# APPENDIX A

**Sample calculation of PID controller parameters using DS based PID controller design**.

Assuming a second order time delay transfer function with inverse response characteristics

$$G_p = \frac{(-0.5s+1)e^{-1s}}{(1s+1)(2s+1)} = \frac{(-0.5s+1)e^{-1s}}{2s^2+3s+1} \qquad \text{(A1)}$$

The above process transfer function can be written into following form $\frac{(fs+g)e^{-\theta s}}{as^2+bs+1}$.

Therefore, $a = 2, b = 3, c = 1, f = -0.5, g = 1 \; and \; k_p = 1, \; \theta = 1$ (A2)

The controller parameters $k_c, \tau_I \; and \; \tau_D$ were estimated by the following experession taken from Eq. 3.36.

$$k_c = \varphi'(0), \quad \tau_I = \frac{k_c}{\varphi(0)} \quad \text{and} \quad \tau_D = \frac{\varphi''(0)}{2k_c} \qquad \text{(A3)}$$

$$\varphi(0) = \frac{N}{D}, \; \varphi'(0) = \frac{N_1 * D - D_1 * N}{D^2} \quad \text{and} \quad \varphi''(0) = \frac{D(N_2 * D - D_2 * N) - 2D_1(N_1 * D - D_1 * N)}{D^3} \qquad \text{(A4)}$$

Where, $N = c$; $D = k_p g(2\lambda + \theta)$; $N_1 = b$; $D_1 = k_p\left[g\left(\lambda^2 - \frac{\theta^2}{2}\right) + f(2\lambda + \theta)\right]$; $N_2 = 2a$; $D_2 = k_p f\left(\lambda^2 - \frac{\theta^2}{2}\right)$ (A5)

The tuning parameter $\lambda = 1.3$ was selected. Therefore, the parameters N, D, $N_1$, $D_1$, $N_2$, and $D_2$ can be calculated from Eq. A2 and A5 as follows:

$N = 1$; $D = 1 \times 1 \times (2 \times 1.3 + 1) = 3.6$; $N_1 = 3$; $D_1 = 1 \times \left[1\left(1.3^2 - \frac{1^2}{2}\right) - 0.5(2 \times 1.3 + 1)\right] = -0.61$; $N_2 = 2 \times 2 = 4$; $D_2 = 1 \times (-0.5)\left(1.3^2 - \frac{1^2}{2}\right) = -0.595$ (A6)

Now, by substituting the parameter of $N, D, N_1, D_1, N_2$, *and* $D_2$ of Eq. (A6) into (A4), the parameters $\varphi(0)$, $\varphi'(0)$, and $\varphi''(0)$ were calculated as follows:

$$\varphi(0) = \frac{1}{3.6} = 0.2778;\ \ \varphi'(0) = \frac{3 \times 3.6 + 0.61 \times 1}{3.6^2} = 0.8804;$$

$$\varphi''(0) = \frac{3.6(4 \times 3.6 + 0.595 \times 1) + 2 \times 0.61(3 \times 3.6 + 0.61 \times 1)}{3.6^3} = 1.4554$$

Therefore, from Eq. (A3)

$$k_c = 0.8804, \quad \tau_I = \frac{0.8804}{0.2777} = 3.1694 \quad \text{and} \quad \tau_D = \frac{\varphi''(0)}{2k_c} = \frac{1.4554}{2 \times 0.8804} = 0.8265$$

# APPENDIX B

### Sample calculation of PID controller parameters using IMC based PID controller design

The transfer function was taken as in form of $G_p = \frac{K_p(1-ps)e^{-\theta s}}{a_1 s^2 + a_2 s + 1}$ for the designing of IMC-PID controller in the present work.

