > \beta, \forall y \in Y^C$

$\Rightarrow x \in \underline{R}_{\alpha,\beta}(Y) \Rightarrow \underline{R}_{\alpha,\beta}(X) \subseteq \underline{R}_{\alpha,\beta}(Y)$

If $x \in \overline{R}_{\alpha,\beta}(X)$, then $\mu_R(x,y) \geq \alpha$ and $\nu_R(x,y) \leq \beta$, for some $y \in X$. Since $X \subseteq Y$, which implies that $x \in \overline{R}_{\alpha,\beta}(Y)$

Hence, $\overline{R}_{\alpha,\beta}(X) \subseteq \overline{R}_{\alpha,\beta}(Y)$.

**Theorem 5.2.9** *Let* $R_1$ *and* $R_2$ *are two* $IF$ *relations such that*

$R_1 \subseteq R_2 \Rightarrow \underline{R_2}_{\alpha,\beta}(X) \subseteq \underline{R_1}_{\alpha,\beta}(X)$ *and* $\overline{R_1}_{\alpha,\beta}(X) \subseteq \overline{R_2}_{\alpha,\beta}(X)$.

**Proof:** Since, $R_1 \subseteq R_2$, hence, $\mu_{R_1}(x,y) \leq \mu_{R_2}(x,y)$ and $\nu_{R_1}(x,y) \geq \nu_{R_2}(x,y), \forall x,y \in U$

Now, let $x \in \underline{R_2}_{\alpha,\beta}(X) \iff \mu_{R_2}(x,y) < \alpha$ and $\nu_{R_2}(x,y) > \beta, \forall y \in X^C \iff x \in \underline{R_1}_{\alpha,\beta}(X) \iff R_1 \subseteq R_2 \Rightarrow \underline{R_2}_{\alpha,\beta}(X) \subseteq \underline{R_1}_{\alpha,\beta}(X)$.

Furthermore, $x \in \overline{R_1}_{\alpha,\beta}(X) \iff \mu_{R_1}(x,y) \geq \alpha$ and $\nu_{R_1}(x,y) \leq \beta$, for some $y \in X \iff \mu_{R_2}(x,y) \geq \alpha$ and $\nu_{R_2}(x,y) \leq \beta$, for some $y \in X \iff x \in \overline{R_2}_{\alpha,\beta}(X) \iff \overline{R_1}_{\alpha,\beta}(X) \subseteq \overline{R_2}_{\alpha,\beta}(X)$.

**Theorem 5.2.10** $\underline{R}_{\alpha,\beta}(X \cap Y) = \underline{R}_{\alpha,\beta}(X) \cap \underline{R}_{\alpha,\beta}(Y)$.

**Proof:** Since $X \cap Y \subseteq X$, hence $\underline{R}_{\alpha,\beta}(X \cap Y) \subseteq \underline{R}_{\alpha,\beta}(X)$ (using theorem 5.7)

Since $X \cap Y \subseteq Y$, hence $\underline{R}_{\alpha,\beta}(X \cap Y) \subseteq \underline{R}_{\alpha,\beta}(Y)$

which implies that $\underline{R}_{\alpha,\beta}(X \cap Y) \subseteq \underline{R}_{\alpha,\beta}(X) \cap \underline{R}_{\alpha,\beta}(Y)$

Now, we have to show that

$\underline{R}_{\alpha,\beta}(X) \cap \underline{R}_{\alpha,\beta}(Y) \subseteq \underline{R}_{\alpha,\beta}(X \cap Y)$

Let $x \in \underline{R}_{\alpha,\beta}(X) \cap \underline{R}_{\alpha,\beta}(Y) \Rightarrow x \in \underline{R}_{\alpha,\beta}(X)$ and $x \in \underline{R}_{\alpha,\beta}(Y)$

$\Rightarrow \mu_R(x,y) < \alpha$ and $\nu_R(x,y) > \beta, \forall y \in X^C$ and $\mu_R(x,z) < \alpha$ and $\nu_R(x,z) > \beta, \forall z \in Y^C$

$\Rightarrow \mu_R(x,u) < \alpha$ and $\nu_R(x,u) > \beta, \forall u \in X^C \cup Y^C$

$\Rightarrow \mu_R(x,u) < \alpha$ and $\nu_R(x,u) > \beta, \forall u \in (X \cap Y)^C$

$\Rightarrow x \in \underline{R}_{\alpha,\beta}(X \cap Y)^C$

Hence, $\underline{R}_{\alpha,\beta}(X) \cap \underline{R}_{\alpha,\beta}(Y) \subseteq \underline{R}_{\alpha,\beta}(X \cap Y)$

**Theorem 5.2.11** $\underline{R}_{\alpha,\beta}(X \cup Y) \supseteq \underline{R}_{\alpha,\beta}(X) \cup \underline{R}_{\alpha,\beta}(Y)$.

