Chapter 3

Intuitionistic fuzzy neighborhood rough set model for feature selection

3.1 Introduction

Feature selection techniques can be divided into two categories, firstly, symbolic method and secondly, numerical method. Symbolic methods consider all features as categorical variables and numerical methods take all the features as real valued variables. If there exist any heterogeneous features, symbolic methods(such as Rough set) based feature selection) use a discretization approach and convert them as symbolic features which may lead to some sort of assumption and cause information loss [52]. Discretization may damage two types of structures, firstly neighborhood structure and secondly ordered structure in real space. Latter problem was handled by fuzzy rough set model, but it was not able to tackle the former problem. Very few researches have been proposed to deal with neighborhood structure of the data sets. Dubois and Prade presented fuzzy rough set model [21] by combining rough set and fuzzy set [100]. This model was generalized and successfully implemented by many researchers for feature selection [39, 80, 81, 19, 88, 43] of real-valued data sets but this model was not able to deal with neighborhood structure of data sets. Qinghua et al [29] proposed neighborhood rough set model and it was successfully implemented to deal with neighborhood structure but was not able to preserve real spaces. Wang et al [86] proposed fuzzy neighborhood rough set model and applied it for feature subset selection. We have extended this model and proposed a novel intuitionistic fuzzy neighborhood rough set model and generalized it for feature selection, which can handle both neighborhood structure and real space structure in more efficient manner. Intuitionistic fuzzy set [1] is an extension of Zadeh fuzzy set. Intuitionistic fuzzy sets have its own ability to better narrating and drawing ambiguities of the objective world than the traditional fuzzy sets because it consider the positive, negative and hesitancy degrees of an object simultaneously [2, 3]. Intuitionistic fuzzy sets have been efficiently applied to solve many of the decision problems [56]. In recent years, some of the intuitionistic fuzzy rough set models [8, 13, 78, 90, 105] have been proposed and implemented for feature selection. However, most of the models consider the intuitionistic fuzzy upper and lower approximations of a decision class using a nearest sample, which fails in case of noisy data set. Our proposed model can easily handle this problem as we are considering neighborhood structure of a sample which clearly indicates the uniqueness of our approach.

In this paper, we first define the intuitionistic fuzzy decision of a sample and characterize its intuitionistic fuzzy information granule by introducing parameterized intuitionistic fuzzy relation. The proposed model is the generalization of fuzzy neighborhood rough set concept, which overcomes the drawback of fuzzy rough set approach by introducing a nearest neighbor of a sample to different intuitionistic fuzzy decision classes. We calculate the reduct of an information system by using degree of dependency approach. Furthermore, We present an attribute reduction algorithm for better understanding of our model. Finally, an illustrative example has been given to demonstrate our approach.

3.2 Intuitionistic Fuzzy neighborhood rough set based approach for feature selection

Let $IFDS = \{X, C \cup q, V_{IF}, IF\}$ where X is a non-empty collection of objects, C is collection of conditional attributes and q is a set of decision attributes. We want to approximate intuitionistic fuzzy decision classes with parameterized intuitionistic fuzzy information granules by defining a model for attribute reduction. Let $\{d_1, d_2.......d_k\}$ is a partition of sample space X into k mutually exclusive decision classes by q . Every

$A \subseteq C$ can induce a intuitionistic fuzzy binary relation P_A on X . If P_A is reflexive, symmetric and transitive, then P_A is called intuitionistic fuzzy similarity relation (intuitionistic fuzzy equivalence relation)[15].

(Example 1:) Feng et al [25] defined similarity between two objects with respect to an attribute as follows:

$$P_a(x_i, x_j) = 1 - \sqrt{\alpha(\mu_a(x_i) - \mu_a(x_j))^2 + \beta(\nu_a(x_i) - \nu_a(x_j))^2 + \gamma(\pi_a(x_i) - \pi_a(x_j))^2}$$

where, $\mu_a(x_i), \nu_a(x_i)$ and $\pi_a(x_i)$ are membership, non-membership and hesitancy degrees of an object with respect to attribute $a \in A$ respectively and α, β and γ are weighted factors. In IFDS, the values of these parameters can be selected according to the requirement of different users along with following conditions

