Chapter 2

Optimal Placement of PMUs in Power System Network for Voltage Stability Estimation under Contingencies

2.1 Introduction

With the advancement in phasor technology, researchers have paid significant attention to utilize it in power system stability monitoring and control. Phasor measurements by Phasor Measurement Units (PMUs) can be utilized to resolve various issues pertaining to power system network [81]. The applications of PMU technology in monitoring, control and protection of power systems is well documented [33].

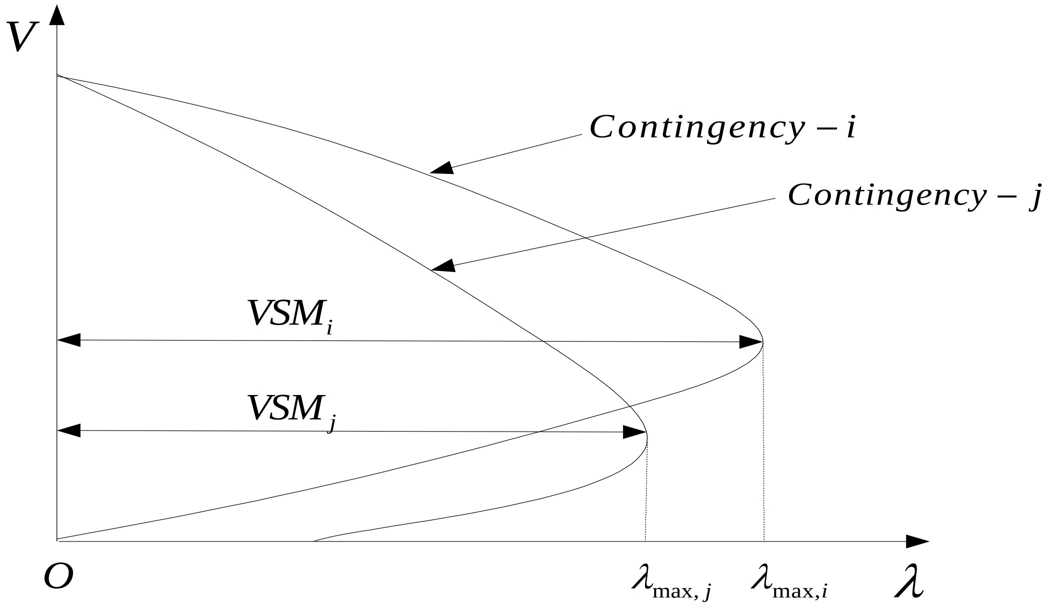
The cost and other associated restrictions prevent most of the utilities to deploy PMUs at all the buses of the network. Therefore, optimal location for the placement of PMUs may be decided considering various aspects such as network observability, vulnerability, transient stability and voltage stability. Network observability based approach may either utilize numerical observability or topological observability [82]. Methods based on numerical observability utilize information (or gain) matrix measurement jacobian that reflects the system configuration and measurement set. Some of the methods considering numerical observability are methods based on simulated annealing [83], Tabu search [84] and Genetic Algorithm [85]. Numerical observability based PMUs placement considering dynamic vulnerability [86, 87], transient stability [88], coherency [89] and state estimation [90, 91], have been considered. A semi-definite programming approach based on numerical observability considering the existence of conventional measurements, zero-injections and impact of PMU channel limit has been considered for the optimal placement of PMUs [92].

Because of huge matrix manipulations, numerical observability based methods are computationally expensive and the measurement matrix may become ill-conditioned in case of large power system networks. Topological observability based methods consider graph theoretic aspects of power networks. Methods based on topological observability include Depth First search method [93], spanning tree method [16] and integer linear programming method [94]. Placement of PMUs considering topological aspects alone may not be suitable to monitor the health of the system. Therefore, system aspects should also be considered in addition to topological aspects for optimal PMUs placement. System aspects should consider criterion such as vulnerability, transient stability, voltage stability and impact of critical contingencies. Optimal placement of PMUs considering topological observability under voltage stability based critical contingencies has been considered [95]. However, effectiveness of the proposed PMUs placement approach has not been established in voltage stability margin estimation under contingencies. A binary integer linear programming based approach considering critical buses based on voltage level, connectivity of the bus to other buses, relevance of the bus to transient and voltage stability has been considered [96]. However, impact of critical contingencies has not been considered in planning optimal location for PMUs placement. Optimal placement of PMUs using binary integer linear programming considering vulnerability of generator and load buses under disturbances has been proposed [17]. However, vulnerable load buses have been found based on Thevenin's model of power system which may not be valid near nose point. Optimal placement of PMUs using binary imperialistic competition algorithm considering impact of contingencies has been proposed [97]. Optimization of number of substations for the placement of PMUs incorporating the transformer tapping and redundancy in the measurement of critical elements has been proposed [98]. Methodologies proposed in [97] and [98] for the placement of Phasor Measurement Units does not consider vulnerability of buses to stability. A method for increasing the observability of interconnected power system network with the help of component reliability for incremental PMUs placement has been suggested [99]. In this an overall system reliability index has been used to determine most suitable location of PMU. However, PMUs placement considering voltage instability effects has not been considered.

In this chapter, optimal placement of PMUs has been considered considering the effect of voltage stability based critical contingencies. Optimal PMU locations have been decided based on binary integer linear programming run for the system intact case and critical contingency cases. In addition to network observability, proposed PMUs placement algorithm is able to effectively estimate voltage stability margin under critical contingencies at different load patterns. Case studies have been carried out on a standard IEEE 14-bus system, New England 39-bus system and a practical 246-bus North Region Power Grid (NRPG) system of India.

2.2 Voltage Stability Based Contingency Ranking

Placement of PMUs should also consider impact of contingencies as an intact case observable network may become unobservable under contingencies. In this work, PMUs have been placed considering impact of voltage stability based critical contingencies. PMUs placement being an offline strategy, it is important to accurately select critical contingencies though the contingency ranking method may be computationally involved. Therefore, in this work, critical contingencies have been identified based on voltage stability margin (distance between the base case operating point and maximum loadability point). Contingency with lower voltage stability margin has been considered more severe as illustrated in Figure 2.1, where contingency-*j* is considered more severe compared to contingency-*i*.



VSM = Voltage Stability Margin

Figure 2.1: Contingency ranking based on voltage stability margin

Critical contingencies may change with the change in operating conditions of the system. In order to see the impact of change in operating conditions in identification of critical contingencies, voltage stability margin has been estimated under contingencies considering wide patterns of load increase by varying real and reactive power demands at buses as per following:

$$P_{D_i} = P_{D_{ib}}(1+\lambda) \qquad (2.1)$$

$$Q_{D_i} = kQ_{D_{ib}}(1+\lambda) \qquad (2.2)$$

where,

P_{D_i} = Real power demand at bus *i*

Q_{D_i} = Reactive power demand at bus *i*

$P_{D_{ib}}$ =Real power demand at bus *i* at the base case operating point

$Q_{D_{ib}}$ =Reactive power demand at bus *i* at the base case operating point

λ = System loading factor common to all the buses

k = A multiplier used to change Q_{Di}/P_{Di} ratio to consider different patterns of load increase

In order to consider wide variations in Q_{Di}/P_{Di} ratio, the multiplier k has been taken as 0.2, 0.5, 1.0 and 1.2, respectively, in the present work. Nose curves (λ-V curves) were plotted for all the single line outage cases for the values of k considered, and voltage stability margin (the distance between the base case operating point and maximum loadability point) were obtained under contingencies for k = 0.2, 0.5, 1.0 and 1.2, respectively. Critical contingencies were identified based on lowest voltage stability margin among margins computed for different values of k considered in this work.