**Proof:** Since $X, Y \subseteq X \cup Y$, hence $\underline{R}_{\alpha,\beta}(X) \subseteq \underline{R}_{\alpha,\beta}(X \cup Y)$ (using theorem 5.7)

and $\underline{R}_{\alpha,\beta}(Y) \subseteq \underline{R}_{\alpha,\beta}(X \cup Y)$

This implies, $\underline{R}_{\alpha,\beta}(X) \cup \underline{R}_{\alpha,\beta}(Y) \subseteq \underline{R}_{\alpha,\beta}(X \cup Y)$

**Theorem 5.2.12** $\overline{R}_{\alpha,\beta}(X \cup Y) = \overline{R}_{\alpha,\beta}(X) \cup \overline{R}_{\alpha,\beta}(Y)$.

**Proof:** Since $X, Y \subseteq X \cup Y$, hence $\overline{R}_{\alpha,\beta}(X), \overline{R}_{\alpha,\beta}(Y) \subseteq \overline{R}_{\alpha,\beta}(X \cup Y)$

$\Rightarrow \overline{R}_{\alpha,\beta}(X) \cup \overline{R}_{\alpha,\beta}(Y) \subseteq \overline{R}_{\alpha,\beta}(X \cup Y)$

Now, $x \in \overline{R}_{\alpha,\beta}(X \cup Y) \Rightarrow \mu_R(x,y) \geq \alpha$ and $\nu_R(x,y) \leq \beta$, for some $y \in X \cup Y$

$\Rightarrow \mu_R(x,y) \geq \alpha$ and $\nu_R(x,y) \leq \beta$, for some $y \in X$ or $\mu_R(x,y) \geq \alpha$ and $\nu_R(x,y) \leq \beta$, for some $y \in Y \Rightarrow x \in \overline{R}_{\alpha,\beta}(X)$ or $x \in \overline{R}_{\alpha,\beta}(Y)$

$\Rightarrow x \in \overline{R}_{\alpha,\beta}(X) \cup \overline{R}_{\alpha,\beta}(Y)$

Hence, $\overline{R}_{\alpha,\beta}(X \cup Y) \subseteq \overline{R}_{\alpha,\beta}(X) \cup \overline{R}_{\alpha,\beta}(Y)$

Hence, we get the required result.

**Theorem 5.2.13** $\overline{R}_{\alpha,\beta}(X \cap Y) \subseteq \overline{R}_{\alpha,\beta}(X) \cap \overline{R}_{\alpha,\beta}(Y)$.

**Proof:** Since $X \cap Y \subseteq X, Y$, hence $\overline{R}_{\alpha,\beta}(X \cap Y) \subseteq \overline{R}_{\alpha,\beta}(X)$ and $\overline{R}_{\alpha,\beta}(X \cap Y) \subseteq \overline{R}_{\alpha,\beta}(Y)$
Therefore, $\overline{R}_{\alpha,\beta}(X \cap Y) \subseteq \overline{R}_{\alpha,\beta}(X) \cap \overline{R}_{\alpha,\beta}(Y)$.

**Definition 5.2.2** *Now positive region of $D$ can be defined as:*

$$POS_P(D) = \cup_{X \in U/D} \underline{R}_{\alpha,\beta}(X)$$

*where, $U/D$ is set of all decision classes.*

**Definition 5.2.3 (54)** *Let $(U, P \cup D)$ be a fuzzy decision system, where $U = \{U_1, U_2, ..., U_n\}$, $P$ is a set of conditional attributes and $D$ is a set of decision attributes. If for $Q \subseteq P$, $POS_Q(D) = POS_P(D)$ and $POS_{Q\setminus\{a\}}(D) \neq POS_Q(D), \forall a \in Q$ , then $Q$ is called positive region preserved reduct.*

## 5.3 Algorithm for reduct computation

In this section, we present a suitable algorithm based on our proposed approach to calculate the reduct as follows:

**Step 1:** Take an $IF$ information system $(U, P \cup D)$.

**Step 2:** Find similarity between each objects using $IF$ relation as defined in Eq.(5.1).

**Step 3:** Input the set $X \subseteq U$ to be approximated.

**Step 4:** Choose parameters $\alpha, \beta \in (0, 1]$.

**Step 5:** Calculate lower approximation of $X$ using definition.

**Step 6:** Calculate positive region using and find the positive region based reduct set.

## 5.4 *Illustrative Example*

As a case study, we consider an $IF$ information system from [59] as given in table 5.1. Now, we calculate positive region based reduct set as follows:
Firstly, we calculate $IF$ relation of two objects for $P = \{A_1, A_2, A_3, A_4, A_5\}$ using Eq.(5.1) as mentioned in table 5.2.

Now, we can find decision classes using table 5.1 as $\{\{U_1, U_2, U_4, U_5\}, \{U_3, U_6, U_7\}\}$
Taking $\alpha = 0.98, \beta = 0.02$, we calculate positive region as follows:

*Table 5.1:* Intuitionistic Fuzzy Information System

| Instances / Attributes | $U_1$ | $U_2$ | $U_3$ | $U_4$ | $U_5$ | $U_6$ | $U_7$ |
|---|---|---|---|---|---|---|---|
| $A_1$ | ⟨0.6,0.3⟩ | ⟨0.7,0.3⟩ | ⟨0.7,0.3⟩ | ⟨0.8,0.2⟩ | ⟨0.7,0.3⟩ | ⟨0.6,0.2⟩ | ⟨0.6,0.3⟩ |
| $A_2$ | ⟨0.8,0.1⟩ | ⟨0.8,0.2⟩ | ⟨0.5,0.2⟩ | ⟨0.8,0.2⟩ | ⟨0.6,0.2⟩ | ⟨0.5,0.2⟩ | ⟨0.7,0.1⟩ |
| $A_3$ | ⟨0.7,0.3⟩ | ⟨0.6,0.3⟩ | ⟨0.6,0.3⟩ | ⟨0.7,0.3⟩ | ⟨0.7,0.3⟩ | ⟨0.6,0.3⟩ | ⟨0.5,0.3⟩ |
| $A_4$ | ⟨0.9,0.1⟩ | ⟨0.7,0.3⟩ | ⟨0.6,0.3⟩ | ⟨0.8,0.2⟩ | ⟨0.6,0.3⟩ | ⟨0.7,0.3⟩ | ⟨0.7,0.1⟩ |
| $A_5$ | ⟨0.8,0.2⟩ | ⟨0.8,0.2⟩ | ⟨0.8,0.2⟩ | ⟨0.8,0.1⟩ | ⟨0.8,0.2⟩ | ⟨0.6,0.2⟩ | ⟨0.6,0.2⟩ |
| D | 1 | 1 | 2 | 1 | 1 | 2 | 2 |

*Table 5.2:* Intuitionistic Fuzzy Relation

| Instances \ Instances | $U_1$ | $U_2$ | $U_3$ | $U_4$ | $U_5$ | $U_6$ | $U_7$ |
|---|---|---|---|---|---|---|---|
| $U_1$ | $\langle 1.0,0.0\rangle$ | $\langle 0.92,0.06\rangle$ | $\langle 1.0,0.0\rangle$ | $\langle 0.88,0.06\rangle$ | $\langle 0.88,0.06\rangle$ | $\langle 0.84,0.06\rangle$ | $\langle 0.86,0.0\rangle$ |
| $U_2$ | $\langle 0.92,0.06\rangle$ | $\langle 1.0,0.0\rangle$ | $\langle 0.92,0.0\rangle$ | $\langle 0.94,0.06\rangle$ | $\langle 0.92,0.0\rangle$ | $\langle 0.88,0.04\rangle$ | $\langle 0.90,0.06\rangle$ |
| $U_3$ | $\langle 0.84,0.06\rangle$ | $\langle 0.92,0.0\rangle$ | $\langle 1.0,0.0\rangle$ | $\langle 0.86,0.06\rangle$ | $\langle 0.96,0.0\rangle$ | $\langle 0.92,0.04\rangle$ | $\langle 0.86,0.06\rangle$ |
| $U_4$ | $\langle 1.0,0.0\rangle$ | $\langle 0.94,0.06\rangle$ | $\langle 0.86,0.06\rangle$ | $\langle 1.0,0.0\rangle$ | $\langle 0.90,0.06\rangle$ | $\langle 0.82,0.06\rangle$ | $\langle 0.84,0.08\rangle$ |
| $U_5$ | $\langle 0.88,0.06\rangle$ | $\langle 0.92,0.0\rangle$ | $\langle 0.96,0.0\rangle$ | $\langle 0.90,0.06\rangle$ | $\langle 1.0,0.0\rangle$ | $\langle 0.88,0.04\rangle$ | $\langle 0.86,0.06\rangle$ |
| $U_6$ | $\langle 0.84,0.06\rangle$ | $\langle 0.88,0.04\rangle$ | $\langle 0.92,0.04\rangle$ | $\langle 0.82,0.06\rangle$ | $\langle 0.88,0.04\rangle$ | $\langle 1.0,0.0\rangle$ | $\langle 0.94,0.06\rangle$ |
| $U_7$ | $\langle 0.86,0.06\rangle$ | $\langle 0.86,0.06\rangle$ | $\langle 0.86,0.06\rangle$ | $\langle 0.86,0.06\rangle$ | $\langle 0.86,0.06\rangle$ | $\langle 0.86,0.06\rangle$ | $\langle 0.86,0.06\rangle$ |