(i) $\alpha \geq \beta > \gamma$

(ii) $\alpha + \beta + \gamma = 1$

(iii) $0 \leq \alpha, \beta, \gamma \leq 1$

Now, intuitionistic fuzzy neighborhood of an object $x_i \in X$ is defined as
$[x_i]_A(x_j) = P_A(x_i, x_j), \forall x_j \in X,$
where, $P_A(x_i, x_j) = inf_{a \in A} P_a(x_i, x_j)$

Example 2: Taking $t_1 = 0.4, t_2 = 0.4$, and $t_3 = 0.2$, we get intuitionistic fuzzy similarity relation matrix P_C of IFDS as given in Table 1.7 by:

$$P_C = \begin{bmatrix} 1 & 0.43 & 0.57 & 0.43 & 0.57 & 0.57 \\ 0.43 & 1 & 0.43 & 0.57 & 0.71 & 0.57 \\ o.57 & 0.43 & 1 & 0.71 & 0.57 & 0.57 \\ 0.43 & 0.57 & 0.71 & 1 & 0.43 & 0.43 \\ 0.57 & 0.71 & 0.57 & 0.43 & 1 & 0.71 \\ 0.57 & 0.57 & 0.57 & 0.43 & 0.71 & 1 \end{bmatrix}$$

On the basis of intuitionistic fuzzy neighborhood of an object, intuitionistic fuzzy decision of x_i is defined as follows:

$$q'_m(x_i) = \frac{|[x_i]_C \cap q_m|}{|[x_i]_C|}, m = 1, 2......k. \tag{3.1}$$

Example 3: From Table 1.7, decision classes are:

$X/q = \{\{x_1, x_3, x_6\}, \{x_2, x_4, x_5\}\}$

Then intuitionistic fuzzy decisions of each object can be calculated using Eq. (3.1) and similarity relation matrix P_C as

$q'_1(x_1) = 0.60, q'_1(x_2) = 0.39, q'_1(x_3) = 0.56, q'_1(x_4) = 0.44, q'_1(x_5) = 0.46, q'_1(x_6) = 0.56.$

$q'_2(x_1) = 0.40, q'_2(x_2) = 0.61, q'_2(x_3) = 0.44, q'_2(x_4) = 0.56, q'_2(x_5) = 0.54, q'_2(x_6) = 0.44.$

Now, we construct a parameterized intuitionistic fuzzy information granule related to x_j as follows:

$$[x_i]_A^\alpha(x_j) = \begin{cases} 0, & if\ P_A(x_i, x_j) < \alpha \\ P_C(x_i, x_j), & if\ P_A(x_i, x_j) \geq \alpha \end{cases} \tag{3.2}$$

where, $\alpha \in [0, 1]$ is a parameter which control the size of intuitionistic fuzzy neighborhood. On the basis of intuitionistic fuzzy information granule, the intuitionistic fuzzy neighborhood lower and upper approximations of q'_m over A can be defined by

$$\begin{aligned} \underline{P_A^\alpha}(q'_m) &= \{x_i \in d_m | [x_i]_A^\alpha \subseteq q'_m\} \\ \overline{P_A^\alpha}(q'_m) &= \{x_i \in d_m | [x_i]_A^\alpha \cap q'_m \neq \phi\} \end{aligned} \tag{3.3}$$

Now, lower and upper approximations of q with respect to $A \subseteq C$ can be defined as follows:

$$\begin{aligned} \underline{P_A^{\alpha}}(q) &= \{\underline{P_A^{\alpha}}(q_1^{'}), \underline{P_A^{\alpha}}(q_2^{'}).............\underline{P_A^{\alpha}}(q_k^{'})\} \\ \overline{P_A^{\alpha}}(q) &= \{\overline{P_A^{\alpha}}(q_1^{'}), \overline{P_A^{\alpha}}(q_2^{'}).............\overline{P_A^{\alpha}}(q_k^{'})\} \end{aligned} \tag{3.4}$$

Furthermore, intuitionistic fuzzy positive region and degree of dependency of q upon A can be defined as:

$$pos_A^{\alpha}(q) = \bigcup_{m=1}^{k} \underline{P_A^{\alpha}}(q_m^{'}) \tag{3.5}$$

$$\Gamma_A^{\alpha}(q) = \frac{|pos_A^{\alpha}(q)|}{|X|} \tag{3.6}$$

A is said to be reduct of C if degree of dependency of q over A is same as degree of dependency of q over C and it has no redundant attribute.