2.3 Optimal Placement of PMUs

Topological observability based method use the graph theoretic approach to find the locations for PMUs placement. For the system using PMUs, the following observability rules are applied to make the system observable:

1. If voltage and current phasor at one end of a branch are known, voltage phasor at the other end of the branch can be calculated using Ohm's law.
2. If voltage phasors at both the ends of a branch are known, branch current can be calculated.
3. If there exists a zero-injection bus with all branch currents known except for one, the unknown branch current can be calculated using Kirchhoff's Current Law (KCL).

The voltage and current phasors obtained by above rules are known as pseudo measurements. Pseudo measurements help in minimizing number of PMUs to be installed to make the network fully observable. Network observability may be measured in terms of Bus Observability Index and Total Observability Index defined as per following:

Bus Observability Index: It is defined as the number of buses connected to a particular bus. It gives a measure of the number of PMUs observing a given bus.

Total Observability Index: It is defined as the sum of bus observability index of all the buses. Total Observability Index gives a measure of number of PMUs observing a given power system network. Higher Total Observability Index ensures network observability under loss of few PMUs.

In this work, optimal placement of PMUs was considered using binary integer linear programming as given below:

Let X be a vector consisting of binary decision variable x_i , where x_i is defined as,

$$x_i = \begin{cases} 1 & \text{if PMU is placed at bus } i \\ 0 & \text{otherwise} \end{cases} \tag{2.3}$$

The binary integer linear programming problem is defined as,

$$\text{Min} \sum_{i=1}^{N} C_i x_i \tag{2.4}$$

Subject to constraints:

$$f_j(X) \geq 1 \; ; \; j = 1, \; 2, \ldots\ldots, N \tag{2.5}$$

where, N = number of buses in the system

C_i = cost of PMU placed at bus i

f_j = observability constraint at bus j calculated using PMUs as well as pseudo-measurements

Binary integer linear programming was run for the system intact case and voltage stability based critical contingency cases. Binary integer linear programming run for a critical contingency yields a set of candidate buses for PMUs placement. Set of candidate buses may in general change with change in critical contingencies. Therefore, in this work optimal buses for PMUs placement was decided by combining set of PMU locations obtained for the system intact case and critical contingency cases.

2.4 Voltage Stability Assessment using PMUs

Effectiveness of proposed PMUs placement approach was tested in estimation of voltage stability margin of critical buses under contingencies. The buses which are most susceptible to voltage instability of the system are identified as critical buses. Identification of critical buses is necessary as such buses need fast control action to ensure voltage stability. In the present work, critical buses have been identified based on sensitivity of bus voltage magnitude (V_i) to loading factor (λ) i.e. $dV_i/d\lambda$. The bus having highest negative $dV_i/d\lambda$ value has been considered as the most critical bus. Critical buses change with change in operating scenario. Therefore, sensitivities $dV_i/d\lambda$ have been calculated for all the buses for the system intact case and voltage stability based critical contingency cases under wide pattern of load increase (viz. k = 0.2, 0.5, 1.0 and 1.2, respectively). The sensitivity $dV_i/d\lambda$ has been calculated at a point close to the nose point for all critical outages and load patterns considered. The set of buses having maximum negative $dV_i/d\lambda$ values for the system intact case and voltage stability based critical contingency cases under different patterns of load increase were considered most critical buses for the system. Nose curves were obtained for all critical buses under different operating conditions utilizing results of PMUs measurements and pseudo measurements. In order to study accuracy of proposed PMUs placement algorithm in voltage stability assessment, nose curves were also obtained for these buses under different operating conditions, based on continuation power flows. Nose curves obtained using proposed approach was compared with offline estimation of nose curves based on continuation power flow method [10].

2.5 Case Studies

Proposed approach of PMUs placement for estimation of voltage stability margin was tested on a standard IEEE 14-bus system, New England 39-bus system and a practical 246-bus Northern Regional Power Grid (NRPG) system of India with the help of Power System Analysis Toolbox (PSAT) software. Results obtained on three systems are presented below:

2.5. 1. IEEE 14-Bus System

IEEE 14-bus system consists of 2 generators (at bus numbers 1 and 2), 3 synchronous condensers (at bus numbers 3, 6 and 8) and 20 transmission lines including 3 transformers. This system has a zero-injection bus at bus number 7. The details of IEEE 14-bus system are shown in Appendix-A.

Nose curves (λ-V curves) were obtained for the system intact case and all the single line outage cases for k = 0.2, 0.5, 1.0 and 1.2, respectively with the help of Power System Analysis Toolbox (PSAT) software. Contingency ranking was done based on voltage stability margin (the distance between the base case operating point and nose point) computed under different values of k considered in this work. Five most severe contingencies based on Voltage Stability Margin (VSM) for k = 0.2, 0.5, 1.0 and 1.2, respectively, have been shown in Table 2.1. Table 2.1 also shows three most severe contingencies obtained in [95] based on voltage stability criterion. It is observed from Table 2.1 that three most severe contingencies obtained by proposed approach is independent of load pattern though the relative order of severity changes in case of k = 1.2. However, contingencies with ranking 4 and 5 depend on pattern of load increase. Three most severe contingencies obtained by proposed approach also match with critical contingencies obtained in [95]. Considering wide pattern of load increase (viz. k = 0.2, 0.5, 1.0 and 1.2, respectively), line outage 1-2, 2-3, 5-6, 2-4, 7-9, 2-5 and 6-13 were considered as most critical contingencies based on voltage stability criterion.

Table 2.1: Critical contingencies for IEEE 14-bus system

R	**Critical contingencies for different values of *k*, and in [95]**								
	k = 0.2		k = 0.5		k = 1.0		k = 1.2		[95]
	L.O.	VSM	L.O.	VSM	L.O.	VSM	L.O.	VSM	L.O.
1	1-2	1.343	1-2	1.340	1-2	1.335	1-2	1.333	1-2
2	2-3	2.280	2-3	2.270	2-3	2.250	5-6	2.212	2-3
3	5-6	2.551	5-6	2.453	5-6	2.271	2-3	2.242	5-6
4	2-4	3.366	2-4	3.340	7-9	2.865	7-9	2.702	-
5	2-5	3.537	7-9	3.348	6-13	3.203	6-13	3.052	-

R = Ranking, L.O. = Line Outage, VSM = Voltage Stability Margin

Binary integer linear programming was run for the system intact case and voltage stability based critical contingency cases (viz. outage of line 1-2, 2-3, 5-6, 2-4, 7-9, 2-5 and 6-13, respectively). Candidate locations for the placement of PMUs for the system intact case and voltage stability based critical contingency cases were obtained based on results of binary integer linear programming, and have been shown in Table 2.2. Candidate locations for the placement of PMUs combining PMUs locations obtained for all critical contingency cases have also been shown in Table 2.2.

Table 2.2: PMU locations for IEEE 14-bus system

Critical Contingency	**PMUs locations**	**Total Observability Index**
Intact Case	2, 6, 9 Total no. of PMUs required are 3	14
1-2	4, 5, 6, 9 Total no. of PMUs required are 4	19

2-3	4, 5, 6, 9 Total no. of PMUs required are 4	19
5-6	2, 6, 9 Total no. of PMUs required are 3	14
2-4	2, 6, 9 Total no. of PMUs required are 3	13
7-9	4, 5, 6, 9 Total no. of PMUs required are 4	17
2-5	2, 6, 9 Total no. of PMUs required are 3	13
6-13	2, 6, 9 Total no. of PMUs required are 3	13
Combining PMU locations obtained for all critical contingencies	2, 4, 5, 6, 9 Total no. of PMUs required are 5	24

It is observed from Table 2.2 that results of binary integer linear programming yields bus numbers 2, 4, 5, 6 and 9 as locations for the placement of PMUs based on combination of PMUs locations obtained for different critical contingency cases. Therefore, bus numbers 2, 4, 5, 6 and 9 were considered as optimal locations for the placement of phasor measurements units. This requires 5 PMUs to be placed in the system.

