PREFACE

In this thesis, we discuss some fixed point problems and some problems closely related to fixed point problems in fuzzy and probabilistic metric spaces. This is not the single context where probabilistic fixed point theory was appeared. In fact, a study of Hans [49] and Spacek [106], on random fixed point theory was initiated in 1950's. The study was vastly grown in the following decades. In a separate endeavour, Sehagal and Barucha-Reid [101] in 1972 studied the Banach's contraction mapping principle in probabilistic metric spaces. The idea behind the construction of this space is the replacement of real metric values with the distribution functions. The concept is primarily attributed to Karl Menger [67]. Several aspects of this space have been developed through the works of several mathematicians, prominent among those are Schweizer, Sklar [99], Wald [112] and many others. In their book, Schweizer and Sklar [99] have comprehensively described many aspects of this space. Our interest lies in the fixed point and associated problems. It is widely recognized that the celebrated result of Banach [9] in 1922 better known as the Banach's contraction mapping principle, is the source of metric fixed point theory. Afterwards several fixed point results are proved for the operators satisfying different types of conditions. Several of these results are either direct generalizations of Banach's result or have been proved by borrowing some ideas from Banach's work. There are also other ideas in metric fixed point theory which are not correlated directly to the above mentioned result. Today this line of study comprises of a vast literature and is considered as an important aspect of functional analysis. Many of these results apart from their theoretical values, have important applications in diverse branches of mathematics and engineering sciences. It is now recognized that fixed point methods are strong methods and tools of applied mathematics.

After the result of Sehgal and Bharucha-Reid [101], as mentioned in the first paragraph, a parallel development has taken place on fixed point theory in probabilistic metric spaces. This development has its own features which are largely due to the inherent flexibilities that the probabilistic metric spaces possess. The theory of $t-$norm has a large involvement in the development of probabilistic metric spaces and, in particular, in the fixed point theory on these spaces. A probabilistic metric space in which a triangular inequality is brought with the help of a $t-$norm is called Menger space. The variations in $t-$norm of the Menger spaces leads to different properties. Fixed point theory is deeply influenced by this feature. Not only the $t-$norms, the flexibility is attributed to many other factors of this notion. As an advantage of this feature, the fixed point theory becomes versatile, sometimes more than its counterpart in ordinary metric spaces. As an instance, we can note that the Banach's contraction mapping principle itself can be probabilistically extended in more than one nonequivalent ways. Apart from the above mentioned Sehgal's contraction, there is an extension of Banach's contraction which is given by Hicks [50] and is very different from the form considered by Sehgal et al. [101]. This contraction is called Hicks contraction or C-contraction which, along with its several modifications and generalizations, have been considered in a good number of papers. A new domain of study is initiated in probabilistic fixed point theory with the introduction of control functions by Choudhury et al. [34]. There was a parallel development in metric spaces initiated by Khan et al. [57] and elaborated through several works [50, 51, 73]. Other types of control functions have been used by several authors like Ciric [26], Jaychimski [53], Fang [39] etc. Particularly, the result of Fang [39] is a culmination of a trend of development in this line.

The probabilistic metric space has been generalized in many directions. These extensions are motivated by the same needs as that of generalizing ordinary metric spaces. Fixed point results in those generalizations have also been developed in recent years. In the sequel to such developments, different probabilistic generalizations of the triangle inequality have been proposed (see [95, 96, 98, 112]) and the study of these triangle inequalities has a center of attraction among researchers working on probabilistic metric spaces. We refer the reader to [99] for further details of development.

In 2006, Mustafa and Sims [70] initiated another class of metric spaces called generalized metric spaces (G-metric spaces). Motivated by Mustafa and Sims [70], recently, Zhou et al. [116] introduced a new probabilistic space called Menger probabilistic G-metric space (Menger PGM-space) which generalizes the Menger PM-space. The development of Menger PGM-space can be seen in [4, 18, 24, 29, 34, 63, 107] in detail.

In this thesis work, we consider only a particular part of this vastly developed branch of probabilistic and fuzzy analysis. Nevertheless, we have tried to focus on problems which have recently emerged and which seek to extend important existing ideas of the fixed point theory. We organize our presentation of the thesis in five different chapters excluding the present one. In the following, we give a brief account of the material included in each of these chapters.

In the first chapter, we give some important mathematical tools which provide the general framework of discussion of the topics considered in this thesis. Some general concepts of feature selection and detailed literature review has also been presented. More specific concepts relevant to particular chapters are described in introduction of the corresponding chapters.

Second chapter of our thesis is intended to some developments in fixed point theory which is an active research area of probabilistic metric spaces. Probabilistic metric spaces are metric structures having uncertainty built within its geometry which has made it into an appropriate context for modeling many real life problems. Theoretical studies on these structures have also appeared in a large way. We define a new contraction mapping in such spaces and show that the contraction has a unique fixed point if such spaces are G-complete with an arbitrary choice of a continuous t-norm. With a minimum t-norm, the result is further extended in any complete probabilistic metric space. The contraction is defined with the help of a control function which is different from several other control functions used in probabilistic fixed point theory by other authors. New methodology has been used to prove the results. An illustrative example is given. This work is a part of probabilistic analysis. The entire chapter in form of a paper has been communicated in the Journal Proceeding of Jangjeon Mathematical Society and is under review.

In the third chapter, we shall focus on some fixed point theorems in Menger PGM-space. The concept of the Menger PGM-space has been introduced by Zhou et al. [116] in 2014. We consider this Menger PGM-space introduced by Zhou et el. [116] by introducing a new type of control function. With this introduced control function, we shall discuss some important lemmas to prove our main results. The established result is supported with an example. The entire chapter in form of a paper has been published in the journal Annals of Fuzzy Mathematics and Informatics (14)4, (2017) 393-405.

The fourth chapter is divided in two parts. In the first part, we have given a fixed point theorem for generalized weak contraction mapping in fuzzy metric space while in the second part, we work out a fixed point result for (φ-ψ)-weak contraction in Menger probabilistic metric space. A combination of analytic and order theoretic approach is used to establish the main theorem. The supporting examples are cited to illustrate the obtained results. The entire chapter has been published in the following proceedings:

(i) Springer Proceedings Communications in Computer and Information Science (CCIS) **(2018), 280-285**.

(ii) Advances in Mathematical Methods and High Performance Computing (Springer Nature) **(In press).**

In the fifth chapter, we use the L-convergence criteria to establish a Hicks type contraction mapping theorem in probabilistic metric spaces. A theorem is established by using a control function which is a recent introduction in literature and a generalization of many other such functions. The fixed point obtained in our theorem is unique. The entire chapter has been communicated and is under review in Turkish Journal of Mathematics.

Sixth chapter summarizes the entire thesis and some future scope of the present work.

The thesis ends with a list of references in which we include the references on which our work is based. Several other important publications in this line of research are also given.