Theorem 3.2.1 *Let IFDS=* $\langle X, C, q\rangle$ *, if* $A \subseteq C$ *then* $P_C \subseteq P_A$ *.*

Proof *It is obvious (since* $P_C = \bigcap_{a\in C} P_a \subseteq \bigcap_{a\in A} P_a = P_A$*).*

Theorem 3.2.2 *If* $\alpha_1 \leq \alpha_2$*, then* $\forall x_i \in X, [x_i]_A^{\alpha_2} \subseteq [x_i]_A^{\alpha_1}$*.*

Proof

$$[x_i]_A^{\alpha_2}(x_j) = \begin{cases} 0, & if \;\; P_A(x_i, x_j) < \alpha_2 \\ P_C(x_i, x_j), & if \;\; P_A(x_i, x_j) \geq \alpha_2 \end{cases} \tag{3.7}$$

Since $\alpha_2 \geq \alpha_1$ *, hence, for* $x_j \in X$

$$[x_i]_A^{\alpha_1}(x_j) = \begin{cases} 0, & if\ P_A(x_i,x_j) < \alpha_1 \leq \alpha_2 \\ P_C(x_i,x_j), & if\ P_A(x_i,x_j) \geq \alpha_1 \end{cases} \tag{3.8}$$

So, using Eq.(3.7) and Eq.(3.8), we get, $[x_i]_A^{\alpha_2}(x_j) \leq [x_i]_A^{\alpha_1}(x_j) \Rightarrow [x_i]_A^{\alpha_2} \subseteq [x_i]_A^{\alpha_1}$

Theorem 3.2.3 *If* $A_1 \subseteq A_2 \subseteq C$ *, then* $pos_{A_1}^{\alpha}(q) \subseteq pos_{A_2}^{\alpha}(q)$

Proof *From theorem 3.2.1, if* $A_1 \subseteq A_2$ *, then* $P_{A_2} \subseteq P_{A_1}$ *, which gives* $[x_i]_{A_2}^{\alpha} \subseteq [x_i]_{A_1}^{\alpha}$ *,* $\forall x_i \in X$ *.*

From the definition of lower approximation, it follows that $\underline{P_{A_1}^{\alpha}}(q_m^{'}) \subseteq \underline{P_{A_2}^{\alpha}}(q_m^{'})$ *. Hence,* $pos_{A_1}^{\alpha}(q) \subseteq pos_{A_2}^{\alpha}(q)$

Theorem 3.2.4 *If* $A \subseteq C$ *and* $\alpha_1 \leq \alpha_2$ *, then* $pos_A^{\alpha_1}(q) \subseteq pos_A^{\alpha_2}(q)$ *.*

Proof *Since* $\alpha_1 \leq \alpha_2 \Rightarrow [x_i]_A^{\alpha_2} \subseteq [x_i]_A^{\alpha_1}$ *,* $\forall x_i \in X$*.. From the definition of lower approximation, we have* $\underline{P_A^{\alpha_1}}(q_m^{'}) \subseteq \underline{P_A^{\alpha_2}}(q_m^{'})$*. Therefore,* $pos_A^{\alpha_1}(q) \subseteq pos_A^{\alpha_2}(q)$*.*

Theorem 3.2.5 *If* $A_1 \subseteq A_2 \subseteq C$*, then* $\Gamma_{A_1}^{\alpha}(q) \leq \Gamma_{A_2}^{\alpha}(q)$ *.*

Proof *It is obvious from theorem 3.2.3.*

Theorem 3.2.6 *If* $A \subseteq C$ *and* $\alpha_1 \leq \alpha_2$ *, then* $\Gamma_A^{\alpha_1}(q) \leq \Gamma_A^{\alpha_2}(q)$*.*

Proof *It is obvious from theorem 3.2.4.*

3.3 Algorithm for feature selection based on Intuitionistic fuzzy neighborhood rough set

G, Collection of all conditional attributes;