Total Observability Index has been calculated for the PMUs locations with system intact case and voltage stability based critical contingency cases. Total Observability Index has also been calculated for the combination of PMUs locations for critical contingency cases. Total Observability Index for different cases have been shown in Table 2.2. It is observed from Table 2.2 that PMU locations obtained combining locations for all critical contingency cases results in a Total Observability Index of 24 which is well above 14 (the total number of buses in the system). This ensures network observability even in case of loss of few PMUs under contingencies.

Set of critical buses were obtained by computing sensitivity of voltage magnitude to loading factor ($dV_i/d\lambda$) for all the load buses at a point close to nose point for the system intact case and voltage stability based critical contingency cases under different patterns of load increase viz. k = 0.2, 0.5, 1.0, and 1.2, respectively. Three most sensitive buses (buses with maximum negative sensitivity value) for the system intact case and voltage stability based critical contingency cases for k = 0.2, 0.5, 1.0, and 1.2, respectively, have been shown in Table 2.3. Based on maximum negative sensitivity value for each of the cases bus numbers 4, 5, 9, 10, 13 and 14 were considered as the most critical buses for the system.

Nose curves (λ-V curves) of critical buses were obtained using combination of PMUs measurements and pseudo measurements for the system intact case and voltage stability based critical contingency cases for k = 0.2, 0.5, 1.0 and 1.2, respectively, with the help of PSAT software. Nose curves of critical buses were also obtained for the considered cases based on offline continuation power flow method using PSAT software. Nose curve of critical bus 4 under line outage 2-3 with k = 1.0, obtained with the help of PMUs and pseudo measurements, and obtained without PMUs measurements with the help of offline continuation power flow method, have been shown in Figure 2.2. Nose curves of critical bus 9 under line outage 7-9 with k = 1.2, obtained with the help of PMUs and pseudo measurements, and obtained without PMUs measurements with the help of offline continuation power flow method, have been shown in Figure 2.3. It is observed from Figure 2.2 and Figure 2.3 that nose curve of critical buses obtained by optimally placed PMU measurements and pseudo measurements match with nose curves estimated by offline continuation power flow method.

Table 2.3: Three most critical buses under contingencies for IEEE 14-bus system

Line Out-age	***k* = 0.2**		***k* = 0.5**		***k* = 1.0**		***k* = 1.2**	
	$dV_i/d\lambda$	**Bus No.**	$dV_i/d\lambda$	**Bus No.**	$dV_i/d\lambda$	**Bus No.**	$dV_i/d\lambda$	**Bus No.**
Intact Case	-1.32	**5**	-1.70	**5**	-8.85	**5**	-1.22	**5**
	-1.15	4	-1.47	4	-7.61	4	-1.06	9
	-0.69	9	-0.97	9	-6.29	9	-1.05	4
1-2	-0.77	**5**	-0.78	**5**	-0.80	**5**	-0.81	**5**
	-0.50	4	-0.51	4	-0.52	4	-0.53	4
	-0.23	9	-0.25	9	-0.27	9	-0.28	9
2-3	-4.94	**4**	-5.98	**4**	-9.14	**4**	-10.32	**4**
	-4.30	5	-5.21	5	-7.96	5	-9.00	5
	-2.25	9	-2.79	9	-4.43	9	-5.09	9
5-6	-0.52	**9**	-0.88	**9**	-0.53	**10**	-0.693	**10**
	-0.51	10	-0.87	10	-0.52	9	-0.688	14
	-0.50	14	-0.85	14	-0.51	14	-0.683	9
2-4	-1.15	**5**	-1.25	**5**	-1.88	**5**	-403	**5**
	-0.98	4	-1.06	4	-1.60	4	-339	4
	-0.49	9	-0.57	9	-0.96	9	-202	9
7-9	-1.70	**9**	-2.05	**9**	-4.32	**9**	-1.18	**9**
	-1.48	10	-1.81	10	-3.85	10	-1.05	10
	-1.30	14	-1.62	14	-3.43	14	-0.95	14
2-5	-5.92	**5**	-2.60	**5**	-1.48	**5**	-1.78	**5**
	-4.99	4	-2.19	4	-1.24	4	-1.48	4
	-2.59	9	-1.22	9	-0.82	9	-1.05	9
6-13	-35	**14**	-10.87	**13**	-3.04	**13**	-0.95	**13**
	-33	13	-10.87	14	-2.93	14	-0.94	14
	-22	5	-5.69	9	-1.56	12	-0.49	12

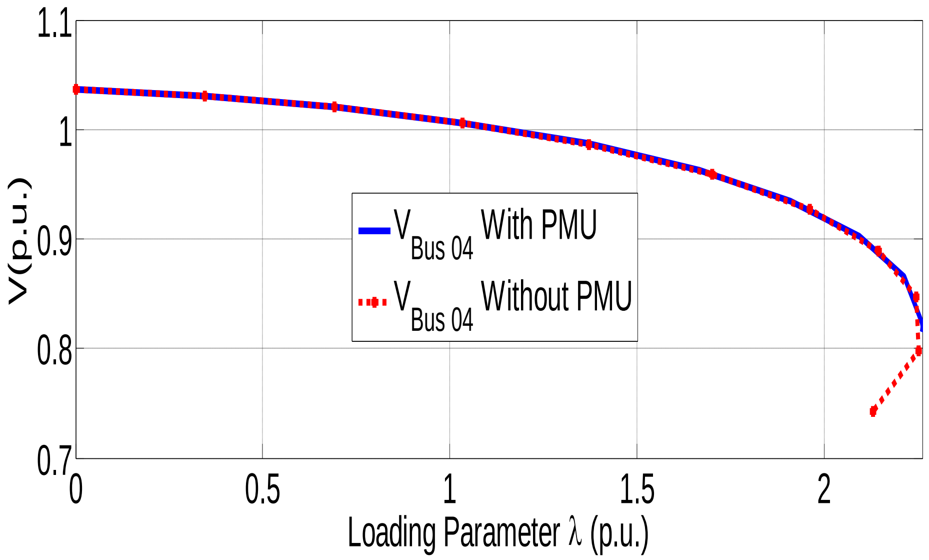


Figure2.2: Nose curve of critical bus 4 under line outage 2-3 with *k* = 1.0 (IEEE 14-Bus system)