The IMC filter was selected as $f(s) = \frac{\beta_2 s^2 + \beta_1 s + 1}{(\lambda s + 1)^4}$

$h = (4\lambda - \beta_1 + \theta + p)$; $\quad x_1 = (\lambda^4 - p\theta\beta_2)/h$; $\quad x_2 = (4\lambda^3 - p\theta\beta_1 + \theta\beta_2 + p\beta_2)/h$ $x_3 = (6\lambda^2 - \beta_2 - \theta p + p\beta_1 + \theta\beta_1)/h$; (B1)

$$x_1 s^3 + x_2 s^2 + x_3 s + 1 = (\gamma s + 1)(a_1 s^2 + a_2 s + 1) \quad \text{(B2)}$$

$$x_1 = \gamma a_1; \quad x_2 = \gamma a_2 + a_1; \quad x_3 = \gamma + a_2 \quad \text{(B3)}$$

The coefficients $\beta_1$, $\beta_2$ and $\gamma$ were calculated by solving Eq. (B1) and Eq. (B3), and given by Eq (B4).

$\beta_1 = \frac{y_4 z_1 - y_2 z_2}{y_1 y_4 - y_2 y_3};$ $\beta_2 = \frac{\beta_1 y_1 - z_1}{y_2}$ $\gamma = \frac{x_1}{a_1}$ (B4)

where,

$y_1 = a_1 p\theta - a_1{}^2; y_2 = a_2 p\theta + a_1(p+\theta); y_3 = a_1(p+\theta) + a_1 a_2; y_4 = a_1 - p\theta$

$z_1 = 4a_1\lambda^3 - a_2\lambda^4 - a_1{}^2(4\lambda + \theta + p);$ (B5.1)

$z_2 = \lambda^4 + a_1 a_2(4\lambda + \theta + p) - a_1(6\lambda^2 - p\theta)$ (B5.2)

Using the above steps the feedback controller $G_c(s)$ was simplified to a standard form of PID and given by Eq. (B6) similar approach was given by to simplify the controller into PID form.

$$G_c(s) = K_c\left(1 + \frac{1}{\tau_I s} + \tau_D s\right)\frac{1}{\alpha s + 1} \quad \text{(B6)}$$

Where, $K_c = \frac{\beta_1}{K_p h}$, $\tau_I = \beta_1$, and $\tau_D = \frac{\beta_2}{\beta_1}$ and $\alpha = \gamma = \frac{x_1}{a_1}$ and λ is the tuning parameter.

Example: assuming the transfer function $G_p = 100(-0.2s+1)e^{-0.2s}/(100s-1)(s-1)$.

Therefore, $k_p = 100,\ p = 0.2, \theta = 0.2, a_1 = 100,\ and\ a_2 = -101$ and $\lambda = 0.9$

Substituting these values into Eq. (B5), $y_1, y_2, y_3, y_4, z_1$, and $z_2$ can be calculated as follows:

$y_1 = 100 \times 0.2 \times 0.2 - 100^2 = -9996,$

$y_2 = -101 \times 0.2 \times 0.2 + 100(0.2 + 0.2) = 35.96,$

$y_3 = 100(0.2 + 0.2) + 100 \times (-101) = -10060,$ $y_4 = 100 - 0.2 \times 0.2 = 99.96,$

$z_1 = 4 \times 100 \times 0.9^3 - (-101) \times 0.9^4 - 100^2(4 \times 0.9 + 0.2 + 0.2) = -3.9642 \times 10^4$

$z_2 = 0.9^4 + 100(-101)(4 \times 0.9 + 0.2 + 0.2) - 100(6 \times 0.9^2 - \ 0.2 \times 0.2) = -4.088 \times 10^4$

Substituting the value of these values into Eq. (B4), then $\beta_1 = 3.91$, $\beta_2 = 15.4524, x_1 = 0.4232$, $h = 0.0898$ and $\alpha = \gamma = 0.0042$ are obtained.

Therefore, $K_c = \frac{\beta_1}{K_p h} = \frac{3.91}{100 \times 0.0898} = 0.4355$ , $\tau_I = \beta_1 = 3.91$, and $\tau_D = \frac{\beta_2}{\beta_1} = \frac{15.4524}{3.91} = 3.952$

The required PID controller with lag filter is $G_c(s) = 0.4355\left(1 + \frac{1}{3.91s} + 3.952s\right)\frac{1}{0.0042s+1}$