M, Collection of all decision attributes;

α , the similarity threshold;

$Z \leftarrow \{\}; \Gamma_{best}^{\alpha} = 0;$

do

$L \leftarrow Z$

$\Gamma^{\alpha}_{prev} = \Gamma^{\alpha}_{best}$

$\forall x \in (G \backslash Z)$

if $\Gamma^{\alpha}_{Z\cup\{x\}}(M) > \Gamma^{\alpha}_{L}(M)$

$L \leftarrow Z \cup \{x\}$

$\Gamma^{\alpha}_{best} = \Gamma^{\alpha}_{L}$

$Z \leftarrow L$

until $\Gamma^{\alpha}_{best} == \Gamma^{\alpha}_{prev}$

return Z

3.4 Illustrative Example

In order to explain our approach, we take an arbitrary intuitionistic fuzzy decision system as given in table 1.7. Now, we can calculate reduct set as follows:

Taking , $\alpha = 0.3$, $A = \{a\}$ and using Eq.(3.3), we get,

$\underline{P^{0.3}_{A}}(q_1^{'}) = \phi$ and $\underline{P^{0.3}_{A}}(q_2^{'}) = \{x_2, x_4\}$

from Eq.(3.4), lower approximation of q with respect to $A \subseteq C$ is

$\underline{P^{0.3}_{A}}(q) = \{\phi, x_2, x_4\}$

Therefore, positive region can be given as follows:

$pos^{0.3}_{A}(q) = \underline{P^{0.3}_{A}}(q_1^{'}) \cup \underline{P^{0.3}_{A}}(q_2^{'}) = \{x_2, x_4\}$

Now, degree of dependency of q upon A is:

$\Gamma^{0.3}_{A}(q) = \frac{2}{6}$

Similarly, for $B = \{b\}, C = \{c\}, D = \{d\}, E = \{e\}$, and $F = \{f\}$ degree of dependencies are:

$\Gamma^{0.3}_{B}(q) = \frac{2}{6}, \Gamma^{0.3}_{C}(q) = \frac{2}{6}, \Gamma^{0.3}_{D}(q) = \frac{4}{6}, \Gamma^{0.3}_{E}(q) = \frac{2}{6}, \Gamma^{0.3}_{F}(q) = \frac{3}{6}.$

Since degree of dependency of q upon D is highest, hence D is the reduct candidate. Now, we add other attributes with reduct candidate $\{d\}$ and calculate other degree of dependencies as follows:

$\Gamma^{0.3}_{\{a,d\}}(q) = \frac{5}{6}, \Gamma^{0.3}_{\{b,d\}}(q) = \frac{5}{6}, \Gamma^{0.3}_{\{c,d\}}(q) = \frac{4}{6}, \Gamma^{0.3}_{\{d,e\}}(q) = \frac{5}{6}, \Gamma^{0.3}_{\{d,f\}}(q) = \frac{5}{6}.$

Since, for $\{a,d\}, \{b,d\}, \{d,e\}, \{d,f\}$, degree of dependencies are same and highest, so we can take any one of them as new reduct candidate. Taking $\{a,d\}$ as a reduct candidate,

we iterate the entire process and get other degree of dependencies as follows:
$\Gamma^{0.3}_{\{a,b,d\}}(q) = \frac{6}{6}, \Gamma^{0.3}_{\{a,c,d\}}(q) = \frac{5}{6}, \Gamma^{0.3}_{\{a,d,e\}}(q) = \frac{4}{6}, \Gamma^{0.3}_{\{a,d,f\}}(q) = \frac{5}{6}$.
Since degree of dependency cannot exceed 1, hence reduct of the given decision system is $\{a, b, d\}$.

3.5 Conclusion

In this paper, we introduced a novel intuitionistic fuzzy neighborhood rough set by defining a parameterized intuitionistic fuzzy relation and dependency of conditional features over decision feature. On the basis of intuitionistic fuzzy decision, we defined lower and upper approximations of decision attribute with respect to a subset of conditional attributes. We then defined positive region along with degree of dependency for attribute reduction. Using this model, a greedy attribute reduction algorithm is given. Finally, we applied our approach to an example data set to get a reduct set.

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