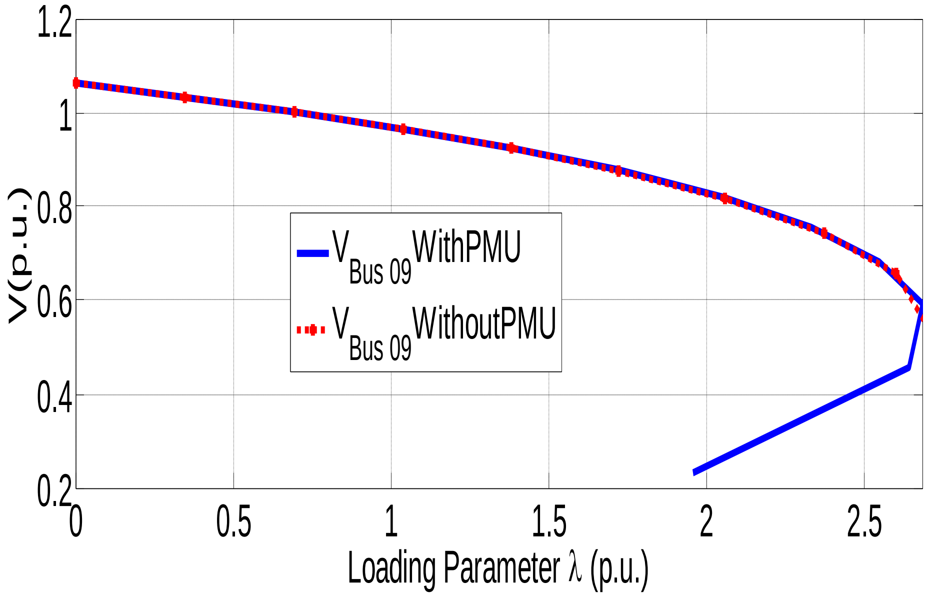


Figure 2.3: Nose curve of critical bus 9 under line outage 7-9 with *k* = 1.2 (IEEE 14-Bus system)

2.5. 2. New England 39-Bus System

The New England 39-bus system has 10 generators and 46 transmission lines/transformers. It has 12 zero-injection buses at bus numbers 1, 2, 5, 6, 9, 10, 11, 13, 14, 17, 19 and 22. The details of New England 39-bus system has been presented in Appendix-B.

Contingency ranking was done based on voltage stability margin (the distance between the base case operating point and nose point) computed under different values of *k* considered in this work. Five most severe contingencies based on Voltage Stability Margin (VSM) for *k* = 0.2, 0.5, 1.0 and 1.2, respectively, have been shown in Table 2.4. Considering wide pattern of load increase (viz. *k* = 0.2, 0.5, 1.0 and 1.2, respectively), line outage 29-38, 22-35, 21-22, 19-33, 10-32, 23-36, 25-37, 20-34, 28-29, 13-14 and 15-16 were considered as most critical contingencies based on voltage stability criterion.

Table 2.4: Critical contingencies for New England 39-bus system

R	Critical contingencies under different values of *k*							
	k = 0.2		*k* = 0.5		*k* = 1.0		*k* = 1.2	
	L.O.	VSM	L.O.	VSM	L.O.	VSM	L.O.	VSM
1	29-38	1.4610	29-38	1.3889	29-38	1.2456	29-38	1.1799
2	22-35	1.6684	22-35	1.5854	10-32	1.4453	10-32	1.3849
3	21-22	1.6689	19-33	1.6046	22-35	1.4503	22-35	1.3993
4	19-33	1.6877	10-32	1.6090	19-33	1.4703	19-33	1.4192
5	10-32	1.7111	21-22	1.6172	21-22	1.5266	15-16	1.4644
6	23-36	1.7791	23-36	1.6854	23-36	1.5376	23-36	1.4816
7	25-37	1.7925	25-37	1.6989	25-37	1.5468	25-37	1.4906
8	20-34	1.8235	20-34	1.7247	20-34	1.5712	21-22	1.4917
9	28-29	1.8592	28-29	1.7841	15-16	1.5796	20-34	1.5122
10	13-14	1.9961	13-14	1.8739	28-29	1.6644	13-14	1.6159

R = Ranking, L.O. = Line Outage, VSM = Voltage Stability Margin

Candidate locations for the placement of PMUs for the system intact case and voltage stability based critical contingency cases were obtained based on results of binary integer linear programming, and have been shown in Table 2.5. Candidate locations for the placement of PMUs combining PMUs locations obtained for all critical contingency cases have also been shown in Table 2.5. It is observed from Table 2.5 that a total of 21 PMUs are to be placed in the system combining locations for each critical contingency. Total Observability Index was calculated with PMU candidate locations for the system intact case, each of the critical contingency cases and for combination of critical contingency cases. It is observed from Table 2.5 that PMU locations obtained combining locations for all critical contingency cases results in a Total Observability Index of 56 which is well above 39 (the total number of buses in the system). This ensures network observability even in case of loss of few PMUs under contingencies.

Table 2.5: PMU locations for New England 39-bus system

Critical Contingency	PMU locations	Total Observability Index
Intact Case	4, 8, 12, 16, 18, 23, 25, 27, 29, 30, 31, 32, 33, 34, 35, 39 Total no. of PMUs required are 16	43
13-14	4, 8, 12, 16, 18, 23, 25, 27, 29, 30, 31, 32, 33, 34, 35, 39 Total no. of PMUs required are 16	41
15-16	3, 8, 12, 15, 21, 23, 25, 27, 29, 30, 31, 32, 33, 34, 35, 39 Total no. of PMUs required are 16	48
20-34	4, 8, 12, 16, 18, 20, 26, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39	50

	Total no. of PMUs required are 17	
21-22	4, 8, 12, 16, 18, 23, 25, 27, 29, 30, 31, 32, 33, 34, 35, 39 Total no. of PMUs required are 16	47
22-35	4, 8, 12, 16, 18, 23, 26, 30, 31, 32, 33, 34, 35, 37, 38, 39 Total no. of PMUs required are 16	45
23-36	4, 8, 12, 16, 18, 24, 26, 30, 31, 32, 33, 34, 35, 36, 37, 38 Total no. of PMUs required are 17	45
25-37	4, 8, 12, 16, 18, 26, 30, 31, 32, 33, 34, 35, 36. 37, 38, 39 Total no. of PMUs required are 16	41
28-29	4, 8, 12, 16, 18, 23, 25, 26, 29, 30, 31, 32, 33, 34, 35, 39 Total no. of PMUs required are 16	47
29-38	4, 8, 12, 16, 18, 26, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39 Total no. of PMUs required are 16	48
Combining PMU locations obtained for all critical contingencies	4, 8, 12, 16, 18, 20, 23, 25, 26, 27, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39 Total no. of PMUs required are 21	56

Set of critical buses were obtained by computing sensitivity of voltage magnitude to loading factor ($dV_i/d\lambda$) for all the load buses at a point close to nose point for the system intact case and voltage stability based critical contingency cases under different patterns of load increase viz. k = 0.2, 0.5, 1.0, and 1.2, respectively. Three most sensitive buses (buses with maximum negative sensitivity value) for the system intact case and voltage stability based critical contingency cases for k = 0.2, 0.5, 1.0, and 1.2, respectively, have been shown in Table 2.6. Based on maximum negative sensitivity value for each of the cases bus numbers 7, 15, 24, 28 and 29 were considered as the most critical buses for the system.