For $X = \{U_1, U_2, U_4, U_5\}, X^C = \{U_3, U_6, U_7\}$

$\underline{R}_{\alpha,\beta}(X) = \{U_4\}$

For $X = \{U_3, U_6, U_7\}, X^C = \{U_1, U_2, U_4, U_5\}$

$\underline{R}_{\alpha,\beta}(X) = \{U_6\}$

$POS_P(D) = \{U_4\} \cup \{U_6\} = \{U_4, U_6\}$

Now, removing attributes one by one from the set $P = \{A_1, A_2, A_3, A_4, A_5\}$, we calculate other positive regions as follows:

For $C = \{A_2, A_3, A_4, A_5\}, POS_C(D) = \{U_4, U_6\}$

For $C = \{A_1, A_3, A_4, A_5\}, POS_C(D) = \{U_4, U_6\}$

For $C = \{A_1, A_2, A_4, A_5\}, POS_C(D) = \{U_4, U_6\}$

For $C = \{A_1, A_2, A_3, A_5\}, POS_C(D) = \{U_4, U_6\}$

For $C = \{A_1, A_2, A_3, A_4\}, POS_C(D) = \{U_4, U_6\}$

Since $POS_{P\setminus\{a\}}(D) = POS_P(D), \forall a \in P$

Hence $P$ cannot be a reduct set.

For $E = \{A_1, A_2, A_3\}, POS_E(D) = \{U_4, U_6\}$

For $E = \{A_2, A_4, A_5\}, POS_E(D) = \{U_4, U_6\}$

For $Q = \{A_1, A_4, A_5\}, POS_Q(D) = \{U_4, U_6\}$

Since $POS_{C\setminus\{a\}}(D) = POS_C(D), \forall a \in C$

Hence $C$ cannot be a reduct set.

For $F = \{A_1, A_4\}, POS_E(D) = \{U_4\}$

For $F = \{A_1, A_5\}, POS_E(D) = \{U_6\}$

For $F = \{A_4, A_5\}, POS_E(D) = \{U_4\}$

Since $POS_Q(D) = \{U_4, U_6\} = POS_P(D)$

And $POS_{Q-\{a\}}(D) \neq POS_Q(D), \forall a \in Q$

Hence, $Q = \{A_1, A_4, A_5\}$ is a reduct set of IF information system as given in table 5.1.

## 5.5 Conclusion

Intuitionistic fuzzy set theory and rough set theory have been proved to be useful mathematical tools to deal with uncertain information. $IF$ set theory can handle uncertainty in much better way when compared to fuzzy set theory as it considers positive, negative and hesitancy degree of an object simultaneously while fuzzy set theory considers only positive degree of an object. $IF$ information systems are an essential type of information systems, which are generalized from fuzzy valued data tables. In this paper, a novel $IF$ rough set model based on $(\alpha, \beta)$-indiscernibility concept was introduced to cope with IF information system. This model was validated by using supporting theorems. Furthermore, a positive region based feature selection technique was proposed by using this model. Moreover, a suitable algorithm was presented to calculate reduct for $IF$ information systems. Finally, we applied our approach on an $IF$ information system and calculate the reduct.

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