Table 2.6: Three most critical buses under contingencies for New England 39-bus system

L.O.	k = 0.2		k = 0.5		k = 1.0		k = 1.2	
	$dVi/d\lambda$	Bus No.	$dVi/d\lambda$	Bus No.	$dVi/d\lambda$	Bus No.	$dVi/d\lambda$	Bus No.
Intact Case	-4.90	**7**	-1.68	**7**	-1.62	**7**	-3.53	**7**
	-4.52	29	-1.55	29	-1.49	8	-3.21	8
	-0.54	8	-0.20	8	-0.19	28	-0.40	28
10-32	-1.88	**29**	-3.51	**29**	-2.17	**7**	-3.23	**7**
	-1.81	7	-3.38	7	-2.10	29	-3.14	29
	-0.14	28	-0.25	28	-0.16	8	-0.24	8
13-14	-1.76	**29**	-3.41	**7**	-1.19	**7**	-1.49	**7**
	-1.53	7	-2.95	29	-1.03	8	-1.29	8
	-1.53	28	-0.23	28	-0.10	28	-0.12	28
15-16	-1.64	**15**	-1.16	**15**	-5.29	**15**	-2.69	**15**
	-1.62	28	-1.15	29	-5.27	29	-2.68	28
	-0.16	29	-0.12	28	-0.46	28	-0.24	12
19-33	-35.20	**29**	-2.02	**29**	-12.00	**7**	-1.86	**7**
	-31.10	7	-1.80	7	-10.67	29	-1.67	8
	-3.24	28	-0.20	28	-1.15	8	-0.19	29
20-34	-2.87	**29**	-14.26	**7**	-3.25	**7**	-7.64	**7**
	-2.55	7	-12.68	29	-2.91	8	-6.82	8
	-0.27	28	-1.33	28	-0.32	29	-0.74	29
21-22	-0.22	**24**	-0.27	**24**	-22.76	**24**	-6.45	**24**
	-0.21	21	-0.26	21	-22.27	21	-6.28	21
	-0.05	16	-0.06	16	-5.18	16	-1.41	16

22-35	-0.88	**29**	-2.17	**29**	-16.28	**7**	-1.99	**7**
	-0.78	7	-1.94	7	-14.50	29	-1.78	8
	-0.08	28	-0.22	28	-1.61	8	-0.21	29
23-36	-2.07	**29**	-4.99	**29**	-2.28	**7**	-3.85	**7**
	-1.84	7	-4.43	7	-2.04	8	-3.44	8
	-0.20	28	-0.47	28	-0.23	29	-0.38	29
25-37	-1.23	**29**	-2.04	**7**	-41.52	**7**	-1.92	**7**
	-1.09	7	-1.81	29	-36.79	8	-1.72	8
	-0.14	28	-0.24	28	-4.93	29	-0.24	29
28-29	-0.36	**28**	-0.51	**28**	-0.99	**28**	-0.91	**28**
	-0.35	26	-0.48	26	-0.94	26	-0.86	26
	-0.18	27	-0.24	27	-0.45	27	-0.37	27
29-38	-32.80	**29**	-2.55	**29**	-1.67	**29**	-1.51	**29**
	-28.80	7	-2.26	7	-1.52	28	-1.39	28
	-3.76	28	-0.36	28	-0.44	26	-0.49	26

Nose curves (λ-V curves) of critical buses were obtained using combination of PMUs measurements and pseudo measurements for the system intact case and voltage stability based critical contingency cases for k = 0.2, 0.5, 1.0 and 1.2, respectively, with the help of PSAT software. Nose curves of critical buses were also obtained for the considered cases based on offline continuation power flow method using PSAT software. Nose curve of critical bus 24 under line outage 21-22 with k = 1.0, obtained with the help of PMUs and pseudo measurements, and obtained without PMUs measurements with the help of offline continuation power flow method, have been shown in Figure 2.4. Nose curves of critical bus 28 under line outage 19-33 with k = 0.5, obtained with the help of PMUs and pseudo measurements, and obtained without PMUs measurements with the help of offline continuation power flow method, have been shown in Figure 2.5. It is observed from Figure 2.4 and Figure 2.5 that nose curve of critical buses obtained by optimally placed PMU measurements and pseudo measurements match with nose curves estimated by offline continuation power flow method.

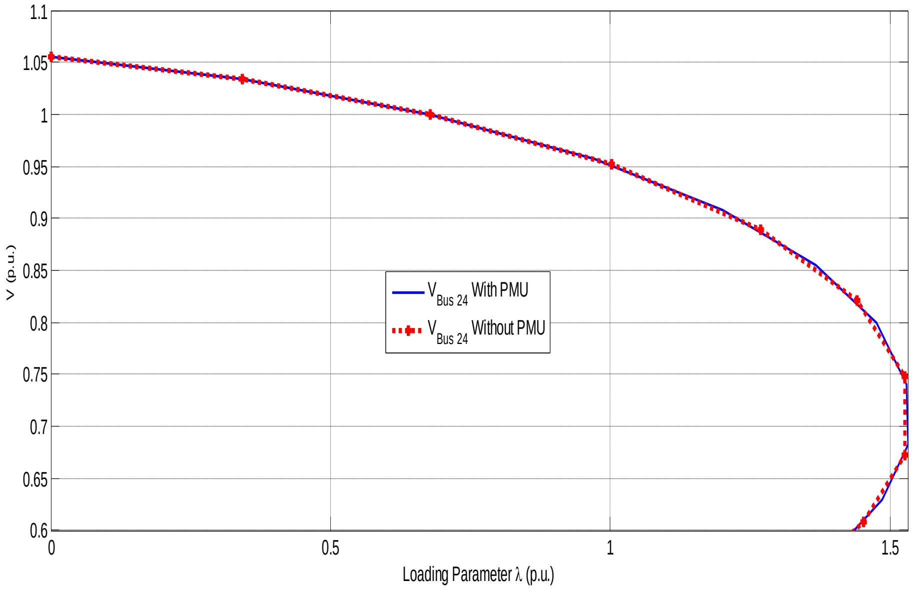


Figure 2.4: Nose curve of critical bus 24 under line outage 21-22 with *k* = 1.0 (New England 39-Bus system)

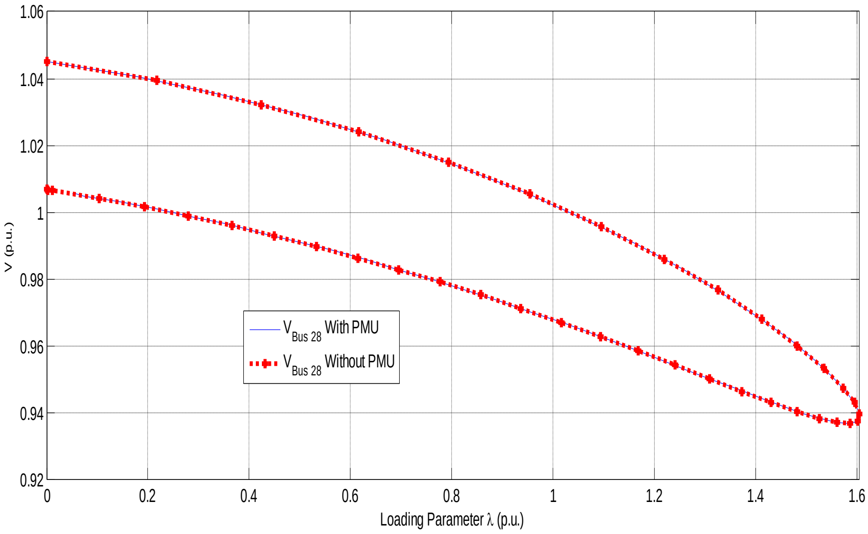


Figure 2.5: Nose curve of critical bus 28 under line outage 19-33 with *k* = 0.5 (New England 39-Bus system)

2.5. 3. NRPG 246-Bus System

The 246-bus Northern Regional Power Grid (NRPG) system is reduced representation of power network of nine states and union territories (Uttar Pradesh, Uttarakhand, Himachal Pradesh, Punjab, Haryana, Rajasthan, Chandigarh, New Delhi and Jammu & Kashmir) of India. It consists of 42 generators at bus numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42 and 376 transmission lines including 36 transformers. It has 15 zero-injection buses with numbers 63, 75, 81, 102, 103, 104, 107, 122, 155, 180, 210, 226, 237, 241, and 244. The details of NRPG 246-bus system have been shown in Appendix-C.

Nose curves (λ-V curves) were obtained for the system intact case and all the single line outage cases for k = 0.2, 0.5, 1.0 and 1.2, respectively with the help of Power System Analysis Toolbox (PSAT) software. Contingency ranking was done based on Voltage Stability Margin (the distance between the base case operating point and nose point) for values of k considered in this work. Ten most severe contingencies based on Voltage Stability Margin (VSM) for k = 0.2, 0.5, 1.0 and 1.2, respectively, have been shown in Table 2.7. Six most critical contingencies obtained in [95] have also been shown in Table 2.7. Considering wide patterns of load increase (viz. k = 0.2, 0.5, 1.0 and 1.2, respectively), line outage 156-158, 40-41, 173-174, 166-173, 181-158, 219-77, 160-164, 168-171, 106-123, 158-160 and 165-171 were considered as most critical contingencies based on voltage stability criterion. It is observed from Table 2.7 that only one critical contingency (line outage 166-173) identified in [95] matches with critical contingencies obtained by proposed approach. This shows that critical contingencies depend on accuracy of contingency ranking methodology, and also on operating conditions of the system. Therefore while planning placement of PMUs which is an offline strategy, due consideration should be given on accuracy of the approach. A set of operating conditions of the network such as change in load patterns should also be considered while finding critical contingencies.

Table 2.7: Critical contingencies for 246-bus NRPG system

R	Critical contingencies under different values of *k*								
	k = 0.2		*k* = 0.5		*k* = 1.0		*k* = 1.2		[95]
	L.O.	VSM	L.O.	VSM	L.O.	VSM	L.O.	VSM	L.O.
1	156-158	0.19	40-41	0.92	156-158	0.34	156-158	0.39	105-245
2	40-41	0.75	173-174	1.25	173-174	1.05	173-174	0.99	75-91
3	166-173	1.10	165-174	1.39	40-41	1.08	165-174	1.09	116-229
4	173-174	1.39	181-158	1.50	165-174	1.16	40-41	1.10	166-173
5	165-174	1.55	219-77	1.51	181-158	1.29	181-158	1.22	188-190
6	219-77	1.63	160-164	1.53	160-164	1.31	160-164	1.23	121-122
7	181-158	1.64	168-171	1.59	168-171	1.36	168-171	1.29	-
8	160-164	1.68	106-123	1.61	219-77	1.37	219-77	1.35	-
9	106-123	1.73	158-160	1.67	165-171	1.47	158-160	1.36	-
10	168-171	1.75	158-34	1.73	106-123	1.48	165-171	1.38	-

R = Ranking, L.O. = Line Outage, VSM = Voltage Stability Margin

Binary integer linear programming was run for the system intact case and voltage stability based critical contingency cases (viz. outage of line 156-158, 40-41, 173-174, 166-173, 181-158, 219-77, 160-164, 168-171, 106-123, 158-160 and 165-171, respectively). Candidate locations for the placement of PMUs for the system intact case and voltage stability based critical contingency cases were obtained based on results of binary integer linear programming. Candidate locations for the placement of PMUs for the system intact case and voltage stability based critical contingency cases have been shown in Table 2.8. Candidate locations for the placement of PMUs combining PMUs locations obtained for all critical contingency cases have also been shown in Table 2.8.

Table 2.8: PMU locations for 246-bus NRPG system

Critical contingency	Optimal PMUs Placement set	Total Observability Index
Intact Case	6, 21, 23, 24, 33, 34, 40, 45, 48, 54, 55, 57, 60, 61, 62, 65, 70, 73, 74, 75, 79, 80, 83, 84, 93, 95, 98, 100, 101, 106, 109, 117, 121, 125, 126, 128, 129, 131, 132, 134, 140, 141, 142, 147, 157, 158, 160, 163, 168, 173, 181, 183, 185, 187, 190, 191, 194, 199, 201, 203, 207, 216, 219, 225, 234, 235, 243, 245 Total no. of PMUs required are 68	311
156-158	6, 7, 23, 34, 40, 42, 45, 48, 54, 55, 57, 60, 61, 62, 65, 68, 70, 73, 75, 78, 80, 83, 93, 94, 95, 98, 100, 101, 106, 117, 121, 125, 126, 128, 129, 131, 132, 134, 140, 141, 142, 153, 157, 160, 167, 168, 169, 174, 181, 183, 185, 187, 190, 191, 194, 199, 201, 202, 203, 206, 217, 219, 234, 235, 239, 243 Total no. of PMUs required are 66	298
40-41	6, 21, 23, 32, 34, 45, 48, 54, 55, 57, 60, 61, 62, 65, 70, 73, 74, 75, 78, 80, 83, 88, 93, 95, 98, 100, 101, 106, 108, 117, 121, 125, 126, 128, 129, 131, 134, 138, 139, 140, 141, 142, 147, 149, 157, 158, 160, 163, 168, 173, 181, 185, 187, 190, 191, 193, 194, 199, 201, 202, 203, 207, 217, 219, 234, 235, 239, 243, 245 Total no. of PMUs required are 69	321
173-174	6, 23, 24, 33, 34, 40, 45, 48, 54, 55, 57, 60, 61, 62, 65, 68, 70, 73, 74, 75, 78, 80, 83, 88, 93, 95, 96, 98, 101, 106, 117, 121, 125, 126, 128, 129, 131, 132, 134, 140, 141, 142, 147, 157, 158, 160, 163, 165, 166, 168, 169, 181, 183, 185, 187, 190, 191, 194, 199, 201, 203, 207, 216,	319

	219, 225, 234, 235, 243, 245 Total no. of PMUs required are 69	
165-174	6, 18, 23, 24, 33, 34, 40, 45, 48, 54, 55, 57, 60, 61, 62, 65, 70, 73, 74, 75, 79, 80, 83, 84, 93, 95, 98, 100, 101, 106, 109, 117, 121, 125, 126, 128, 129, 131, 132, 134, 141, 142, 144, 147, 157, 158, 160, 167, 168, 169, 173, 181, 185, 187, 190, 191, 194, 199, 201, 203, 207, 216, 219, 225, 234, 235, 243, 245 Total no. of PMUs required are 68	309
219-77	6, 8, 21, 23, 24, 34, 40, 45, 48, 54, 55, 57, 60, 61, 62, 65, 70, 73, 74, 75, 79, 80, 83, 84, 93, 95, 98, 100, 101, 106, 109, 117, 121, 125, 126, 128, 129, 132, 134, 140, 141, 142, 147, 157, 158, 160, 163, 168, 173, 181, 183, 185, 187, 190, 191, 194, 199, 201, 202, 203, 207, 216, 217, 219, 234, 235, 243, 245 Total no. of PMUs required are 68	316
181-158	6, 10, 18, 23, 24, 33, 34, 40, 45, 48, 54, 55, 57, 60, 61, 62, 65, 68, 70, 73, 74, 75, 79, 80, 83, 84, 93, 95, 98, 100, 101, 106, 117, 121, 125, 126, 128, 129, 132, 134, 141, 142, 144, 147, 157, 158, 160, 167, 168, 169, 174, 181, 185, 187, 190, 191, 194, 199, 201, 203, 207, 216, 219, 225, 234, 235, 243, 245 Total no. of PMUs required are 68	305
160-164	6, 8, 18, 21, 23, 24, 33, 34, 40, 45, 48, 54, 55, 57, 60, 61, 62, 65, 70, 73, 74, 75, 79, 80, 83, 84, 93, 95, 98, 100, 101, 106, 109, 117, 121, 125, 126, 128, 129, 132, 134, 141, 142, 144, 147, 157, 158, 160, 163, 168, 173, 181, 185, 187, 190, 191, 194, 199, 201, 203, 207, 216, 219, 225, 234, 235, 243, 245 Total no. of PMUs required are 68	312
106-123	6, 14, 23, 24, 33, 34, 40, 45, 48, 54, 55, 57, 60, 61, 62, 65, 70, 73, 74, 75, 79, 80, 83, 84, 95, 98, 100, 101, 106, 109, 116, 119, 121, 125, 126, 129, 131, 132, 134, 140, 141, 142, 147, 157, 158, 160, 163, 168, 170, 173, 181, 183, 185, 187, 190, 191, 194, 199, 201, 203, 207, 216, 219, 225, 234, 235, 243, 245 Total no. of PMUs required are 68	313

168-171	6, 23, 24, 33, 34, 40, 45, 48, 54, 55, 57, 60, 61, 62, 65, 68, 70, 73, 74, 75, 78, 80, 83, 88, 93, 95, 98, 100, 101, 106, 117, 121, 125, 126, 128, 129, 131, 132, 134, 140, 141, 142, 147, 157, 158, 160, 163, 165, 166, 168, 169, 181, 183, 185, 187, 190, 191, 194, 199, 201, 203, 207, 216, 219, 225, 234, 235, 243, 245 Total no. of PMUs required are 69	318
158-160	6, 8, 22, 23, 33, 34, 40, 45, 48, 54, 55, 57, 60, 61, 62, 65, 70, 73, 74, 75, 78, 80, 83, 88, 93, 95, 96, 98, 101, 106, 109, 117, 121, 125, 126, 128, 129, 132, 134, 140, 141, 142, 147, 157, 158, 160, 167, 168, 169, 173, 181, 183, 185, 187, 190, 191, 194, 199, 201, 203, 207, 216, 219, 225, 234, 235, 243, 245 Total no. of PMUs required are 68	309
165-171	6, 10, 21, 23, 24, 33, 34, 40, 45, 48, 54, 55, 57, 60, 61, 62, 65, 68, 70, 73, 74, 75, 79, 80, 83, 84, 93, 95, 98, 100, 101, 106, 117, 121, 125, 126, 128, 129, 132, 134, 140, 141, 142, 147, 157, 158, 160, 163, 168, 173, 181, 183, 185, 187, 190, 191, 194, 199, 201, 203, 207, 216, 219, 225, 234, 235, 243, 245 Total no. of PMUs required are 68	310
Combining PMU locations obtained for all critical contingencies	6, 7, 8, 10, 14, 18, 21, 22, 23, 24, 32, 33, 34, 40, 42, 45, 48, 54, 55, 57, 60, 61, 62, 65, 68, 70, 73, 74, 75, 78, 79, 80, 83, 84, 88, 93, 94, 95, 96, 98, 100, 101, 106, 108, 109, 116, 117, 119, 121, 125, 126, 128, 129, 131, 132, 134, 140, 141, 142, 144, 147, 153, 157, 158, 160, 163, 165, 166, 167, 168, 169, 170, 173, 174, 181, 183, 185, 187, 190, 191, 193, 194, 199, 201, 202, 203, 206, 207, 216, 217, 219, 225, 234, 235, 239, 243, 245 Total no. of PMUs required are 97	419

It is observed from Table 2.8 that results of binary integer linear programming yields 97 PMU locations at bus numbers 6, 7, 8, 10, 14, 18, 21, 22, 23, 24, 32, 33, 34, 40, 42, 45, 48, 54, 55, 57, 60, 61, 62, 65, 68, 70, 73, 74, 75, 78, 79, 80, 83, 84, 88, 93, 94, 95, 96, 98, 100, 101, 106, 108, 109, 116, 117, 119, 121, 125, 126, 128, 129, 131, 132, 134, 140, 141, 142, 144, 147, 153, 157, 158, 160, 163, 165, 166, 167, 168, 169, 170, 173, 174, 181, 183, 185, 187, 190, 191, 193, 194, 199, 201, 202, 203, 206, 207, 216, 217, 219, 225, 234, 235, 239, 243 and 245, based on combination of PMUs locations obtained for different critical contingency cases. Therefore, these buses were considered as optimal locations for the placement of phasor measurements units.

Total Observability Index has been calculated for the PMU locations with system intact case and voltage stability based critical contingency cases. Total Observability Index has also been calculated for the combination of PMUs locations for critical contingency cases. Total Observability Index for different cases have been shown in Table 2.8. It is observed from Table 2.8 that PMU locations obtained combining locations for all critical contingency cases results in a Total Observability Index of 419 which is well above 246 (the total number of buses in the system). This ensures network observability even in case of loss of few PMUs under contingencies.

Set of critical buses were obtained by computing sensitivity of voltage magnitude to loading factor $(dV_i/d\lambda)$ for all the load buses at a point close to nose point for the system intact case and voltage stability based critical contingency cases under different patterns of load increase viz. k = 0.2, 0.5, 1.0, and 1.2, respectively. Table 2.9 shows three most sensitive buses (buses with maximum negative sensitivity value) for the system intact case and voltage stability based critical contingency cases for k = 0.2, 0.5, 1.0 and 1.2, respectively. Based on maximum negative sensitivity value for each of the cases, bus numbers 156, 164, 171, 173 and 174 were considered as the most critical buses for the system.

Table 2.9: Three most critical buses under contingencies for 246-bus NRPG system

L.O.	k = 0.2		k = 0.5		k = 1.0		k = 1.2	
	$dVi/d\lambda$	Bus No.	$dVi/d\lambda$	Bus No.	$dVi/d\lambda$	Bus No.	$dVi/d\lambda$	Bus No.
Intact Case	-3.5	**156**	-194	**156**	-36	**156**	-2.6	**174**
	-3.1	158	-172	158	-31	158	-1.9	173
	-1.0	172	-44	172	-8	174	-1.4	156
156-158	-0.062	**174**	-0.079	**174**	-0.139	**174**	-0.16	**174**
	-0.057	171	-0.071	173	-0.125	171	-0.14	171
	-0.052	173	-0.071	171	-0.114	173	-0.13	173
40-41	-0.088	**156**	-0.131	**174**	-0.213	**174**	-0.25	**174**
	-0.087	174	-0.128	156	-0.205	156	-0.23	156
	-0.078	158	-0.115	171	-0.182	171	-0.21	171
173-174	-18.8	**174**	-5.42	**174**	-32.4	**174**	-8.8	**174**
	-1.5	165	-0.45	165	-2.4	165	-0.7	165
	-1.1	171	-0.41	171	-1.8	171	-0.6	171
166-173	-2.2	**173**	-3.86	**173**	*	*	*	*
	-2.1	174	-3.73	174	*	*	*	*
	-0.2	165	-0.31	165	*	*	*	*
181-158	-3.31	**156**	-31.7	**156**	-9.51	**156**	-35	**156**
	-3.14	158	-29.7	158	-8.78	158	-32	158
	-0.30	160	-2.4	160	-0.71	160	-2	160
219-77	-0.170	**156**	-0.200	**156**	-0.276	**174**	-0.33	**174**
	-0.158	174	-0.196	174	-0.265	156	-0.31	156
	-0.156	158	-0.180	158	-0.235	158	-0.27	158
160-164	-11.5	**164**	-37.6	**164**	-179	**164**	-3.0	**164**
	-9.9	163	-31.5	163	-145	163	-2.5	163

	-3.4	167	-10.4	167	-46	167	-0.9	167
168-171	-8.46	**171**	-7.19	**171**	-40.7	**171**	-44.1	**171**
	-0.75	165	-0.68	174	-3.3	165	-3.4	165
	-0.74	174	-0.63	165	-2.9	174	-3.1	174
106-123	-0.187	**156**	-0.222	**156**	-0.32	**174**	-0.38	**174**
	-0.173	174	-0.218	174	-0.30	156	-0.35	156
	-0.170	158	-0.200	158	-0.27	158	-0.31	158
158-160	-10.8	**156**	-21.4	**156**	-49	**156**	-150	**156**
	-10.2	158	-19.9	158	-45	158	-136	158
	-2.08	172	-3.9	172	-8	172	-25	172
165-171	-27.4	**171**	-10.3	**171**	14	**171**	-17.9	**171**
	-16.1	178	-5.8	168	-7.41	178	-9.47	178
	-15.9	168	-5.7	178	-7.39	168	-9.45	168

*continuation power flow diversed

Nose curves (λ-V curves) of critical buses were obtained using combination of PMUs measurements and pseudo measurements for the system intact case and voltage stability based critical contingency cases for k = 0.2, 0.5, 1.0 and 1.2, respectively, with the help of PSAT software. Nose curves of critical buses were also obtained for the considered cases based on offline continuation power flow method using PSAT software. Nose curve of critical bus 171 under line outage 168-171 with k = 0.5 obtained with the help of PMUs and pseudo measurements, and obtained without PMUs measurements with the help of offline continuation power flow method, have been shown in Figure 2.6. Nose curves of critical bus 164 under line outage 160-164 with k = 0.5 obtained with the help of PMUs and pseudo measurements, and obtained without PMUs measurements with the help of offline continuation power flow method, have been shown in Figure 2.7. Nose curves of critical bus 174 under line outage 166-173 with k = 0.2 obtained with the help of PMUs and pseudo measurements, and obtained without PMUs measurements with the help of offline continuation power flow method have been shown in Figure 2.8. It is observed from Figure 2.6, Figure 2.7 and Figure 2.8 that nose curve of critical buses obtained by optimally placed PMU measurements and pseudo measurements match with nose curves estimated by offline continuation power flow method.

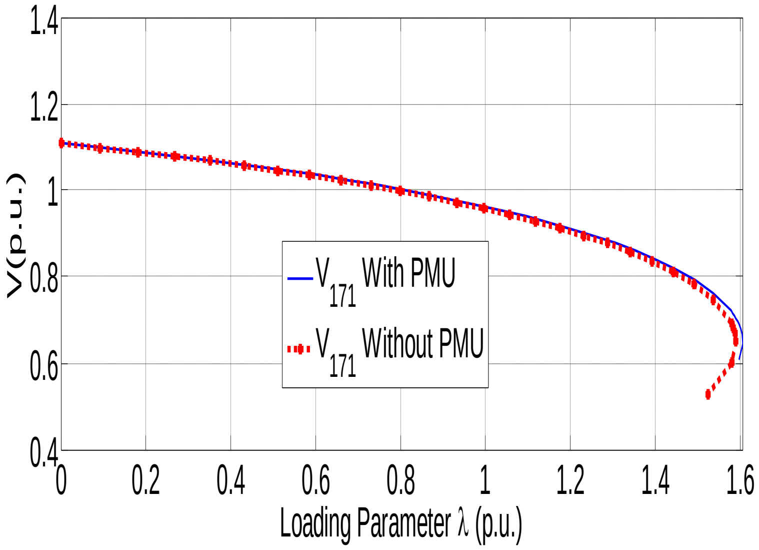


Figure 2.6: Nose curve of critical bus 171 under line outage 168-171 with *k* = 0.5 (NRPG 246-bus system)

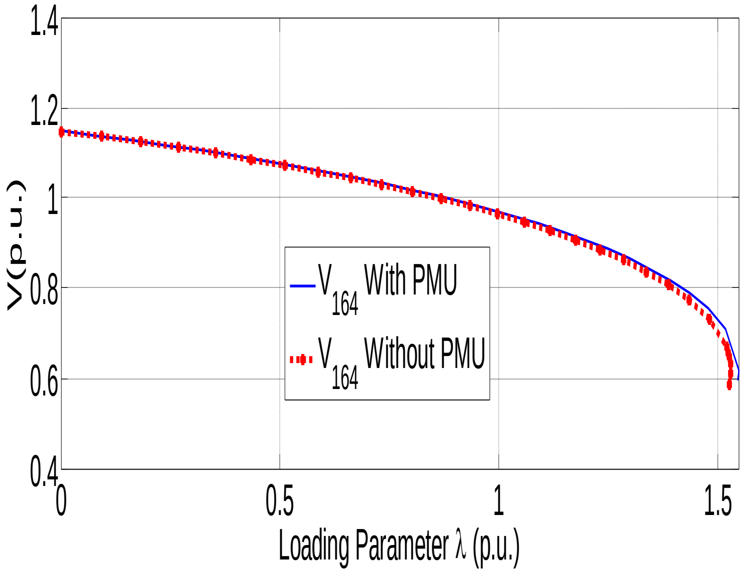


Figure 2.7: Nose curve of critical bus 164 under line outage 160-164 with *k* = 0.5 (NRPG 246-bus system)

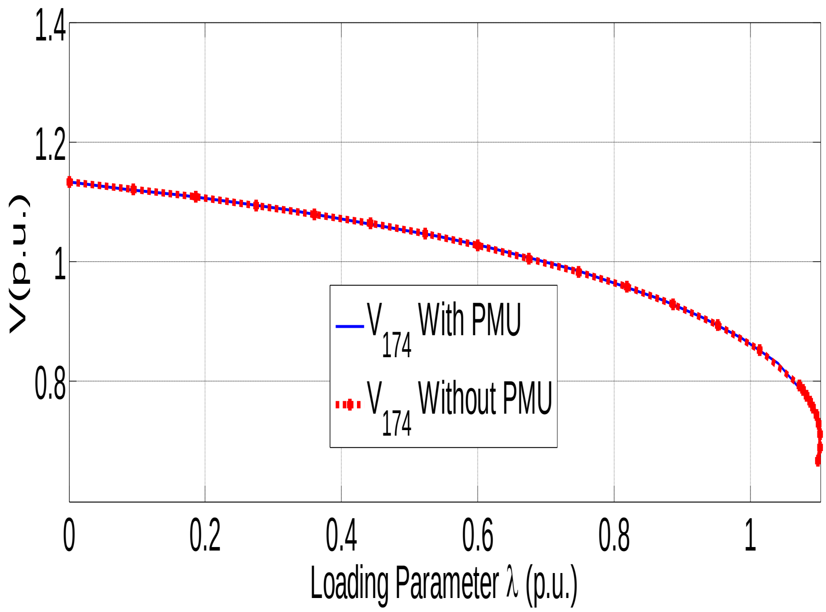


Figure 2.8: Nose curve of critical bus 174 under line outage 166-173 with *k* = 0.2 (NRPG 246-bus system)

2.6 Conclusions

This chapter suggests optimal placement of phasor measurement units to monitor voltage stability of power systems under changing operating conditions. PMUs have been placed based on results of binary integer linear programming run for the system intact case and voltage stability based critical contingency cases. Critical contingencies have been selected based on lowest Voltage Stability Margin (the distance between the base case operating point and the nose point) under various load patterns. Nose curves estimated under different operating conditions by optimally placed PMU measurements closely match with offline continuation power flow based nose curves. Simulations results on IEEE 14-bus system, New England 39-bus system and a practical 246-bus Northern Regional Power Grid (NRPG) system of India validate effectiveness of proposed approach of PMUs placement in assessment of voltage stability margin under contingencies at different load patterns.